Let the firm’s demand curve be defined as follows:

\[ X = S \left[ \frac{1}{n} - b(P - \bar{P}) \right] \]  

(1)

where \( X \) is firm sales, \( S \) is total sales, \( n \) is the number of firms. \( P \) is the price charged by the firm and \( \bar{P} \) is the average industry price.

Derivation of equilibrium will follow in three steps.
1. Derive the number of firms as a function of average cost (\( n \) as a function of \( AC \)).
2. Derive the average industry price as a function of the number of firms (\( \bar{P} \) as a function of \( n \)).
3. Establish conditions for exit and entry, viz., if the average industry price exceeds (is below) the average cost, the firms enter (exit) the industry (\( \bar{P} > AC \), firms enter, and if \( \bar{P} < AC \), firms exit).

**Step 1.** Set \( P = \bar{P} \). Note that if all firms charge the same price, then equation (1) becomes \( X = S/n \).

Recall average cost is given by:

\[ AC = \left( \frac{F}{X} \right) + c \]  

(2)

Substituting (1) into (2) yields:

\[ AC = \left( \frac{F}{S/n} \right) + c = n \times \left( \frac{F}{S} \right) + c \]  

(3)

Therefore the more firms in the industry, the higher is average cost.

**Step 2.** Recall the demand curve:

\[ X = S \left[ \frac{1}{n} - b(P - \bar{P}) \right] \]  

(1)

Assume each firm takes \( \bar{P} \) as given; rewrite (1):

\[ X = \frac{S}{n} + Sb\bar{P} - SbP = A - BP \]  

(1’)

where
The profit maximizing firm always sets the marginal revenue equal to marginal cost, \( MR = MC \). We know for linear demand curves:

\[
MR = \frac{\partial R}{\partial X} = P - \frac{X}{B} = P - \left( \frac{X}{Sb} \right)
\]  

(4)

So:

\[
MR = P - \left( \frac{X}{Sb} \right) = c = MC
\]

(5)

where \( c \) is marginal cost.

Solving for the firm’s optimal price yields:

\[
P = c + \frac{X}{Sb}
\]

(6)

Since all firms charge the same price, \( X = S/n \), then:

\[
P = c + \frac{X}{Sb} = c + \frac{(S / n)}{Sb} = c + \frac{1}{bn}
\]

(7)

Therefore the more firms, the lower price that will be charged by firms.

**Step 3.** We have two countervailing effects. What determines the industry price? Notice by equation (3), as the size of the market \( S \) increases, the average cost decreases. So on the supply side, this would be represented by a shift of the cost curve; by equation (7), we know that as firm number increases, the price falls, and this is a movement along the pricing curve:
Figure 1.

Notice in the long run, price must equal average cost (this is the entry/exit condition). Set price equal to AC.

\[ P = AC \Rightarrow \frac{1}{nb} + c = \frac{Fn}{S} + c \]  

(8)

Solving:

\[ \frac{1}{b} = \frac{Fn^2}{S} \Rightarrow \frac{S}{bF} = n^2 \Rightarrow n = \sqrt{\frac{S}{bF}} \]  

(9)

Therefore, a doubling in market size leads to an approximately 1.4 increase in the number of firms. What about the price charged by the representative firm? From (7), substituting in (9):

\[ P = \frac{1}{bn} + c = \left( \frac{1}{b} \right) \times \frac{1}{\sqrt{\frac{S}{bF}}} + c \Rightarrow P = \sqrt{\frac{F}{Sb}} + c \]  

(10)

And sales, \( X \), are given by the following, when \( P = \bar{P} \):

\[ X = \frac{S}{n} = \frac{S}{\sqrt{\frac{S}{bF}}} = \sqrt{SbF} \]  

(11)