Expenditure Switching/Reducing

This set of notes extends the Keynesian model to incorporate the elasticities approach explicitly.

1. Derivation

<table>
<thead>
<tr>
<th>Eq.No.</th>
<th>Equation</th>
<th>Description</th>
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<tbody>
<tr>
<td>(1)</td>
<td>$Y = AD$</td>
<td>Output equals aggregate demand – an equilibrium condition</td>
</tr>
<tr>
<td>(2)</td>
<td>$AD = C + I + G + EX - IM$</td>
<td>Definition of aggregate demand</td>
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<td>(3)</td>
<td>$C = \bar{C}O + c(Y - T)$</td>
<td>Consumption function, $c$ is the marginal propensity to consume</td>
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<td>(4)</td>
<td>$T = \bar{T}A + tY$</td>
<td>Tax function; $\bar{T}A$ is lump sum taxes, $t$ is tax rate.</td>
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<td>(5)</td>
<td>$I = \bar{I}N$</td>
<td>Investment function</td>
</tr>
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<td>(6)</td>
<td>$G = \bar{G}O$</td>
<td>Government spending on goods and services</td>
</tr>
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<td>(7)</td>
<td>$EX = \bar{EXP} + vq$</td>
<td>Export spending</td>
</tr>
<tr>
<td>(8)</td>
<td>$IM = \bar{IMP} + mY - nq$</td>
<td>Import spending</td>
</tr>
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</table>

where $v$ is the sensitivity of exports to the real exchange rate, and $-n$ is the sensitivity of imports to the real exchange rate.

Substitute (3)-(8) into (2), and substitute (2) into (1):

(9) \quad Y = AD = \bar{C}O + c(Y - \bar{T}A - tY) + \bar{I}N + \bar{G}O + \bar{EXP} - \bar{IMP} - mY + (n + v)q

Collect up terms:

(10) \quad Y = \bar{A} + \bar{EXP} - \bar{IMP} + (cY - ctY - mY) + (n + v)q \quad \text{where} \quad \bar{A} = \bar{C}O - c\bar{T}A + \bar{I}N + \bar{G}O

Shift “$Y$” terms to the left hand side:

(11) \quad Y - (cY - ctY - mY) = \bar{A} + \bar{EXP} - \bar{IMP} + (n + v)q \Rightarrow 

Y[1 - c(1 - t) + m] = \bar{A} + \bar{EXP} - \bar{IMP} + (n + v)q

Divide both sides by the term in the square bracket to obtain equilibrium income, $Y_0$:

(12) \quad Y_0 = \left(\frac{1}{1 - c(1 - t) + m}\right)[\bar{A} + \bar{EXP} - \bar{IMP} + (n + v)q] \quad \text{let} \quad \bar{\alpha} = \left(\frac{1}{1 - c(1 - t) + m}\right)
Interpretation of (12): equilibrium income is a multiple of the amounts of “autonomous” spending and the real exchange rate. The first component is familiar -- the higher the level of autonomous spending, the higher the equilibrium level of income. Notice also that lump sum taxes enter in negatively, so the higher lump sum taxes, the lower equilibrium income is. The second component is new – changes in real exchange rates (which are the same as the change in the nominal exchange rate when the price levels are fixed) affect the international components of aggregate demand, namely exports and imports.

To think about how changes in autonomous spending or exchange rates affect equilibrium income, think about a change of income ($\Delta Y$) as being attributable to changes in each of those autonomous spending components. Take equation (12):

(13) $\Delta Y = \bar{\alpha}(\Delta A + \Delta EXP - \Delta IMP + (n + v)\Delta q)$

So if, for instance, the only autonomous spending component that changes is government spending (so $\Delta A = \Delta GO$, and $\Delta EXP = 0 = \Delta IMP$) and the real exchange rate is constant ($\Delta q = 0$), then:

(14) $\Delta Y = \bar{\alpha}\Delta GO \rightarrow \Delta Y / \Delta GO = \bar{\alpha}$

Notice that by this reasoning, all the multipliers can be derived. Now consider changes in the real exchange rate. Then:

(15) $\Delta Y = \bar{\alpha}(n + v)\Delta q \rightarrow \Delta Y / \Delta q = \bar{\alpha}(n + v)$

2. Expenditure reduction versus expenditure switching

To figure out what happens to the trade balance in response to changes in government spending, take the definition of the trade balance:

(16) $TB \equiv EX - IM = (\bar{EXP} + \nu q) - (\bar{IMP} + mY - nq)$

Break up the changes in the trade balance in the changes in the constituent parts,

(17) $\Delta TB = \Delta EXP - \Delta IMP - m\Delta Y + (n + v)\Delta q$

If the only thing that changes is government spending, then substitute (14) into (17), and setting $\Delta EXP = 0 = \Delta IMP$ and $\Delta q = 0$:

(18) $\Delta TB = m[\bar{\alpha}\Delta GO] < 0$

In other words, the effect of an increase in government spending is a deterioration in the trade balance, holding everything else constant. Thus, decreasing government spending would improve the trade balance. This is the “expenditure reduction” channel.
On the other hand, holding GO constant, one can substitute (15) into (17) to obtain:

\[ \Delta TB = (n + v)\Delta q > 0 \]

In words, the effect of a real exchange rate depreciation \((\Delta q > 0)\) is an improvement in the trade balance. This occurs because exports are now cheaper, so demand for exports rises, while imports become more expensive for home residents, so imports fall. This is the “expenditure switching” channel.

How does this relate to the “Swan Diagram” (Figure 18.3) on page 343 of the textbook? Take equation (17),

\[ \Delta TB = \Delta EXP - \Delta IMP - m\Delta Y + (n + v)\Delta q \]

substitute in (14) and (15):

\[ \Delta TB = \Delta EXP - \Delta IMP - m(\alpha\Delta GO + \beta(n + v)\Delta q) + (n + v)\Delta q \]

Starting from initially balanced trade, setting autonomous exports and imports constant, and requiring the change in the trade balance to be zero, yields:

\[ 0 = -m\alpha\Delta GO - m\beta(n + v)\Delta q + (n + v)\Delta q \]

Solving for the change in \(q\) (which is the same as the change in \(E\)):

\[ \begin{align*}
\text{(22.a)} \quad & \Delta E = \frac{m\alpha}{(1 - m\alpha)(n + v)} \Delta GO \\
\text{(22.b)} \quad & \left. \frac{\Delta E}{\Delta GO} \right|_{TB=\text{const}} = \frac{m\alpha}{(1 - m\alpha)(n + v)} > 0
\end{align*} \]
Notice that the slope of the BB curve is the change in $E$ for a change in $GO$, holding $TB$ constant, or in other words, the expression in (22.b). Notice the YY curve in 18.5 can be derived by taking equation (13) with the change in income set to zero, setting $\Delta EXP = 0 = \Delta IMP$. Then

\begin{equation}
0 = \Delta Y = \bar{\alpha}[\Delta GO + (n + v)\Delta q]
\end{equation}

Once again letting $\Delta q = \Delta E$, and solving:

\begin{equation}
\frac{\Delta E}{\Delta GO} = -\frac{1}{n + v} < 0
\end{equation}

That is, the YY curve has slope $-1/(n+v)$. This means in words, for higher levels of government spending, a lower exchange rate (a stronger currency) is required to keep output constant.

Putting the two together, one obtains the diagram in the textbook.

Hence, one can hit both internal and external equilibrium with two instruments (the exchange rate and government spending).

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