Present Value Calculations and Asset Markets

The Present Value Model

We assume for now Rational Expectations. Hence the expectations operator refers to the conditional mathematical expectations operator. In one-period:

\[ P_t = \frac{D_{t+1}}{1 + k_e} + \frac{E_t P_{t+1}}{1 + k_e} \]  

(1)

Where \( k_e \) is the required return on equity, \( E_t(Z_{t+1}) = E(Z | \text{information available at time } t+1) \). Assume at time \( t \), that \( D_t \) is known.

Then the **Generalized Dividend Valuation Model** is given by

\[ P_t = \frac{D_{t+1}}{1 + k_e} + \frac{E_t D_{t+2}}{(1 + k_e)^2} + \ldots + \frac{E_t D_{t+n}}{(1 + k_e)^n} + \frac{E_t P_{t+n}}{(1 + k_e)^n} \]  

(2)

Note that this expression (2) implies, under certain conditions:

\[ P_t = \frac{D_{t+1}}{1 + k_e} + \frac{E_t D_{t+2}}{(1 + k_e)^2} + \ldots + \frac{E_t D_{t+n}}{(1 + k_e)^n} = \sum_{n=0}^{\infty} \frac{E_t D_{t+n}}{(1 + k_e)^n} \]  

(3)

Equation (2) rules out “bubbles”. The **Gordon Growth Model** assumes that dividends are expected to grow deterministically at rate \( g \), such that \( D_{t+n} = (1 + g)^n \times D_t \)

\[ P_t = \frac{D_t \times (1 + g)^1}{(1 + k_e)^1} + \frac{D_t \times (1 + g)^2}{(1 + k_e)^2} + \ldots + \frac{D_t \times (1 + g)^\infty}{(1 + k_e)^\infty} \]  

(4)

\[ P_t = D_t \times \left[ \frac{(1 + g)^1}{(1 + k_e)^1} + \frac{(1 + g)^2}{(1 + k_e)^2} + \ldots + \frac{(1 + g)^\infty}{(1 + k_e)^\infty} \right] = \frac{D_t}{(k_e - g)} \]  

(5)

In general, \( D \) will not grow in a smooth deterministic fashion, nor will \( k_e \) be constant. As a consequence, the fluctuations in prices will not move one for one with contemporaneous dividends.

In the below figures, monthly data from Robert Shiller’s website (http://www.econ.yale.edu/~shiller/data/ie_data.xls) are used to highlight the relationships between stock prices, dividends and interest rates. In Figure 1, real (CPI deflated) stock prices and dividends are shown; and in Figure 2 real prices and ten year interest rates.
Figure 1: Real (CPI deflated) Standard and Poor index (left scale), and real dividends (right scale). Source: Robert Shiller, [http://www.econ.yale.edu/~shiller/data/ie_data.xls](http://www.econ.yale.edu/~shiller/data/ie_data.xls).

Figure 2: Real (CPI deflated) Standard and Poor index (left scale), and ten year interest rate (right scale). Source: Robert Shiller, [http://www.econ.yale.edu/~shiller/data/ie_data.xls](http://www.econ.yale.edu/~shiller/data/ie_data.xls).
As predicted, the stock price index covaries positively with dividends (dividends are highly serially correlated, so a movement up in dividends persists), and is negatively related to the interest rate. Note that dividing both sides of equation 5 by D leads to the relationship that the price/dividend ratio is inversely related to the interest rate (minus the growth rate of dividends).

\[
\frac{P_t}{D_t} = \left[ \frac{(1 + g)^1}{(1 + k_e)^1} + \frac{(1 + g)^2}{(1 + k_e)^2} + \ldots + \frac{(1 + g)^\infty}{(1 + k_e)^\infty} \right] = \frac{1}{(k_e - g)}
\] (5')

The posited inverse relationship is shown in Figure 3.

Figure 3: Standard and Poor price to dividend ratio (left scale), and ten year interest rate (right scale). Source: Robert Shiller, [http://www.econ.yale.edu/~shiller/data/ie_data.xls](http://www.econ.yale.edu/~shiller/data/ie_data.xls).