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Notes on the Monetary Model of Exchange Rates (rev'd)

1. The Flexible-Price Monetary Approach

Let the spot exchange rate be given as

(1) $s = \log(S)$

Where S is measured in US dollars per foreign currency unit. Assume uncovered interest rate parity (UIP), which is implied by perfect capital substitutability [1].

(2) $i_t - i_t^* = \Delta s_{t+1}^e \equiv s_{t,t+1}^e - s_t$

The object on the right-hand side of the equation is "expected depreciation", which is typically modeled as the mathematical expectation of the log spot exchange rate at time t, based on time t information set (Φ_t) minus the time *t* log-spot exchange rate.

(3) $i_t - i_t^* = \Delta s_{t+1}^e = E_t(s_{t+1}) - s_t$

The next relation is purchasing power parity (PPP) in log-levels.

(4) $s_t = p_t - p_t^*$

Finally, assume stable money demand functions in the two countries:

(5)
$$(m_t - p_t)^d = \varphi y_t - \lambda i_t$$
$$(m_t^* - p_t^*)^d = \varphi^* y_t^* - \lambda^* i_t^*$$

where the d superscripts indicate "demand". Rearranging, assuming money supply equals money demand,

(5')
$$p_t = m_t - \varphi y_t + \lambda i_t$$
$$p_t^* = m_t^* - \varphi^* y_t^* + \lambda^* i_t^*$$

[1] In this course, we will take perfect capital *substitutability* to be as Frankel (1983) defined it: government bonds issued in different currencies by different government are perfectly substitutable, such that uncovered interest rate parity (UIP) holds. Perfect capital *mobility* is defined as the condition where there are no actual or incipient government restrictions on movements of capital. Then covered interest parity holds, or alternatively, the covered interest differential equals zero.

And using (4) (PPP) one obtains:

(6) $$s_t = (m_t - m_t^*) - \varphi(y_t - y_t^*) + \lambda(i_t - i_t^*)$$

Where $\lambda = \lambda^*, \varphi = \varphi^*$

Note that this step requires that *stock* (as well as flow) equilibrium holds. That is, the trade balance is zero.

It is useful to contrast the results of the monetarist model with that of the old-fashioned (Keynesian) version of the Mundell-Fleming model of exchange rates. The model is essentially an IS-LM model augmented with an ad hoc balance of payments equilibrium condition, called the BP=0 schedule.

$$BP = [current\ account] + [private\ financial\ account]$$

$$BP = \left[\overline{EXP} - \overline{IMP} - m(Y - Y^*) + n\left(\frac{SP^*}{P}\right)\right] + [\overline{KA} - \kappa(i - i^*)]$$

Re-arranging:

$$S = \left(\frac{P}{nP^*}\right)[\overline{IMP} - \overline{EXP} - \overline{KA} + m(Y - Y^*) - \kappa(i - i^*)]$$

Notice the difference between equation (6) and equation (ii); the monetary model implies:
(a) Higher relative income induces a *stronger* currency.
(b) A higher relative interest rate induces a *weaker* currency.
Both of these predictions are opposite of those obtained by the Mundell-Fleming model.

The reasons for these differences are obvious. Regarding (a), in Mundell-Fleming, higher income induces higher imports, *ceteris paribus*, and hence a weaker currency. In the monetary model, a higher income induces a higher money demand relative to supply, and hence a stronger currency. Regarding (b), in Mundell-Fleming, a higher interest rate causes a capital inflow, by the *ad hoc* KA function. In the monetary approach, a higher interest rate causes a lower money demand, relative to money supply, and hence a weaker currency.

2. A Present Value Formulation of the FPMA

Some additional insights can be garnered by re-expressing the monetary model in terms of current and expected future values of the "fundamentals". Note by UIP ,

(2) $$i_t - i_t^* = \Delta s_{t+1}^e \equiv s_{t,t+1}^e - s_t$$

Assuming perfectly flexible prices, the Fisherian model of interest rates should hold:

(7) $$i_t = r_t + \pi^e_{t,t+1}$$

Combining (2) and (7) yields:

(8) $$i_t - i_t^* = \Delta s^e_{t+1} \equiv \pi^e_{t,t+1} - \pi^{e*}_{t,t+1}$$

Hence equation (6) can be re-expressed as:

(9) $$s_t = \widetilde{M}_t + \lambda(s^e_{t,t+1} - s_t) = \widetilde{M}_t + \lambda s^e_{t,t+1} - \lambda s_t$$

Where $\widetilde{M}_t \equiv (m_t - m_t^*) - \varphi(y_t - y_t^*)$

By manipulating this expression

$$s_t + \lambda s_t \equiv s_t(1+\lambda) = \widetilde{M}_t + \lambda s^e_{t,t+1}$$

$$s_t = \left(\frac{1}{1+\lambda}\right)\widetilde{M}_t + \left(\frac{\lambda}{1+\lambda}\right)s^e_{t,t+1}$$

Imposing rational expectations yields and expression for the future expected spot rate in period period *t+1*:

(10) $$E_t(s_{t+1}) = \left(\frac{1}{1+\lambda}\right)E_t\widetilde{M}_{t+1} + \left(\frac{\lambda}{1+\lambda}\right)E_t s_{t+2}$$

substituting equation (10) into (9) yields:

(11) $$s_t = \left(\frac{1}{1+\lambda}\right)\widetilde{M}_t + \left(\frac{\lambda}{(1+\lambda)^2}\right)E_t\widetilde{M}_{t+1} + \left(\frac{\lambda}{1+\lambda}\right)\left(\frac{\lambda}{1+\lambda}\right)E_t s_{t+2}$$

but consider:

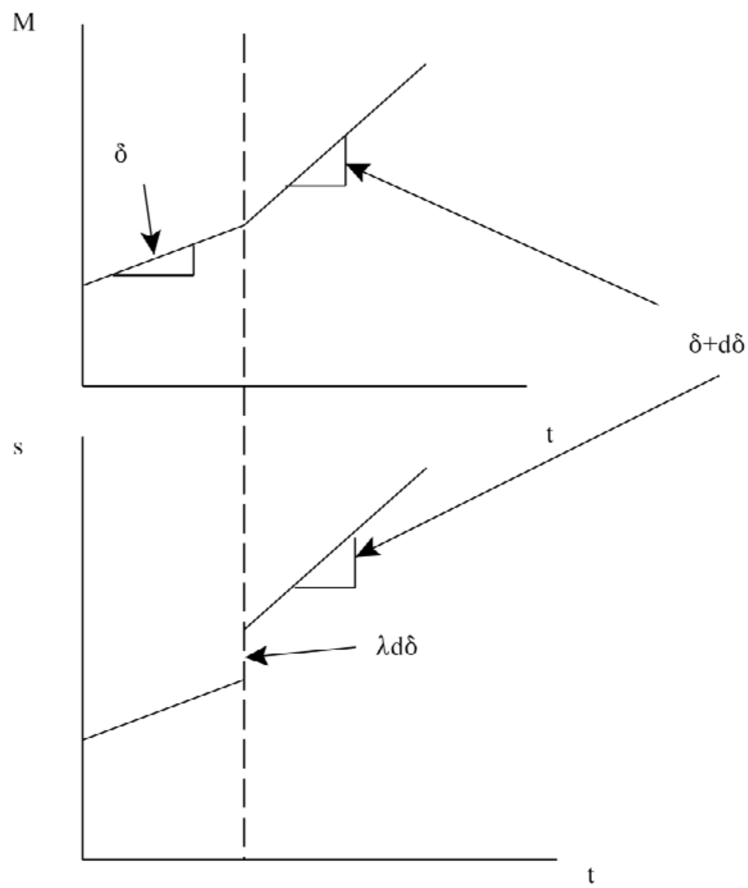
(12) $$E_t(s_{t+2}) = \left(\frac{1}{1+\lambda}\right)E_t\widetilde{M}_{t+2} + \left(\frac{\lambda}{1+\lambda}\right)E_t s_{t+3}$$

So that by substituting iteratively, one obtains:

(13) $$s_t = \left(\frac{1}{1+\lambda}\right)\sum_{\tau=0}^{\infty}\left(\frac{\lambda}{(1+\lambda)}\right)^{\tau} E_t\widetilde{M}_{t+\tau}$$

Assuming that the future expected spot rate in the infinite future discounted equals zero.

Hence, the log spot exchange rate is equal to the present discounted value of the fundamentals from now to the infinite future, where the discount factor [λ/(1+λ)] is a function of the interest semi-elasticity of money demand. This is a common form for rational expectations solutions to take; the current value of an asset depends upon all expected future values of that asset; but that in turn depends upon the values of the fundamentals expected from now to the infinite future (of course with declining weights).



The magnification effect means that an increase in the growth rate of the money stock, holding constant the actual level of the current fundamentals, causes an immediate and discontinuous depreciation, and then a more rapid rate of depreciation thereafter.

3. Sticky-Price Monetary Approach to Exchange Rates

3.1. Overview

The flexible price monetary approach (which some termed monetar*ist*, in earlier, more ideologically charged days) yields some very strong predictions. One of the most controversial is that increasing interest differential will be associated with weakening currencies. In the context of a model with purchasing power parity holding in both the long run and short run, this result makes sense; positive interest differentials arise from positive inflation differentials (via the Fisher relation). The more rapid a currency loses value against a basket of real goods, the more rapid a currency loses value against another currency, given that PPP links prices of home and foreign real goods.

(14) $s_t = (m_t - m_t^*) - \varphi(y_t - y_t^*) + \lambda(i_t - i_t^*)$

The positive relationship between the interest differential and the exchange rate runs counter to casual empiricism, at least as far as the developed economies are concerned (the high-inflation LDCs such as Argentina and Brazil are another matter). Hence we consider allowing the PPP condition to hold only in the long run.

3.2. Derivation of the Frankel Model

The assumption of long run PPP is denoted as follows, where "overbars" indicate long run variables:

(15) $\overline{s_t} = \overline{p_t} - \overline{p_t}^*$

Hence, rewrite the flexible price monetary model equation for the exchange rate (14) is re-written:

(16) $\overline{s_t} = (\overline{m_t} - \overline{m_t^*}) - \varphi(\overline{y_t} - \overline{y_t^*}) + \lambda(\overline{\pi_t} - \overline{\pi_t^*})$

where the inflation rates stand in for long run interest rates, given the Fisher relation holds in the long run.

Now introduce "overshooting": exchange rates tend to revert back towards the long run value at some rate θ. That is, if exchange rates are too high, relative to some long run value, they will then tend to fall toward the long run value. This suggests the following mechanism:

(17) $\Delta s_t \equiv s_{t+1} - s_t = -\theta(s_t - \bar{s}_t) + \pi_{t+1} - \pi_{t+1}^*$

In words, if the exchange rate is undervalued, the exchange rate will appreciate. The θ parameter is the rate of reversion. If θ = 0.5, then a 0.10 (10%) undervaluation induces a 0.05 (5%) exchange rate appreciation in the subsequent period, holding everything else constant. The inflation rates are added because the more rapid the inflation rate, the faster the exchange rate is losing value against the other currency, everything else held constant.

Assuming "rational expectations", that is *on average* people's expectations match what actually happens, then

(18) $$\Delta s^{e}_{t,t+1} - s_t = -\theta(s_t - \bar{s}_t) + \pi^{e}_{t,t+1} - \pi^{e*}_{t,t+1}$$

However, by uncovered interest parity, the left-hand side of equation (5) is also equal the interest differential:

(19) $$i_t - i^{*}_t = -\theta(s_t - \bar{s}_t) + \pi^{e}_{t,t+1} - \pi^{e*}_{t,t+1}$$

Rearranging and solving for *s*:

(20) $$(i_t - \pi^{e}_{t,t+1}) - (i^{*}_t - \pi^{e*}_{t,t+1}) = -\theta(s_t - \bar{s}_t)$$

$$s_t = \bar{s}_t - \left(\frac{1}{\theta}\right)[(i_t - \pi^{e}_{t,t+1}) - (i^{*}_t - \pi^{e*}_{t,t+1})]$$

we have an expression for the long run *s*; substituting that in:

$$\boxed{s_t = (m_t - m^{*}_t) - \varphi(y_t - y^{*}_t) - (1/\theta)(i_t - i^{*}_t) + \left(\lambda + \frac{1}{\theta}\right)(\pi^{e}_{t,t+1} - \pi^{e*}_{t,t+1})}$$

This expression can be rewritten as:

$$\boxed{s_t = (m_t - m^{*}_t) - \varphi(y_t - y^{*}_t) - (1/\theta)(r_t - r^{*}_t) + \lambda(\pi^{e}_{t,t+1} - \pi^{e*}_{t,t+1})}$$

Since the real interest rate shows up in this expression, this model is sometimes called the "real interest differential" model.

Interpretation: The current exchange rate depends positively on current money stocks, and inflation rates, and negatively on income levels and interest rates. This result regarding interest rates differs from the flex-price monetary model because in the short run, inflation rate differentials can differ from interest rate differentials.

A common error in using this model is to trace out the following logic: higher real interest rates in the US induce investors to shift their capital to the US, resulting in a capital inflow. The capital inflow causes a greater demand for US dollars, thereby appreciating the currency. This interpretation cannot literally be correct since, as noted above, the trade balance is always zero so that the capital account is also always zero. Recall also that uncovered interest parity always holds, so investors are always indifferent between holding US versus foreign assets.

4. Empirical Evidence

How well do the data fit this model? In the long run, the models work fairly well. Consider the data from Alquist and Chinn (2008). We rely upon quarterly data for the United States, Canada, U.K., and the Euro Area over the 1970q1 to 2004q4 period. The exchange rate, money, price and income (real GDP) variables are drawn primarily from the IMF's International Financial Statistics. M1 is used for the money variable, with the exception of the UK, where M4 is used. For the money stocks, exchange rates, and interest rates, end-of-quarter rates are used.

Table 1: DOLS Estimates of the Monetary Model

	EUR	GBP	CAD	JPY
Money	−0.154	−0.224	0.029	0.422
	(0.263)	(0.192)	(0.058)	(0.395)
Output	−1.035	−2.696*	−2.938***	1.604
	(0.899)	(1.567)	(0.512)	(1.181)
Interest rates	−1.602	−0.351	−2.558*	−7.713
	(1.495)	(1.441)	(1.419)	(2.393)
Inflation	9.942**	1.314	3.523***	3.080
	(4.317)	(1.314)	(1.181)	(6.000)
Adj. R-sq.	0.56	0.20	0.59	0.59
Sample	81Q4-05Q4	75Q4-05Q4	75Q2-05Q4	80Q4-05Q2
T	95	119	121	99

Notes: Point estimates from DOLS(2,2). Newey-West HAC standard errors in parentheses. *(**)(***) indicates statistical significance at the 10% (5%) (1%) level.

Statistically, this model is hard pressed to outperform a random walk. That tells us something – that there is tremendous uncertainty in the evolution of exchange rates – but does not necessarily deny the usefulness of these models in understanding the movements of exchange rates *ex post*.

References

Alquist, Ron and Menzie Chinn, 2008, "Conventional and Unconventional Approaches to Exchange Rate Modeling and Assessment," *International Journal of Finance and Economics* 13: 2-13.

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