1. Supply, Demand, and Exchange Rates: Floating

2. Supply, Demand, and Exchange Rates: Fixed
3. Devaluation ($E' > E$)

3.1. Imports

3.2 Exports
4. The Marshall-Lerner Conditions

Let the US be the home country, and the foreign country be Japan. As noted on page 295 of Caves, et al. (2007), the US trade balance, $TB^*$, in units of foreign currency, e.g., yen (¥), is given by:

$$TB^* = \left( \frac{P}{E} \right) \times EX - \bar{P}^* \times IM$$

(0)

(where $EX$ is the same as the $X$ in the textbook, and $IM$ is the same as the $M$ in the textbook). Overbars indicate fixed values. Multiply both sides by $E$, and divide by $P$, to obtain:

$$\frac{E \times TB^*}{P} = \frac{TB}{P} = EX - \left( \frac{E\bar{P}^*}{P} \right) \times IM$$

(1)

$TB$ is the trade balance in nominal domestic currency terms; $TB/P$ is the trade balance denominated in US widgets. Define the trade balance ($\tilde{TB}$) in domestic widget terms as the difference between exports ($EX$) denominated in US widgets and imports ($IM'$) denominated in US widgets:

$$\frac{TB}{P} \equiv \tilde{TB} = EX - IM'$$

(2)

However, imports are the foreign (*) country’s exports, so:

$$IM' = (E\bar{P}^*/P) \times \bar{EX}^* \equiv qEX^*$$

(3)

where $q$ is a real exchange rate, measured in terms of number of home widgets needed to purchase one foreign widget. Then the trade balance is functionally defined as:

$$\tilde{TB} = EX - qEX^*$$

(4)

(As in the textbook, we take the income levels as exogenously fixed.) To determine the response of the trade balance to a change in the real exchange rate, take the partials with respect to the real exchange rate:

$$\frac{\partial \tilde{TB}}{\partial q} = \frac{\partial EX}{\partial q} - \left( q \frac{\partial EX^*}{\partial q} + EX^* \right)$$

$$\frac{\partial \tilde{TB}}{\partial q} = \left( \frac{\partial EX}{\partial q} - q \frac{\partial EX^*}{\partial q} \right) - EX^*$$

(5)

$$\frac{\partial \tilde{TB}}{\partial q} = \text{(volume effect)} + \text{(value effect)}$$

One wants to know if $\frac{\partial \tilde{TB}}{\partial q}$ is greater than or less than zero. Solve for:

$$0 < \frac{\partial EX}{\partial q} - q \frac{\partial EX^*}{\partial q} - EX^*$$

(6)
Multiply both sides by the quantity \((q/EX)\) to obtain:

\[
0 < \frac{\partial EX}{\partial q} \frac{q}{EX} - q \frac{\partial EX^*}{\partial q} \frac{q}{EX} - EX^* \frac{q}{EX} \tag{7}
\]

Define the first term as \(\varepsilon_{EX}\), the export supply-demand elasticity. Further note that if initial trade is balanced, then \(EX = qEX^*\) such that \(q/EX = 1/EX^*\).

\[
0 < \varepsilon_{EX} - \frac{\partial EX^*}{\partial q} \frac{q}{EX^*} - 1 \tag{8}
\]

Finally, define the import demand elasticity:

\[-\frac{\partial EX^*}{\partial q} \frac{q}{EX^*} \equiv \varepsilon_{IM}\]

then one obtains the Marshall-Lerner-Robinson condition:

\[1 < \varepsilon_{IM} + \varepsilon_{EX} \tag{9}\]

which is appropriate for export demand supply and import supply elasticities that are infinite.

Caveats:
1. What if export demand supply and import supply elasticities are not infinite?
2. What if initial trade is not balanced?
3. Recall, prices of exports are fixed in domestic currency terms; prices of the foreign country’s exports are fixed in foreign currency terms.