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Present Value Calculations and Asset Markets

The Present Value Model

We assume for now Rational Expectations. Hence the expectations operator refers to the conditional mathematical expectations operator. In one-period:

$$P_t = \frac{D_{t+1}}{1+k_e} + \frac{E_t P_{t+1}}{1+k_e} \qquad (1)$$

Where k_e is the required return on equity, $E_t(Z_{t+1}) = E(Z \mid \text{information available at time t+1})$. Assume at time t, that D_t is known.

Then the **Generalized Dividend Valuation Model** is given by

$$P_t = \frac{D_{t+1}}{1+k_e} + \frac{E_t D_{t+2}}{(1+k_e)^2} + \ldots + \frac{E_t D_{t+n}}{(1+k_e)^n} + \frac{E_t P_{t+n}}{(1+k_e)^n} \qquad (2)$$

Note that this expression (2) implies, under certain conditions:

$$P_t = \frac{D_{t+1}}{1+k_e} + \frac{E_t D_{t+2}}{(1+k_e)^2} + \frac{E_t D_{t+3}}{(1+k_e)^3} + \ldots + \frac{E_t D_{t+\infty}}{(1+k_e)^\infty} = \sum_{n=1}^{\infty} \frac{E_t D_{t+n}}{(1+k_e)^n} \qquad (3)$$

Equation (2) rules out "bubbles". The **Gordon Growth Model** assumes that dividends are expected to grow deterministically at rate g, such that $D_{t+n} = (1+g)^n \times D_t$

$$P_t = \frac{D_t \times (1+g)^1}{(1+k_e)^1} + \frac{D_t \times (1+g)^2}{(1+k_e)^2} + \ldots + \frac{D_t \times (1+g)^\infty}{(1+k_e)^\infty} \qquad (4)$$

$$P_t = D_t \times \left[\frac{(1+g)^1}{(1+k_e)^1} + \frac{(1+g)^2}{(1+k_e)^2} + \ldots + \frac{(1+g)^\infty}{(1+k_e)^\infty} \right] = \frac{D_t}{(k_e - g)} \qquad (5)$$

In general, D will not grow in a smooth deterministic fashion, nor will k_e be constant. As a consequence, the fluctuations in prices will not move one for one with contemporaneous dividends.

In the below figures, monthly data from Robert Shiller's website (http://www.econ.yale.edu/~shiller/data/ie_data.xls) are used to highlight the relationships between stock prices, dividends and interest rates. In Figure 1, real (CPI deflated) stock prices and dividends are shown; and in Figure 2 real prices and ten year interest rates.

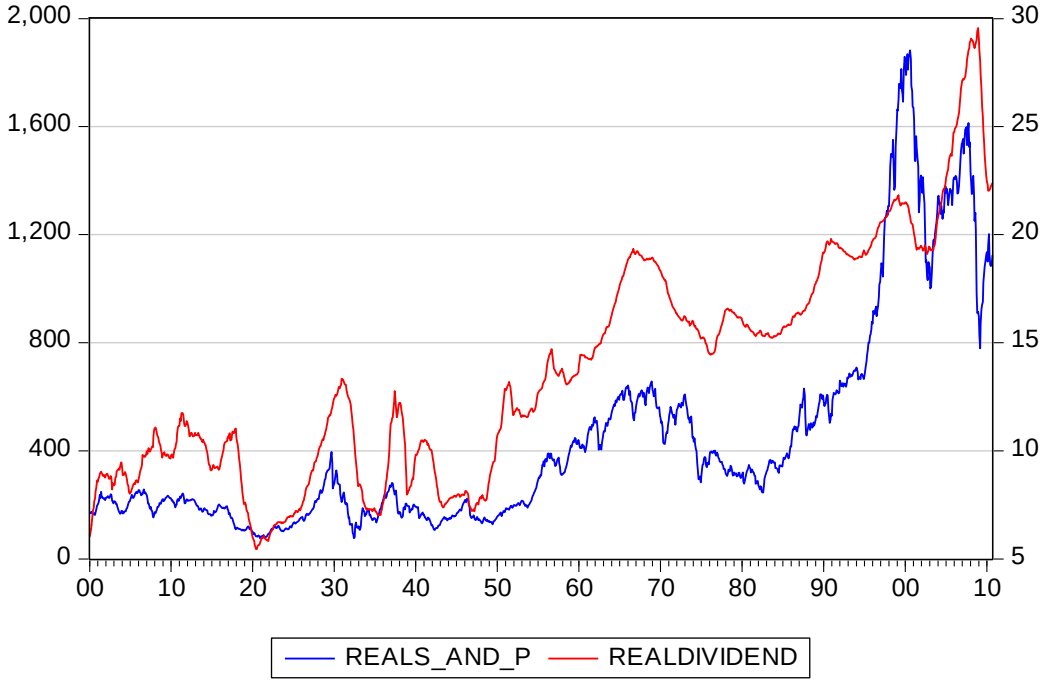

Figure 1: Real (CPI deflated) Standard and Poor index (left scale), and real dividends (right scale). Source: Robert Shiller, http://www.econ.yale.edu/~shiller/data/ie_data.xls .

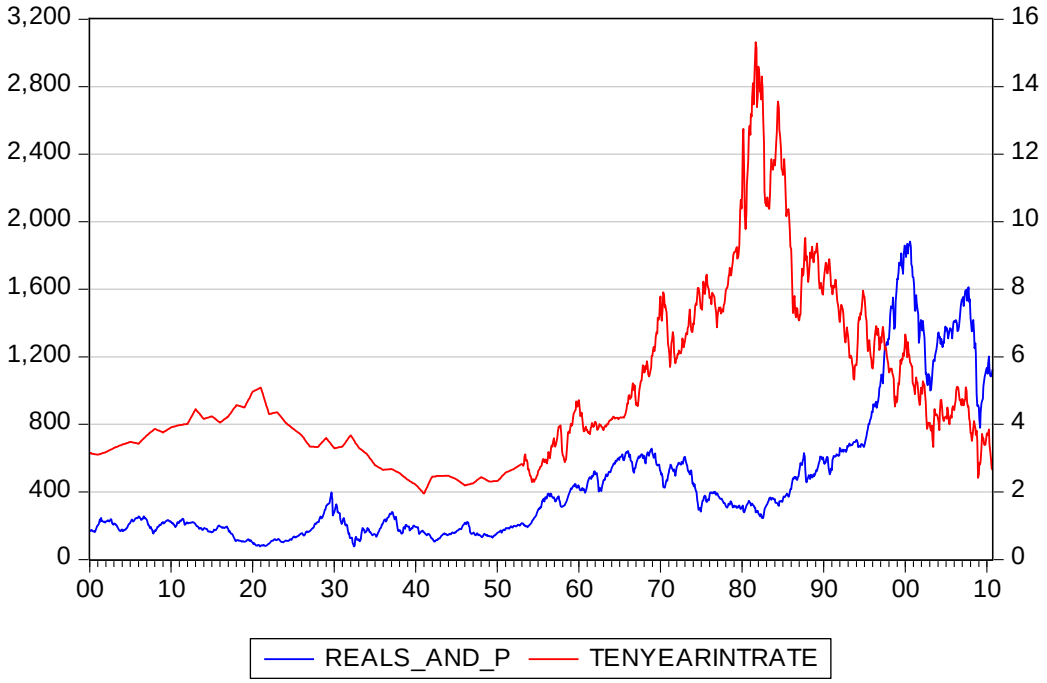

Figure 2: Real (CPI deflated) Standard and Poor index (left scale), and ten year interest rate (right scale). Source: Robert Shiller, http://www.econ.yale.edu/~shiller/data/ie_data.xls .

As predicted, the stock price index covaries positively with dividends (dividends are highly serially correlated, so a movement up in dividends persists), and is negatively related to the interest rate. Note that dividing both sides of equation 5 by D leads to the relationship that the price/dividend ratio is inversely related to the interest rate (minus the growth rate of dividends).

$$\frac{P_t}{D_t} = \left[\frac{(1+g)^1}{(1+k_e)^1} + \frac{(1+g)^2}{(1+k_e)^2} + \ldots + \frac{(1+g)^\infty}{(1+k_e)^\infty} \right] = \frac{1}{(k_e - g)} \qquad (5')$$

The posited inverse relationship is shown in Figure 3.

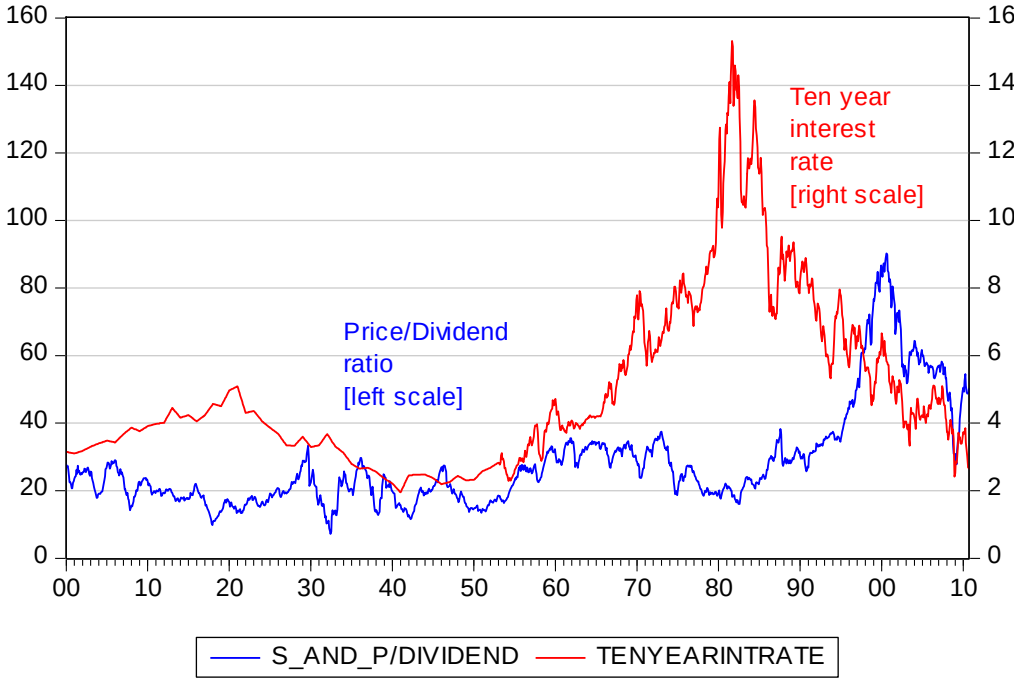

Figure 3: Standard and Poor price to dividend ratio (left scale), and ten year interest rate (right scale). Source: Robert Shiller, http://www.econ.yale.edu/~shiller/data/ie_data.xls .

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