Tax Rate Change Multiplier

Equilibrium income, \( Y_0 \), is given by:

\[
(1) \quad Y_0 = \left( \frac{1}{1-c(1-t)+m} \right) [\overline{A} + \overline{EXP} - \overline{IMP}] \quad \text{let} \quad \alpha = \left( \frac{1}{1-c(1-t)+m} \right)
\]

where \( \overline{A} \equiv CO - cTA + IN + G0 \)

Multipliers are given by the total differential of equation (1):

\[
(2) \quad \Delta Y = \alpha [\Delta A + \Delta EXP - \Delta IMP]
\]

Notice that if lump sum taxes rise, then:

\[
(3) \quad \Delta Y = -\alpha c \Delta TA \quad \text{and hence} \quad \frac{\Delta Y}{\Delta TA} = -\alpha c
\]

What happens if the marginal tax rate changes? Then one has to use the product and chain rules.

\[
(4) \quad \Delta Y = \alpha \frac{\partial [\overline{A} + \overline{EXP} - \overline{IMP}]}{\partial t} \Delta t + \frac{\partial \alpha}{\partial t} [\overline{A} + \overline{EXP} - \overline{IMP}] \Delta t
\]

Note the first term is zero since the term in the brackets does not depend on \( t \). Moreover, recalling \( \overline{\alpha} = (1-c(1-t)+m)^{-1} \) then

\[
\frac{\partial \alpha}{\partial t} = (-1) \times \left( \frac{1}{1-c(1-t)+m} \right)^2 \times (c)
\]

And hence:

\[
(5) \quad \Delta Y = \frac{\partial \alpha}{\partial t} [\overline{A} + \overline{EXP} - \overline{IMP}] \Delta t = -c(\alpha^2)[\overline{A} + \overline{EXP} - \overline{IMP}] \Delta t
\]

So the multiplier is:

\[
\frac{\Delta Y}{\Delta t} = -c(\alpha^2)[\overline{A} + \overline{EXP} - \overline{IMP}]
\]
This result can be interpreted graphically.

\[ AD = (1 - t_0)Y - mY + \bar{A} + EXP - IMP \]

\[ AD = (1 - t_1)Y - mY + \bar{A} + EXP - IMP \]

\[ \Delta Y = -c\alpha^2 \Delta t \]

**Figure 1:** Change in income due to a change in marginal tax rate

Note that \( t_1 = t_0 + \Delta t \).