Equilibrium income, $Y_0$, is given by:

$$Y_0 = \left(\frac{1}{1-c(1-t)+m}\right)[\bar{A} + \bar{EXP} - \bar{IMP}]$$

let $\alpha = \left(\frac{1}{1-c(1-t)+m}\right)$

where $\bar{A} = \bar{C} - c\bar{TA} + \bar{IN} + \bar{GO}$

Multipliers are given by the total differential of equation (1):

$$\Delta Y = \alpha [\Delta A + \Delta EXP - \Delta IMP]$$

Notice that if lump sum taxes rise, then:

$$\Delta Y = -\alpha c \Delta TA$$

and hence

$$\frac{\Delta Y}{\Delta TA} = -\alpha c$$

What happens if the marginal tax rate changes? Then one has to use the product and chain rules.

$$\Delta Y = \alpha \frac{\partial}{\partial t} [\bar{A} + \bar{EXP} - \bar{IMP}] \Delta t + \alpha \frac{\partial}{\partial t} [\bar{A} + \bar{EXP} - \bar{IMP}] \Delta t$$

Note the first term is zero since the term in the brackets does not depend on $t$. Moreover, recalling $\alpha = (1-c(1-t)+m)^{-1}$ then

$$\frac{\partial \alpha}{\partial t} = (-1) \times \left(\frac{1}{1-c(1-t)+m}\right)^2 \times (c)$$

And hence:

$$\Delta Y = \alpha \frac{\partial}{\partial t} [\bar{A} + \bar{EXP} - \bar{IMP}] \Delta t = -c(\alpha^2) [\bar{A} + \bar{EXP} - \bar{IMP}] \Delta t$$

So the multiplier is:

$$\frac{\Delta Y}{\Delta t} = -c(\alpha^2) [\bar{A} + \bar{EXP} - \bar{IMP}]$$
This result can be interpreted graphically.

\[
AD = c(1 - t_0)Y - mY + \overline{A} + \overline{EXP} - \overline{IMP}
\]

\[
AD = c(1 - t_1)Y - mY + \overline{A} + \overline{EXP} - \overline{IMP}
\]

\[\Delta Y = -c\alpha^2 \Delta t\]

**Figure 1:** Change in income due to a change in marginal tax rate

Note that \(t_1 = t_0 + \Delta t\).