The IS-LM-TB=0 Model

This set of notes discusses the IS-LM-TB=0 model, which appends an external sector to the standard IS-LM model. Private capital flows are set to zero (or a constant). Policy under fixed exchange rates is examined.

1. The Model

To allow for a role for money, let’s first modify the model. On the real side of the economy, everything is the same.

(1) $Y = AD$ 
Output equals aggregate demand – an equilibrium condition
(2) $AD ≡ C + I + G + EX - IM$ 
Definition of aggregate demand
(3) $C = YO + c(Y - T)$ 
Consumption function, $c$ is the marginal propensity to consume
(4) $T = TA + tY$ 
Tax function; $TA$ is lump sum taxes, $t$ is tax rate.
(5) $I = IN - bi$ 
Investment function
(6) $G = GO$ 
Government spending on goods and services
(7) $EX = EXP + vq$ 
Export spending
(8) $IM = IMP + mY - nq$ 
Import spending

The only essential difference is that investment spending now depends on the interest rate. The coefficient $b$ is the interest sensitivity of investment. Since income now depends on interest rates, which is endogenous, then solving equations (1)-(8) yields an equation of a line.

(9) $Y = \alpha[\bar{A} + EXP - IMP + (n + v)q - bi]$ $<IS \ curve>$
(9') $i = \frac{\bar{A} + EXP - IMP + (n + v)q}{b} - \left( \frac{1 - c(1 - t) + m}{b} \right)Y$ $<IS \ curve>$

This expression means that for lower levels of interest rates, investment, a component of aggregate demand, is higher, and thus income is also higher.

The monetary sector is unchanged from before.

<table>
<thead>
<tr>
<th>Eq.No.</th>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10)</td>
<td>$\frac{M^d}{P} = \frac{M^*}{P}$</td>
<td>Equilibrium condition</td>
</tr>
<tr>
<td>(11)</td>
<td>$\frac{M^*}{P} = \frac{M}{P}$</td>
<td>Money supply</td>
</tr>
<tr>
<td>(12)</td>
<td>$\frac{M^d}{P} = kY - hi$</td>
<td>Money demand</td>
</tr>
</tbody>
</table>

Substitute (11) and (12) into (10), and rearrange to obtain:
Equilibrium at any given time is given by the solution to (9) and (13), i.e., the intersection of the IS and LM curves.

Now we derive the $TB = 0$ schedule. This is built upon the Balance of Payments, except that now $KA$ (private capital inflows) are set to zero (or a constant). Recall the definition of the Balance of Payments accounting identity.

\begin{equation}
CA + KA + ORT = 0
\end{equation}

where the “balance of payments”, $BP$, is given by:

\begin{equation}
CA + KA = BP = TB + KA
\end{equation}

For $KA = 0$, when $TB = 0$, then $ORT = 0$. In other words, external equilibrium in this model holds when the trade balance is zero. Substituting in the expression for the trade balance (exports minus imports) into (15), one obtains:

\begin{equation}
TB = [(EXP + vq) - (IMP - mY - nq)] = 0
\end{equation}

\begin{equation}
Y = \left(\frac{1}{m}\right)[EXP - IMP + (v + n)q]
\end{equation}

Notice that the slope of this curve is infinite, and that anything that changes the autonomous components of exports, imports as well as the real exchange rate will change the position of the schedule.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{IS-LM-TB=0}
\caption{IS-LM-TB=0 in equilibrium}
\end{figure}
2. Fiscal and Monetary Policies (fixed exchange rates)

In the fixed exchange rate situation, \( q \) is not changed unless the government devalues or revalues the currency. This simplifies the mechanics of the model, and so we examine this situation first.

Shifts in the IS and LM curves occur for the same reasons as before. Consider what happens if one increases government spending.

\[\text{Fig. 2: Expansionary fiscal policy}\]

In this case, equilibrium income and interest rates rise. Notice that output is higher than that consistent with external equilibrium (i.e., \( TB=0 \)). As a consequence, the balance of payments is in deficit, so \( ORT > 0 \), and foreign exchange reserves are decreasing. In the absence of sterilized intervention, the LM curve will shift back in until output returns to its original level. This linkage between the trade deficit and the money supply is sometimes referred to as the Monetary Approach to the Balance of Payments (or the Mundell income reserve flow mechanism). To see how this works, recall that the money supply is a multiple of the “money base” (\( MB \), liabilities of the central bank):

\[ MB = Res + NDA \]

Where \( Res \) is the stock of foreign exchange reserves, and \( NDA \) is net domestic assets (usually the stock of government securities owned by the central bank). The money base is the central bank liabilities that are backed up by central bank assets (foreign exchange reserves and government securities).

Since the expansionary fiscal policy induces a trade deficit, then \( Res \) will decline over time. In the absence of offsetting increases in \( NDA \) (“sterilization”), then the money base will decline, reducing the money supply, and shifting the LM curve in, until the original level of income is restored. Note, however, if the central bank sterilizes the inflow, then the LM remains where it is – at least until foreign exchange reserves are exhausted.

It is instructive to consider what happens if a monetary expansion is undertaken.
Figure 3: Expansionary monetary policy

In this case, the resulting equilibrium interest rate $i_3$ is less than required for external equilibrium. As a consequence, there is a balance of payments deficit, ORT > 0, and foreign exchange reserves are decumulated. In the absence of offsetting sterilization by the central bank, the money supply shrinks, and the LM curve shifts back. This process stops only when the interest rate is back at $i_0$. In other words, the monetary policy is undone.

3. Devaluation

What happens when there’s a devaluation in the IS-LM=TB=0 model? This statement is the same as $\Delta q > 0$.

Figure 4: Devaluation
Upon devaluation, both the TB=0 and IS curves shift. For the TB=0 curve:

\[ Y = \left( \frac{1}{m} \right) \left[ EXP - IMP + (v + n)q \right] \]

Note: The fact that \( v \) and \( n \) are considered positive means that the Marshall-Lerner conditions hold.

The horizontal shift in the TB=0 curve is given by:

\[ \Delta Y = \left( \frac{1}{m} \right) \left[ \Delta EXP - \Delta IMP + (v + n)\Delta q \right] \]

since the autonomous components of exports and imports are constant:

\[ \Delta Y = \left( \frac{1}{m} \right) [(v + n)\Delta q] \quad (1) \]

What about the IS curve?

\[ Y = \left( \frac{1}{1 - c(1 - t) + m} \right) \left[ A + EXP - IMP + (n + v)q - bi \right] \quad \text{let} \quad \alpha = \left( \frac{1}{1 - c(1 - t) + m} \right) < \text{IS} \]

This movement horizontally is given by thinking about a change in the right hand side objects, holding the interest rate constant so \( \Delta i = 0 \):

\[ \Delta Y = \alpha \left[ \Delta A + \Delta EXP - \Delta IMP + (n + v)\Delta q - b\Delta i \right] \]

\[ \Delta Y = \alpha [(n + v)\Delta q] \quad (2) \]

Notice that for a change in the real exchange rate, the TB=0 shift in (1) and the IS shift in (2) differ only by the terms in front of the square brackets, then how far each curve shifts is a function of those terms. Since

\[ \alpha = \left( \frac{1}{1 - c(1 - t) + m} \right) < \left( \frac{1}{m} \right) \]

The TB=0 curve shifts further (in this case rightward) than does the IS curve.