Problem Set 1 Answers

Due in lecture on Monday, October 11\textsuperscript{th}.

1. IS-LM. Suppose the economy is described by the following two equations:

\begin{align*}
\text{(1)} & \quad i = \frac{A_0}{b} - \frac{(1-c(1-t))}{b} Y <\text{IS curve}> \\
\text{Where } A_0 & \equiv a_0 - c(TA_0) + IN_0 + GO_0 + NX_0 \\
\text{(2)} & \quad i = \frac{\mu_0}{h} - \left(\frac{1}{h}\right) \left(\frac{M_0}{P_0}\right) + \left(\frac{k}{h}\right) Y <\text{LM curve}>
\end{align*}

1.1 Show equilibrium, graphically. Assume $k/h > 0$, $-(1-c(1-t))/b < 0$.

1.2 Show the impact of an increase in government spending.
1.3 Show the impact of an increase in money supply.

1.4 Show what the central bank has to do in order to keep interest rates constant. What is the size of the increase in output?
2. Modified IS-LM. Suppose the IS curve is as in equation (1), but the LM curve is given by:

\[
(3) \quad i = \mu_0 - \left(\frac{1}{h}\right) \left(\frac{M_0}{P_0}\right) + \left(\frac{j}{h}\right) \left(\frac{M_0}{P_0} + B_0\right) + \left(\frac{k}{h}\right) Y
\]

2.1 Show the impact of an increase in government spending, starting from initial budget balance. Assume \(k/h > 0\).
2.2 Show what happens if the central bank simultaneously increases the money supply.

![Diagram showing LM and IS curves with money supply increase]

2.3 How does your answer to problem 2.1 change if $h = \infty$?

This means the slope of the LM is flat; furthermore, the LM shifts up 0 by any change in bonds, so deficits do not induce upward shifts in the LM curve.

![Diagram showing LM and IS curves with $h = \infty$]

This means the increase in output equals the result that one would obtain from the standard Keynesian model. In other words, when money and bonds are highly substitutable, then there is no transactions or portfolio crowding out, and fiscal policy is quite effective.
3. AD-AS. Suppose aggregate demand and the Phillips Curve are given by:

\[ Y = \hat{a} [A_0 + \left( \frac{b}{h} \right) \left( \frac{M_0}{P} \right) - \left( \frac{b}{h} \right) \mu_0] \tag{4} \]

where \( \hat{a} \equiv \frac{1}{1-c(1-c)+(bk/h)} \)

\[ \pi_i = f \left( \frac{Y_{t-1} - Y^*}{Y^*} \right) + Z_i \tag{5} \]

3.1 Draw the AD-AS diagram, assuming initial long run and short run equilibrium.

3.2 Show what happens if autonomous spending decreases, both initially, and over time.

In the figure below, output falls to \( Y_1 \), and the price levels stays constant to begin with. Then over time, output rises, as the price level falls. In the end, output returns to potential, and prices fall to \( P_{\text{final}} \).
3.3 Show what happens if autonomous spending decreases, but so too does $Y^*$. In this case, output falls to $Y_1$ again, and in period 1, the price level again stays constant. But over time, the price level falls only to $P_{\text{Final}}$ (which is higher than the level in 3.2); in addition, output does not return to its pre-shock level, but to $Y_F$ (which equals the new level of potential output).
3.4 Redo 3.3, assuming the government and the central bank seek to restore income to its initial level.

4. What is the yield to maturity of a $1000 face value discount bond maturing in one year that sells for $900? Show your algebraic work, "boxing-in" your answers.

What is the yield to maturity of a $1000 face value discount bond maturing in one year that sells for $900? Show your algebraic work, boxing-in your answers.

\[ 11.1\% = \frac{1000 - 900}{900} = \frac{100}{900} = 0.111. \]

5. Calculation of real interest rates.
Take the 3 month interest rates for the U.S., the Euro area, and Japan reported in the tables below (drawn from the Oct. 2nd issue of the Economist). Calculate the real interest rate, assuming that the expected (annualized) inflation rate for the next 3 months equals the most recently recorded inflation rate. [For future reference: These data are updated weekly, and are available at: http://www.economist.com, and are also reported in the hard copy version of the magazine.]

The real interest rate is defined thus:

\[ r_t = i_t - \pi_{t+1}^e \]

Where \( r_t (i_t) \) is the real (nominal) interest rate for time t, for one period maturity, and \( \pi_{t+1}^e \) is the expected inflation as of time t, for the period ending at t+1.
So, for the three economies (using the assumption that expected inflation for the next three months equals the actual lagged one year inflation):

US:  \[0.23 - 1.1 = -0.87\]

Euro area:  \[0.89 - 1.6 = -0.71\]

Japan:  \[0.19 - (-0.9) = 1.09\]