

Math Review

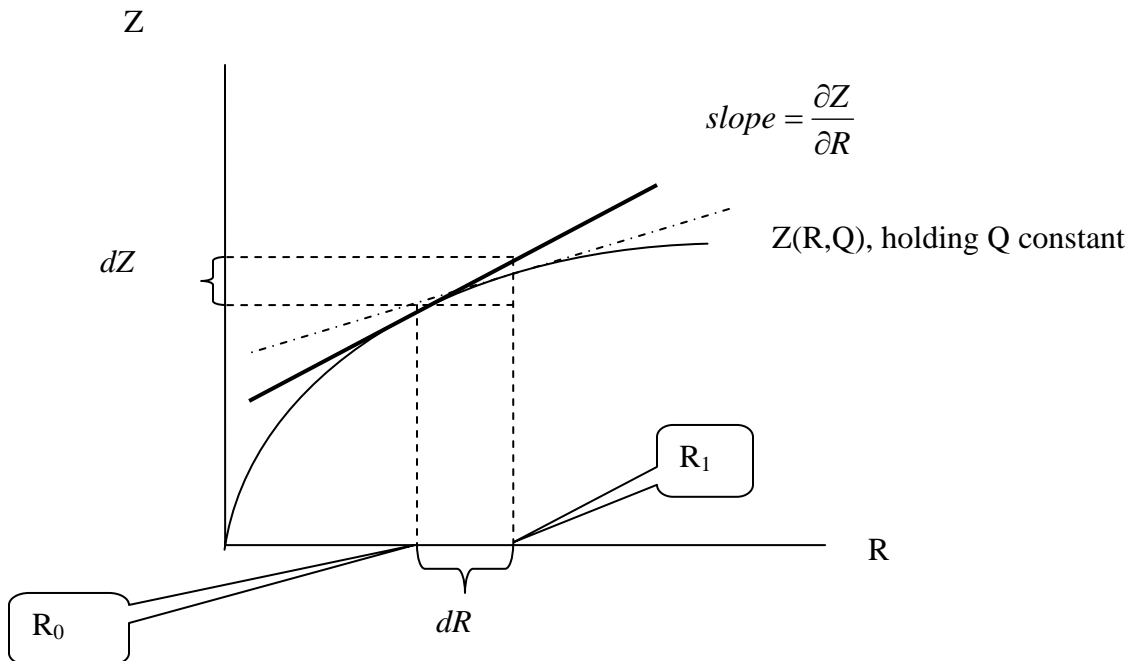
Outline:

1. Total Differentials and Slopes
2. Changes, Elasticities and Growth Rates

1. Total Differential and Slopes

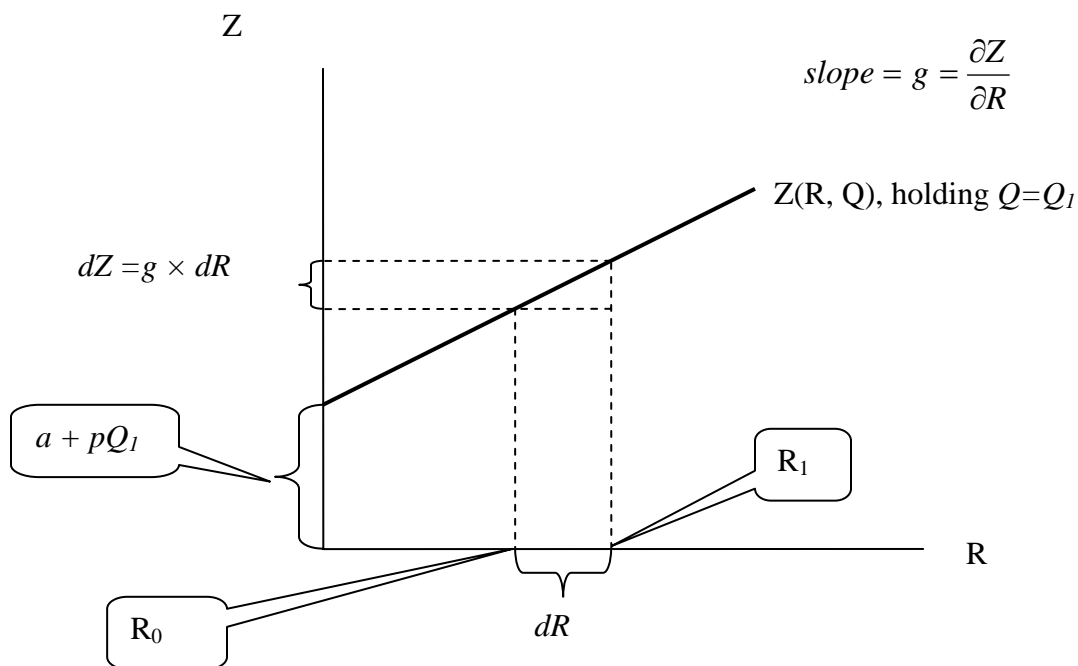
Suppose $Z = Z(R, Q)$. Then the total differential of Z is:

$$dZ = \frac{\partial Z}{\partial R} dR + \frac{\partial Z}{\partial Q} dQ$$



Notice that the slope differs depending upon whether it is evaluated at R_0 or at R_1 .

When the functions are linear, such as $Z = a + gR + pQ$, then the partial differentials are constant, and one gets a picture such as the following:



What's the relevance of this? Well, consider the import equation:

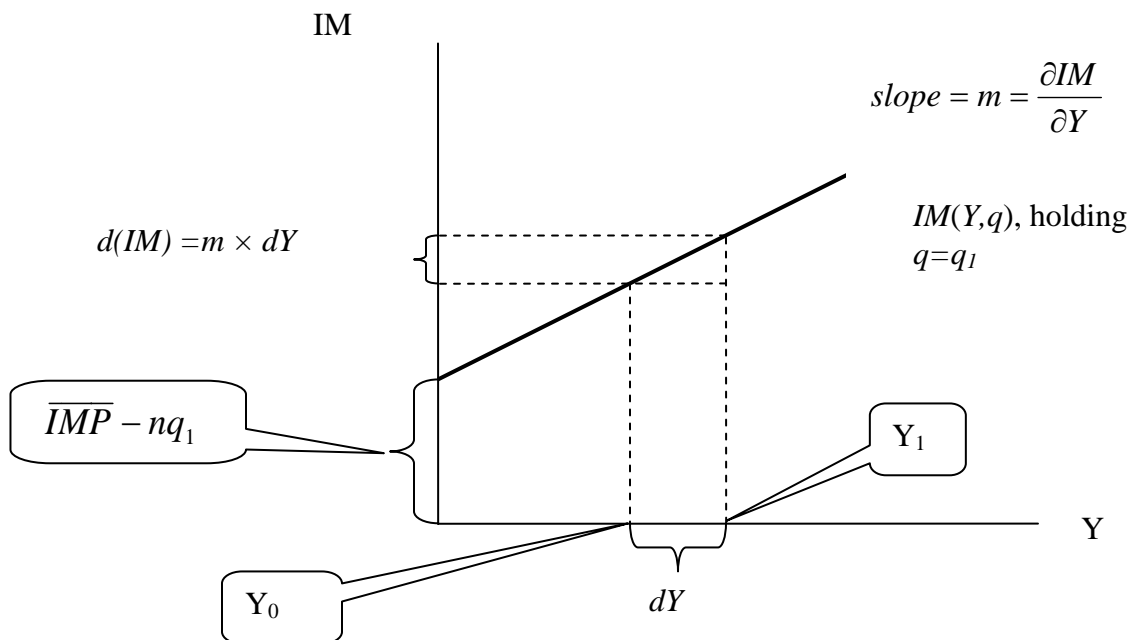
$$IM = \overline{IMP} + mY - nq$$

Take the total differential of this equation:

$$d(IM) = d(IMP) + m \times dY - n \times dq$$

Which in this class we have been writing using the "delta's" in place of the "d's",

$$\Delta IM = \Delta IMP + m\Delta Y - n\Delta q$$



2. Changes, Elasticities, Growth Rates

It is important to note that the slopes in our linear models have been changes for changes. The slopes have important interpretations. For instance, in the import equation:

$$IM = \overline{IMP} + mY - nq$$

The marginal propensity to import (also known as in the income sensitivity of imports) is $m = \partial IM / \partial Y$, which is the change in imports (in real dollars) for a one unit change in GDP (in real dollars). Note that the real exchange rate sensitivity of imports is $-n = \partial IM / \partial q$, is the change in imports (in real dollars) for a one unit change in the real exchange rate (in US widgets per foreign widget).

How are these slope coefficients or “sensitivities” related to elasticities. Elasticities are “percent change for percent change”. Hence, the income elasticity of imports would be:

$$\varepsilon_{Y,IM} = \frac{(\partial IM / \ddot{IM})}{(\partial Y / \ddot{Y})} = \frac{\partial IM}{\partial Y} \frac{\ddot{Y}}{\ddot{IM}}$$

Where the over-dots indicate “average values”. Hence, while the income sensitivity of imports is related to the income sensitivity of imports, they are not the same.

While we are discussing changes, it might be useful to explain how growth rates (in percent terms) are calculated. In most official statistics, a growth rate of a variable Z is calculated as:

$$growth_rate_Z_t \equiv \left(\frac{Z_t}{Z_{t-1}} \right) - 1 \equiv \left(\frac{Z_t - Z_{t-1}}{Z_{t-1}} \right) \equiv \left(\frac{\Delta Z_t}{Z_{t-1}} \right)$$

If the data are of annual frequency, then this calculation will lead to an annual growth rate. If the data are sampled more frequently than annually, then the above calculation will lead to a growth rate consistent with the data frequency. For instance, if the data is monthly, then this calculation will lead to the month-on-month growth rate. Typically, in order to make rates comparable, we calculate then on an *annualized* basis. In order to do this when the data is monthly, you calculate the following:

$$annualized_growth_rate_Z_t \equiv \left(\frac{Z_t}{Z_{t-1}} \right)^{12} - 1 \quad (\text{where there are 12 months in a year})$$

If the data were at a quarterly frequency, then the corresponding calculation would be:

$$annualized_growth_rate_Z_t \equiv \left(\frac{Z_t}{Z_{t-1}} \right)^4 - 1 \quad (\text{where there are 4 quarters in a year})$$

Notice that in all these cases, growth rates are calculated based relative to the starting point (i.e., relative to the value at time t-1). Of course, one could with equal validity calculate relative to the ending value (i.e., the value at time t). One common technique is to measure relative to an average of the initial and ending values.

This can be accomplished by measuring growth rates in continuously compounded rates. At this juncture, natural logs (the \log_e or \ln operator) comes in useful.

The time derivative of a logged variable is equal to the growth rate in percent terms. Hence,

$$\textit{continuously_compounded_growth_rate_}Z_t \equiv d(\log(Z_t)) \equiv \log(Z_t) - \log(Z_{t-1})$$

Using log differences is convenient for annualizing growth rates because the annualized growth rate for a month-on-month change can be calculated as:

$$\textit{continuously_compounded_annualized_growth_rate_}Z_t \equiv d(\log(Z_t)) \times 12$$

And the annualized quarter-on-quarter change, using quarterly data:

$$\textit{continuously_compounded_annualized_growth_rate_}Z_t \equiv d(\log(Z_t)) \times 4$$