Public Affairs 854
Spring 2013
University of Wisconsin-Madison

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Social Sciences 7418

Bubbles in Exchange Rates

1. Bubbles: Standard Exposition

Take the monetarist approach to the exchange rate:

(1) $$s_t = (m_t - m_t^*) - \varphi(y_t - y_t^*) - \lambda\,(i_t - i_t^*) \equiv \widetilde{M}_t - \lambda\,(i_t - i_t^*)$$

Assume rational expectations:

(2) $$i_t - i_t^* = E_t s_{t+1} - s_t$$

Hence equation (1) can be re-expressed as:

(3) $$s_t = \widetilde{M}_t + \lambda E_t s_{t+1} - \lambda s_t$$

By manipulating this expression

(4) $$s_t = \left(\frac{1}{1+\lambda}\right)\widetilde{M}_t + \left(\frac{\lambda}{1+\lambda}\right)E_t s_{t+1}$$

Imposing rational expectations yields an expression for the future expected spot rate in period period *t+1*:

(5) $$E_t(s_{t+1}) = \left(\frac{1}{1+\lambda}\right)E_t\widetilde{M}_{t+1} + \left(\frac{\lambda}{1+\lambda}\right)E_t s_{t+2}$$

substituting equation (5) into (4) yields:

(7) $$s_t = \left(\frac{1}{1+\lambda}\right)\widetilde{M}_t + \left(\frac{\lambda}{(1+\lambda)^2}\right)\widetilde{M}_{t+1} + \left(\frac{\lambda}{1+\lambda}\right)\left(\frac{\lambda}{1+\lambda}\right)E_t s_{t+2}$$

but consider:

(8) $$E_t(s_{t+2}) = \left(\frac{1}{1+\lambda}\right)E_t\widetilde{M}_{t+2} + \left(\frac{\lambda}{1+\lambda}\right)E_t s_{t+3}$$

So that by substituting iteratively, one obtains:

(9)
$$s_t = \underbrace{\left(\frac{1}{1+\lambda}\right)\sum_{\tau=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^{\tau} E_t\widetilde{M}_{t+\tau}}_{fundamentals} + \underbrace{\underset{\tau\Rightarrow\infty}{Limit}\left(\frac{\lambda}{(1+\lambda)}\right)^{\tau} E_t s_{t+\tau}}_{bubble\_term}$$

If the second term goes to zero as τ approaches infinity, then the exchange rate is the present discounted value of the fundamentals from now to the infinite future. Otherwise, the exchange rate includes a bubble term.

Notice that this bubble interpretation does not rely upon irrational behavior. The results are entirely consistent with rational expectations.

2. Rational Stochastic Bubbles

What about bubbles that don't last forever, as considered above. One can consider what are rational stochastic bubbles – bubbles that can "pop" with some probability. The idea of such bubbles doesn't seem so unreasonable, especially in the context of certain events. Consider the experience of the mid-1980's.

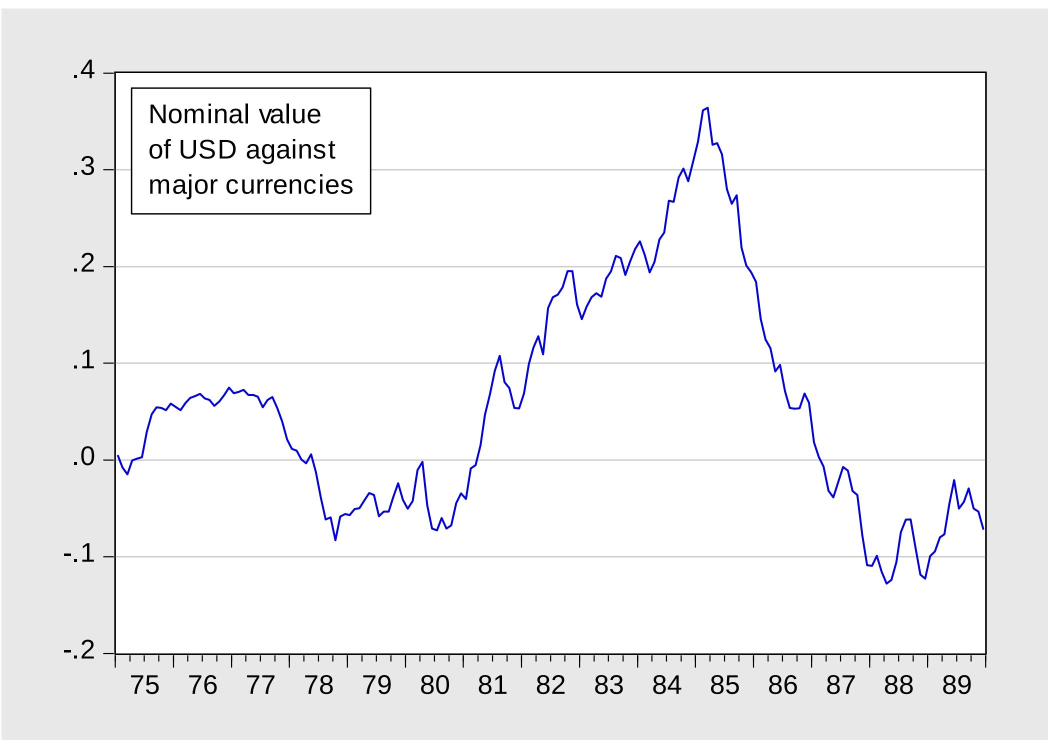

Figure 1: Log nominal trade weighted USD, evaluated against a basket of major currencies. Source: Federal Reserve Board.

Frankel in "The Dazzling Dollar" (*BPEA* 1985), considers the possibility that at any given instant the exchange rate S_t will revert to $\bar{S}_t$, the exchange rate that is consistent with the fundamentals, with probability P_t, and with probability $(1 - P_t)$, the exchange rate will appreciate at exponential rate a_t.

(10) $$i_t - i_t^* = FD_t = P_t \ln\left(\frac{\bar{S}_t}{S_t}\right) + (1 - P_t)(-a_t)$$

Solve for the implicit probability of collapse:

(11) $$P_t = \frac{FD_t + a_t}{\ln\left(\frac{\bar{S}_t}{S_t}\right) + a_t}$$

Frankel estimates $\bar{S}_t$ as an average and a_t as a regression coefficient of the exchange rate against a time trend. He then calculates the implicit probability of collapse using (11).

Table 3. Implicit Probability of Collapse in Deutsche Mark-Dollar Rate under Bubble Hypothesis, January 1981–March 1985

Month	*Nominal appreciation of dollar* $-\ln S_t$ (*percent*)[b]	*Real "over-valuation" of dollar* $-\ln(S_t/\bar{S}_t)$ (*percent*)[b]	*Forward discount* FD_t (*percent per year*)	*Probability of collapse*[a] P_t	*Cumulated probability of no collapse* $\prod_{t=1}^{T}(1-P_t)$
January 1981	−13.09	8.61	10.88	0.168	0.83
February 1981	−6.80	15.17	5.75	0.071	0.77
March 1981	−8.26	13.69	2.68	0.061	0.73
April 1981	−5.69	16.24	3.83	0.057	0.68
May 1981	0.13	22.51	7.04	0.053	0.65
June 1981	3.78	26.61	5.85	0.042	0.62
July 1981	6.52	29.87	7.14	0.041	0.59
August 1981	8.90	32.68	6.79	0.036	0.57
September 1981	2.75	27.04	5.12	0.039	0.55
October 1981	−1.67	22.43	4.69	0.045	0.53
November 1981	−2.81	21.09	2.78	0.041	0.50
December 1981	−1.32	22.56	2.33	0.036	0.49
January 1982	0.14	23.64	3.79	0.040	0.47
February 1982	3.38	26.87	5.32	0.040	0.45
March 1982	3.92	27.32	5.83	0.041	0.43
April 1982	4.71	28.27	6.34	0.041	0.41
May 1982	0.97	24.77	6.57	0.047	0.39
June 1982	5.96	29.88	6.38	0.039	0.38
July 1982	7.43	31.76	5.28	0.034	0.37
August 1982	8.02	32.61	2.61	0.026	0.36
September 1982	9.02	33.43	3.81	0.028	0.35
October 1982	10.06	34.35	3.58	0.027	0.34
November 1982	11.14	35.00	2.89	0.025	0.33
December 1982	5.59	28.77	3.32	0.032	0.32

Month	Nominal appreciation of dollar $-\ln S_t$ (percent)[b]	Real "over-valuation" of dollar $-\ln (S_t/\bar{S}_t)$ (percent)[b]	Forward discount FD_t (percent per year)	Probability of collapse[a] P_t	Cumulated probability of no collapse $\prod_{t=1}^{T} (1-P_t)$
January 1983	4.32	27.40	3.78	0.034	0.31
February 1983	6.00	28.99	3.36	0.031	0.30
March 1983	5.16	28.33	4.49	0.035	0.29
April 1983	6.44	30.01	4.61	0.034	0.28
May 1983	7.51	31.41	3.99	0.031	0.27
June 1983	10.81	34.69	4.61	0.029	0.26
July 1983	12.42	36.37	5.31	0.030	0.25
August 1983	15.64	39.57	5.10	0.027	0.25
September 1983	15.39	39.56	4.27	0.025	0.24
October 1983	12.91	37.32	4.24	0.026	0.23
November 1983	15.94	40.35	3.92	0.024	0.23
December 1983	18.35	42.66	4.06	0.023	0.22
January 1984	20.60	45.05	4.10	0.022	0.22
February 1984	16.64	41.24	4.09	0.024	0.21
March 1984	12.68	37.44	4.93	0.028	0.21
April 1984	14.51	39.58	5.32	0.027	0.20
May 1984	18.38	43.69	5.63	0.025	0.20
June 1984	18.03	43.31	5.95	0.026	0.19
July 1984	21.92	47.68	6.44	0.025	0.19
August 1984	23.23	49.56	6.59	0.024	0.18
September 1984	28.03	54.75	6.28	0.021	0.18
October 1984	29.40	55.84	5.15	0.019	0.17
November 1984	26.87	53.14	3.94	0.018	0.17
December 1984	30.53	56.72	3.22	0.016	0.17
January 1985	32.63	58.42	2.67	0.015	0.17
February 1985	36.61	62.40	2.62	0.014	0.16
March 1985	38.60	64.39	3.07	0.014	0.16

a. From equation 2, with trend logarithmic appreciation of 7.80 percent per year.
b. 1973–79 = 0.

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