Notes on the Elasticities Approach

Let the US be the home country, and the foreign country be Japan. As noted on page 295 of Caves, et al. (2007), $TB^*$, in units of foreign currency, e.g., yen (¥), is given by:

$$TB^* = \left( \frac{P}{E} \right) \times EX - \frac{P^*}{P} \times IM$$

(where $EX$ is the same as the $X$ in the textbook, and $IM$ is the same as the $M$ in the textbook). Overbars indicate fixed values. Multiply both sides by $E$, and divide by $P$, to obtain:

$$ET \frac{TB^*}{P} P TB_{\times} = EX - \left( \frac{E}{P} \right) \times IM$$

(1)

$TB$ is the trade balance in nominal domestic currency terms; $TB/P$ is the trade balance denominated in US widgets. Define the trade balance ($TB$) in domestic widget terms as the difference between exports ($EX$) denominated in US widgets and imports ($IM'$) denominated in US widgets:

$$\frac{TB}{P} \equiv TB = EX - IM'$$

(2)

However, imports are the foreign (*)& country's exports, so:

$$IM' = \left( \frac{EP^*}{P} \right) \times EX^* \equiv qEX^*$$

(3)

where $q$ is a real exchange rate, measured in terms of number of home widgets needed to purchase one foreign widget. Then the trade balance is functionally defined as:

$$TB = EX - qEX^*$$

(4)

(As in the textbook, we take the income levels as exogenously fixed.) To determine the response of the trade balance to a change in the real exchange rate, take the partials with respect to the real exchange rate:
\[
\frac{\partial \bar{TB}}{\partial \bar{q}} = \left( \frac{\partial EX}{\partial \bar{q}} - q \frac{\partial EX^*}{\partial \bar{q}} \right) + EX^*
\]

\[
\frac{\partial \bar{TB}}{\partial \bar{q}} = \left( \frac{\partial EX}{\partial \bar{q}} - q \frac{\partial EX^*}{\partial \bar{q}} \right) - EX^*
\]

\[
\frac{\partial \bar{TB}}{\partial \bar{q}} = (\text{volume effect}) + (\text{value effect})
\]

One wants to know if \( \frac{\partial \bar{TB}}{\partial \bar{q}} \) is greater than or less than zero. Solve for:

\[
0 < \frac{\partial EX}{\partial \bar{q}} - q \frac{\partial EX^*}{\partial \bar{q}} - EX^*
\]

(6)

Multiply both sides by the quantity (\( q/EX \)) to obtain:

\[
0 < \frac{\partial EX}{\partial \bar{q}} \frac{q}{EX} - q \frac{\partial EX^*}{\partial \bar{q}} \frac{q}{EX} - EX^* \frac{q}{EX}
\]

(7)

Define the first term as \( \varepsilon_{EX} \), the export supply elasticity. Further note that if initial trade is balanced, then \( EX = qEX^* \) such that \( q/EX = 1/EX^* \).

\[
0 < \varepsilon_{EX} - \frac{\partial EX^*}{\partial \bar{q}} \frac{q}{EX^*} - 1
\]

(8)

Finally, define the import elasticity:

\[
-\frac{\partial EX^*}{\partial \bar{q}} \frac{q}{EX^*} \equiv \varepsilon_{IM}
\]

then one obtains the Marshall-Lerner-Robinson condition:

\[
1 < \varepsilon_{IM} + \varepsilon_{EX}
\]

(9)

which is appropriate for export demand and import supply elasticities that are infinite.

Caveats:

1. What if export demand and import supply elasticities are not infinite?
2. What if initial trade is not balanced?
3. Recall, prices of exports are fixed in domestic currency terms; prices of the foreign country’s exports are fixed in foreign currency terms.