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**The National Saving-Investment Identity,**
**and the Keynesian Model of Equilibrium and the Trade Balance**

This set of notes discusses the National Saving Identity, and links that identity to the Keynesian model of national income determination and trade balance ("net export") determination. It then shows how to solve for multipliers.

## 1. National Saving-Investment Identity

The National Saving-Investment Identity indicates what must be true in terms of accounting. Let:

| | |
|---|---|
| $C + S + T \equiv Y$ | How income can be allocated (Y is the same as GDP) |
| $Y \equiv C + I + G + EX - IM$ | How spending can be categorized |
| $C + S + T \equiv C + I + G + (EX - IM)$ | Combining two definitions of GDP |
| $(S - I) + (T - G) \equiv (EX - IM)$ | where $TB \equiv (EX - IM)$ is the "trade balance", or "net exports" |
| $NS - I \equiv (EX - IM)$ | where $NS \equiv S + (T - G)$ |

Notice that national saving minus investment must equal net exports by accounting; however, there is no way to impute causality in this set of manipulations (e.g., a trade deficit "causes" a budget deficit). For that we need a model.

## 2. An Expanded Model

Note that in the equations below, while C, I, G, and so forth look the same as in Section 1 above, they are different conceptually. In particular, they are *planned* levels of consumption, investment and government spending respectively, which might differ from the *ex post*, or actual, levels of those variables. In the model below, actual investment equals planned only when the economy is in equilibrium. In this case, there is no unintended inventory accumulation or decumulation.

| Eq.No. | Equation | Description |
|---|---|---|
| (1) | $Y = AD$ | Output equals aggregate demand – an equilibrium condition |
| (2) | $AD \equiv C + I + G + EX - IM$ | Definition of aggregate demand |
| (3) | $C = \overline{CO} + c(Y - T)$ | Consumption function, *c* is the marginal propensity to consume |
| (4) | $T = \overline{TA} + tY$ | Tax function; $\overline{TA}$ is lump sum taxes, *t* is tax rate. |
| (5) | $I = \overline{IN}$ | Investment function |
| (6) | $G = \overline{GO}$ | Government spending on goods and services |
| (7) | $EX = \overline{EXP}$ | Exports, simplification of $X = X_d(E, Y^*)$ where *E*, *Y** fixed |
| (8) | $IM = \overline{IMP} + mY$ | Import spending, simplification of $M = M_d(E, Y)$ where *E* fixed |

Substitute (3)-(8) into (2), and substitute (2) into (1):

(9) $Y = AD = \overline{CO} + c(Y - \overline{TA} - tY) + \overline{IN} + \overline{GO} + \overline{EXP} - \overline{IMP} - mY$

Collect up terms:

(10) $Y = \overline{A} + \overline{EXP} - \overline{IMP} + (cY - ctY - mY)$ where $\overline{A} \equiv \overline{CO} - c\overline{TA} + \overline{IN} + \overline{GO}$

Shift "*Y*" terms to the left hand side:

(11) $Y - (cY - ctY - mY) = \overline{A} + \overline{EXP} - \overline{IMP} \rightarrow Y[1 - c(1-t) + m] = \overline{A} + \overline{EXP} - \overline{IMP}$

Divide both sides by the term in the square bracket to obtain equilibrium income, $Y_0$:

(12) $\boxed{Y_0 = \left(\frac{1}{1 - c(1-t) + m}\right)[\overline{A} + \overline{EXP} - \overline{IMP}]}$ let $\overline{\alpha} \equiv \left(\frac{1}{1 - c(1-t) + m}\right)$

Notice that if there are no taxes, $t$=0 and $\overline{TA} = 0$, so 1-$c$ = $s$ and (12) becomes identical to (17.7) in the textbook.

(17.7') $Y_0 = \left(\frac{1}{s + m}\right)[\overline{A} + \overline{EXP} - \overline{IMP}]$ where $\overline{A} \equiv \overline{CO} - c\overline{TA} + \overline{IN} + \overline{GO}$

Interpretation of (12): equilibrium income is a multiple of the amounts of "autonomous" spending. The higher the level of autonomous spending, the higher the equilibrium level of income. Notice also that lump sum taxes enter in negatively, so the higher lump sum taxes, the lower equilibrium income is.

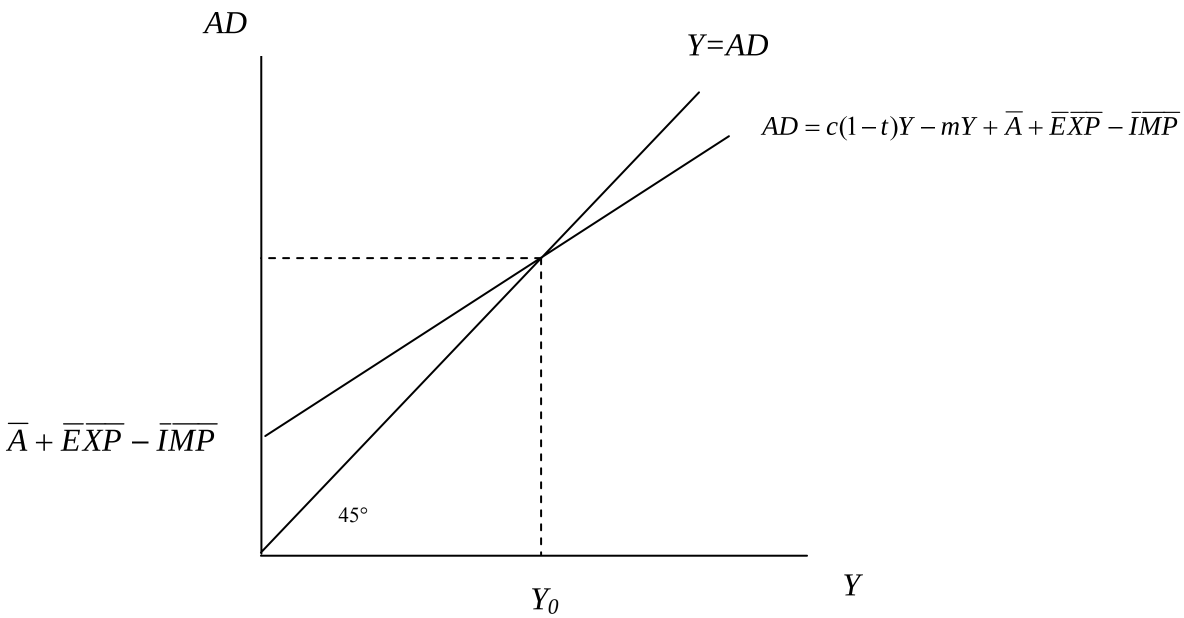


**Figure 1:** Equilibrium in the Keynesian Cross

## 3. Effects of changes in autonomous spending on income

To think about how changes in autonomous spending – government spending ( $\overline{GO}$ ), investment spending ( $\overline{IN}$ ), export spending ( $\overline{EXP}$ ) and import spending ( $\overline{IMP}$ ) – affect equilibrium income, think about a change of income ( $\Delta Y$ ) as being attributable to changes in each of those autonomous spending components. Take equation (12):

(13) $\Delta Y = \overline{\alpha}[\Delta A + \Delta EXP - \Delta IMP]$

So if, for instance, the only autonomous spending component that changes is government spending (so $\Delta A = \Delta GO$ , and $\Delta EXP = 0 = \Delta IMP$ ), then:

(14) $\Delta Y = \overline{\alpha}\Delta GO$

This result can be interpreted graphically.

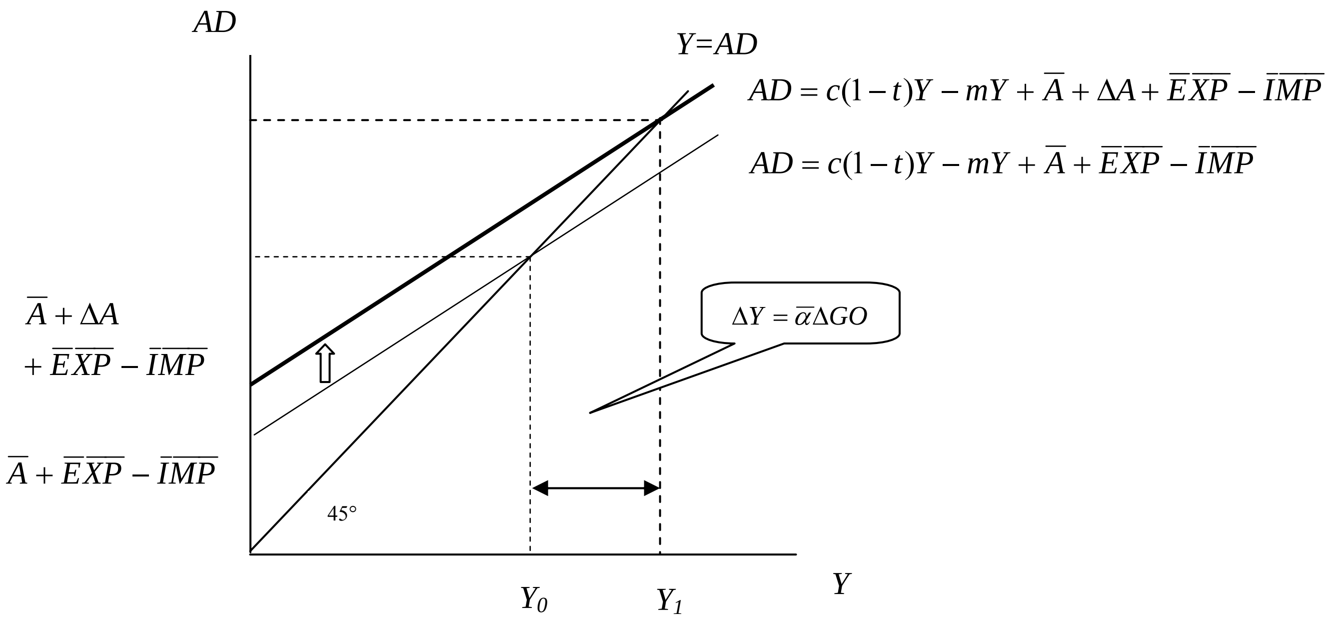


**Figure 2:** Change in income due to a change in autonomous spending

Note that $Y_1 = Y_0 + \Delta Y$. You should also understand that $\overline{A}' \equiv \overline{A} + \Delta A$.

Returning to (14), consider what the change in income for a change in government spending is. That can be obtained by dividing both sides by $\Delta GO$ :

$$\frac{\Delta Y}{\Delta GO} = \overline{\alpha} \equiv \frac{1}{1 - c(1-t) + m}$$

Note that a general expression for the change in autonomous domestic spending ($\Delta A$) is:

$\Delta A \equiv \Delta CO - c\Delta TA + \Delta IN + \Delta GO$ (and recalling that we're setting $\Delta EXP = 0 = \Delta IMP$)

Keeping this in mind, see if you can solve for the lump sum tax multiplier, $\frac{\Delta Y}{\Delta TA}$.

**4. Effects of changes in government spending on the trade balance and budget balance**

To figure out what happens to the trade balance in response to changes in government spending again), take (17.6) from the textbook:

(17.6) $TB \equiv X - M = \overline{X} - (\overline{M} + mY)$

Re-expressed in our notation:

(15) $TB \equiv EX - IM = \overline{EXP} - (\overline{IMP} + mY)$

Break up the changes in the trade balance in the changes in the constituent parts,

(16) $\Delta TB = \Delta EXP - \Delta IMP - m\Delta Y$

If, once again, the only thing that changes is government spending, then substitute (14) into (16), and setting $\Delta EXP = 0 = \Delta IMP$:

(17) $\Delta TB = -m[\overline{\alpha}\Delta GO] < 0$

In other words, the effect of an increase in government spending is a deterioration in the trade balance, holding everything else constant.

What about the budget balance? Use the definition of a budget balance, and substitute in (4) and (6):

(18) $BuS \equiv T - G = \overline{TA} + tY - \overline{G}\overline{O}$

Take the total differential:

(19) $\Delta BuS = \Delta TA + t\Delta Y - \Delta GO$

However, notice that Y changes in response to government spending, so substituting in yields:

(20) $\Delta BuS = t\overline{\alpha}\Delta GO - \Delta GO = (t\overline{\alpha} - 1)\Delta GO < 0$

Hence, an increase in government spending in this model causes both the budget and trade balances to deteriorate. This is the "causal" idea behind the "twin deficits" hypothesis.