Present Value, Future Value, and Bonds

For one period:

\[ FV_t = PV_t + PV_t \times i = PV_t (1 + i) \]

\[ PV_t = \frac{FV_t}{(1 + i)} \]

In general:

\[ FV_{n,t} = PV_t (1 + i)^n \]

\[ PV_t = \frac{FV_{n,t}}{(1 + i)^n} \]

When the interest rate and payments are not constant:

\[ PV_t = \frac{X_{t+1}}{(1 + i_t)} + \frac{X_{t+2}}{(1 + i_t) \times (1 + i_{t+1})} + ..... + \frac{X_{t+n}}{(1 + i_t) \times (1 + i_{t+1}) \times (1 + i_{t+2}) \times (1 + i_{t+n-1})} \]

And when the future is not known:

\[ PV_t = \frac{\sum X_{t} + \sum \sum X_{t}}{(1 + i_t)} + \frac{\sum \sum X_{t}}{(1 + i_t) \times (1 + i_{t+1})} + ..... + \frac{\sum \sum \sum X_{t}}{(1 + i_t) \times (1 + i_{t+1}) \times (1 + i_{t+2}) \times (1 + i_{t+n-1})} \]

Types of bonds:

- Zero-coupon or discount bonds
- Fixed payment loans
- Coupon bonds
- Consols

For one year discount bonds,

\[ 1 + i_t = \frac{\text{FaceValue}_t}{P_{DB,t}} \quad \text{or} \quad i_t = \frac{\text{FaceValue}_t}{P_{DB,t}} - 1 \]

For fixed payment loan, where fixed payments and interest rates are constant:

\[ P_{FPL,t} = \frac{\text{FixedPaym't}}{(1 + i)} + \frac{\text{FixedPaym't}}{(1 + i)^2} + ... + \frac{\text{FixedPaym't}}{(1 + i)^n} \]

For coupon bonds where the coupons and interest rates are constant,

\[ P_{CB,t} = \frac{\text{CouponPaym't}}{(1 + i)} + \frac{\text{CouponPaym't}}{(1 + i)^2} + ... + \frac{\text{CouponPaym't}}{(1 + i)^n} + \frac{\text{FaceValue}}{(1 + i)^n} \]

For consols,

\[ P_{Consol,t} = \frac{\text{CouponPaym't}}{(1 + i)} + \frac{\text{CouponPaym't}}{(1 + i)^2} + ... + \frac{\text{CouponPaym't}}{(1 + i)^n} = \frac{\text{CouponPaym't}}{(i)^n} \]
The value of a bond varies inversely with the interest rate used to calculate the present value of the promised payment.

Types of interest rates:
- Yield to maturity
- Current yield
- Holding period yield

Yield to maturity for coupon bond is the interest rate that solves:

\[
P_{CB,t} = \frac{CouponPaym't}{(1+i)} + \frac{CouponPaym't}{(1+i)^2} + \ldots + \frac{CouponPaym't}{(1+i)^n} + \frac{FaceValue}{(1+i)^n}
\]

Current yield on a coupon bond is:

\[
= \frac{CouponPaym't}{P_{CB,t}}
\]

Holding period yield on a coupon bond held for one period is:

\[
= \frac{CouponPaym't}{P_{CB,t}} + \frac{P_{CB,t+1} - P_{CB,t}}{P_{CB,t}}
\]

Where the first term is the current yield, and the second is the capital gains.

The expected (or ex ante) real interest rate is given by the Fisher equation:

\[
i_t = r_t + \pi^e_{t+1}
\]

\[
r_t = i_t - \pi^e_{t+1}
\]

The ex post real interest rate is given by:

\[
r_t^{expost} = i_t - \pi^e_{t+1}
\]

Determination of interest rates (by way of determination of bond prices)

Figure 6.2 A Shift in the Supply of Bonds

Figure 6.3 A Shift in the Demand for Bonds