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Bubbles

Suppose there is a stable money demand equation:

$$(m_t - p_t)^d = -\eta i_{t+1} \qquad \textbf{(1)}$$

where the absence of *y* indicates that income has been held constant, and its effect subsumed into a constant (suppressed for convenience). Notice further that this notation differs from previous, in that the interest rate for period t is denoted by i_{t+1}. By the Fisher relation, this conventional money demand equation can be re-written as a Cagan money demand equation:

$$m_t^d - p_t = -\eta E_t(p_{t+1} - p_t) \qquad \textbf{(1)}$$

where once again, the r_t term has been subsumed into a constant, and suppressed for notational ease.

Set:

$$m_t^d = m_t \qquad \textbf{(2)}$$

that is, there is money market equilibrium and money supply is set exogenously. Further assume the process defining money demand is nonstochastic (for the moment):

$$m_t - p_t = -\eta(p_{t+1} - p_t) \qquad \textbf{(3)}$$

Solving for p_t yields:

$$-p_t = -m_t - \eta p_{t+1} + \eta p_t$$
$$(1+\eta)p_t = m_t + \eta p_{t+1}$$

$$p_t = \left(\frac{1}{1+\eta}\right)m_t + \left(\frac{\eta}{1+\eta}\right)p_{t+1} \qquad \textbf{(6)}$$

Notice that this is a recursive expression; it can be rewritten for the price level, lead by

one period, viz.

$$p_{t+1} = \left(\frac{1}{1+\eta}\right)m_{t+1} + \left(\frac{\eta}{1+\eta}\right)p_{t+2}$$

Substituting this latter equation into equation (6) yields:

$$p_t = \left(\frac{1}{1+\eta}\right) \times \left[m_t + \left(\frac{\eta}{1+\eta}\right) m_{t+1}\right] + \left(\frac{\eta}{1+\eta}\right)^2 p_{t+2}$$

this leads, via repeated substitution to:

$$p_t = \underbrace{\left(\frac{1}{1+\eta}\right)\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} m_s}_{Fundamentals} + \underbrace{\underset{T\Rightarrow\infty}{limit}\left(\frac{\eta}{1+\eta}\right)^{T} p_{t+T}}_{Bubble\ \ Term} \qquad \textbf{(7)}$$

Clearly, equation (7) differs from the expression derived in Lecture 4 for the present value form of the flex-price monetary model. Only if the "no-bubble" restriction,

$$\underset{T\Rightarrow\infty}{limit}\left(\frac{\eta}{1+\eta}\right)^{T} p_{t+T} = 0 \qquad \textbf{(8)}$$

is imposed will the two expressions be the same. The problem is that in general, this restriction cannot be imposed, so that one cannot in turn rule out the following expression for the price level:

$$p_t = \left(\frac{1}{1+\eta}\right)\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} m_s + b_0\left(\frac{1+\eta}{\eta}\right)^{t} \qquad \textbf{(11)}$$

where b_0 is an initial deviation of p_0 from the value implied by the fundamentals,

2. Meese's Tests for Foreign Exchange Bubbles [1]

2.1. A Joint Test for Bubbles

Once again, assume a money demand equation:

$$m_t - p_t = a_1 y_t - a_2(i_t - i_t^*) \qquad \textbf{(1)}$$

where now *m* and *p* are expressed in *relative* (to foreign country) terms. Assume uncovered

[1] A similar paper which undertakes testing for bubbles is Woo (1987).

interest parity (UIP), which is perfect capital substitutability.

$$i_t - i_t^* = E(s_{t+1}|\Phi_t) - s_t \tag{2}$$

The object on the right-hand side of the equation is "expected depreciation", which is modeled as the mathematical expectation of the log spot exchange rate at time t, based on time t information set (Φ_t) minus the time t log-spot exchange rate. The next relation indicates that deviations from purchasing power parity (PPP) in log-levels follows a random walk:

$$s_t - p_t = u_t, \quad u_t = u_{t-1} + \epsilon_t \tag{3}$$

where the ϵ is a white noise error term. This is a slightly odd assumption, as it implies the real exchange rate follows a random walk. However, Meese motivates it as an approximation to slow mean reversion to PPP (as would occur in the Dornbusch-Frankel model with very sticky prices).

Substituting (2) and (3) into (1) yields:

$$s_t = m_t - a_1 y_t + a_2[(E(s_{t+1}|\Phi_t) - s_t] + u_t \tag{4}$$

Defining $b = a_2/(1 + a_2)$, $0 < b < 1$, then equation (4) can be rewritten as:

$$s_t = (1-b)(m_t - a_1 y_t) + b[E(s_{t+1}|\Phi_t)] + (1-b)u_t \tag{5}$$

Since there is evidence of unit roots in nominal exchange rates, Meese first-differences

the series being examined.

$$\Delta s_t = (1-b)(\Delta m_t - a_1 \Delta y_t) + b[E(s_{t+1}|\Phi_t) - E(s_t|\Phi_{t-1})] + (1-b)\epsilon_t \tag{6}$$

Let the "fundamentals" be defined as

$$\Delta x_t \equiv (\Delta m_t - \alpha_1 \Delta y_t) = c\Delta x_{t-1} + \delta, \quad |c| < 1 \tag{7}$$

In other words, the fundamentals follow an AR(1) process.

If the no-bubbles, or transversality, condition is imposed, then one obtains the usual

present value expression for the exchange rate.

$$\Delta s_t^* \equiv \Delta s_t = \Delta x_t + \frac{bc}{1-bc}(\Delta x_t - \Delta x_{t-1}) + \epsilon_t$$
$$\Delta x_t = c\Delta x_{t-1} + \delta \tag{12}$$

On the other hand, if the transversality condition in (9) is violated then any solution of the form:

$$s_t = s_t^* + d_t,$$
$$where \quad E(d_{t+1}|\Phi_t) = (1/b)d_t \tag{13}$$

will be satisfied.

Meese exploits the fact that McCallum's procedure, as applied to equation (6), yields consistent estimates, while maximum likelihood applied to equation (12) will in general yield inconsistent estimates if the bubble term is correlated with the RHS variables. Then a Hausman specification error test can then be applied, where the test statistic is given by:

$$\frac{T(\hat{b}_{IV} - \hat{b})^2}{\frac{b^2(1+c)^2}{c^2} + \sigma_\epsilon^2 \frac{(1+c)^3(1-bc)^2[(1-bc)^2+2b^2(1-c)]}{\sigma_\delta^2 2(1-c^2)c^2}} \tag{19}$$

which is distributed χ^2 with one degree of freedom. Such an approach does not impose a specific form on the bubble, and as such is a more general test for bubbles. Another difference between

Meese and other tests is that instead of estimating the income elasticity of money demand, he searches over a grid of plausible parameter values.

Note that in order to implement estimation of (12), Δy and Δm must be represented by lag polynomials of the same order, and that Δx must be strongly exogenous with respect to Δs.

Examining the DM/$, £/$ and ¥/$ exchange rates, he in general he finds that the bubble test statistic always rejects the null of identical coefficients. Since his coefficient estimates and the diagnostic statistics for the US-Japan regressions are so poor, he considers these test results weak, and hence does not report them. This omission reflects the joint nature of the test, discussed further below.