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Some linear and nonlinear thoughts on exchange rates

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After discussing the inconclusive results of linear structural models as applied to exchange rates, this paper assesses the possibilities of using a particular form of nonlinear estimation, called Alternating Conditional Expectations, as: (i) a diagnostic tool, and (ii) a forecasting method. It contrasts the forecast performance of various linear (in levels, in differences, error correction) and nonlinear (in levels, in differences) specifications of a sticky-price monetary model, augmented by a relative wealth variable. The diagnostic results are as follows: the optimal transformations are almost always nonlinear, and often nonmonotonic. Forecasting results: in-sample and out-of-sample nonlinear forecasts yield substantial improvements over a random walk. However, in exercises with forecasts from rolling regressions, the random walk specification still dominates (over one-quarter forecast horizons), albeit only marginally and insignificantly so. Nonlinear specifications do slightly better than linear competitors at four-quarter horizons.

Exchange rates have proven depressingly difficult to track, even in *ex post* historical simulations. After initial success with quasi-structural monetary and portfolio balance models, it has become increasingly apparent that economists' understanding of what factors actually determined exchange rates is amazingly limited. Moreover, there is by now a plethora of econometric and simulation evidence that exchange rates are well approximated as martingale processes.

Recent research has shifted to nonstructural linear long-run relationships ('cointegration') and nonlinear relationships in the second moment, such as ARCH models (see Diebold, 1988). A new area of research is in nonlinear models of the exchange rate, focussing on the first moment (see Diebold and Nason, 1990; Meese and Rose, 1991; Schinasi and Swamy, 1989).

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The purpose of this paper is to discuss the prospects for linear and nonlinear modelling of the exchange rate, basing comparative results on estimations of variants of the sticky-price monetary model.[1] At this juncture, one may ask why nonlinear models are of interest. One answer is that theory provides us the set of 'fundamentals', but not the *form* of the relationships whereby the fundamentals determine the exchange rate. The superiority of the random walk in forecasting exercises such as Meese and Rogoff (1988) may not be so much an indictment of structural models, as much as one of *linear* structural models.

The plan of the paper is as follows. First the cointegration literature is briefly surveyed in Section I. The somewhat disappointing results there will provide the departure point for nonlinear approaches. Second, after discussing what types of economic models might yield nonlinear relationships, I implement an estimation technique which searches for optimal and possibly nonlinear functions in Section II. This is done via a procedure that searches for transformations that linearize the relationship between two or more variables. Third, in Section III, I then assess the comparative forecasting properties of several nonlinear specifications versus several linear specifications. Section IV summarizes the results.

I. Linearity in the long run

Recent work on linear exchange rate models has focused on cointegration, the idea of 'common trends' in macroeconomic time series as operationalized by Engle and Granger (1987).[2] Cointegration is intimately related to the error correction model (ECM) of Hendry *et al.* (1984), wherein only a proportion of the current period's disequilibrium is corrected in the next period.

These concepts are potentially of interest to researchers in international finance and open economy macroeconomics because, as evidenced in the work of Meese and Singleton (1982), exchange rates seem to follow random walks, and hence follow processes integrated of order one. However, first differencing the data to induce stationarity (and hence avoid 'spurious correlations') is not necessarily appropriate. For instance, a VAR in the first differences of the data is mis-specified if cointegrating relationships obtain between some of the data in levels. That is because running the regressions in first differences under such conditions imposes the restriction that changes in the dependent variable do not respond to disequilibria (Engle and Granger, 1987, p. 259).[3] Most empirical analyses of exchange rates, both real and nominal, have failed to find evidence of cointegration with the conventionally defined fundamentals (Baillie and Selover, 1987). An exception is Kaminsky (1987).[4]

In tracking the 1980–88 dollar rise and fall, real net wealth, measured as the sum of the cumulated current account and government debt, helps predict the path of the exchange rate.[5] This suggests alternative cointegrating variables than those indicated by the Dornbusch–Frankel and Hooper–Morton models examined by Boothe and Glassman (1987). Under the null that the first differences of both variables follow AR(4) processes, some positive results are obtained (Table 1).

Using the Augmented Dickey–Fuller (ADF) test, one finds evidence of cointegration at conventional significance levels for one case: the $/DM rate. It is somewhat surprising that more evidence of cointegration is not to be found.[6]

TABLE 1. Tests for cointegration.

Augmented Dickey–Fuller (ADF) tests on residuals (Data for 1974.2–1988.4; $T=59$)			
Regression	\$/DM rate	\$/¥ rate	DM/¥ rate
s vs. $(rw\text{-}rw^*)$	2.22	2.21	1.06
s vs. rw, rw^*	3.39[a]	2.29	2.44

Notes:
Critical values for ADF tests ($T=50$) under:
H_0: all processes follow random walks: 3.28 (10 per cent); 3.67 (5 per cent); 4.32 (1 per cent), for two series.
H_0: all processes follow ARI(4,1) processes: 2.90 (10 per cent); 3.29 (5 per cent); 4.12 (1 per cent), for two series.
H_0: all processes follow random walks: 3.73 (10 per cent); 4.11 (5 per cent); 4.85 (1 per cent) for three series.
H_0: all process follow ARI(4,1) processes: 3.36 (10 per cent); 3.75 (5 per cent); 4.45 (1 per cent) for three series.
s is the log exchange rate, rw is the log of real wealth, and * denotes a foreign country variable.
[a] Significant at 10 per cent or less under ARI(4,1) null.
Source for critical values: Engle and Yoo (1987).

However, it is important to recall that there are only 14 years of data in the set, probably too short a period to detect mean reversion. Frankel and Meese (1987) only find mean reversion in over a hundred years of data, a period that encompasses both fixed and flexible exchange rate regimes. Moreover, recall that the ADF test is under the null that the residuals are integrated. Hence the usual caveat applies—failure to reject the null hypothesis is not the same as acceptance of the null.

II. Nonlinear modeling: motivation and empirical application

II.A. Theory

Most rational expectations models of the exchange rate have as an implication that the current exchange rate is a *linear* function of current fundamentals (in which a random walk process drives the fundamentals). However, such implied linearities break down in some recent theoretical models in which the authorities are committed to some sort of target zone scheme.

Krugman (1988) and Froot and Obstfeld (1989) examine this case, where the form of the nonlinearities is fairly obvious. Froot and Obstfeld model the implications of reflecting and absorbing barriers[7] and, using techniques of stochastic calculus, find that the conventional formulation of exchange rates as a linear function of current fundamentals is only a special case of the more general model, wherein the exchange rate is linearly and nonlinearly related to the fundamentals. The greater the barrier credibility, or the tighter the band, the more 'S'-shaped the relationship between the exchange rate and the fundamentals. Krugman has termed this phenomenon the 'bias in the band.'[8]

II.B. Overview of econometric techniques

There is a tremendous literature on nonlinear estimation techniques. One useful

survey is contained in Hackl (1989). There are (at least) three major conceptions of nonlinearities: (i) nonlinearities due to discrete regime shifts; (ii) nonlinearities due to time-varying coefficients; and (iii) nonlinearities arising because the data generating process is inherently nonlinear.

The first sort of nonlinearity is associated with regime change models such as those originally suggested by Goldfeld and Quandt (1973). The basic problem lies in determining the timing of regime shifts endogeneously, rather than by imposing it *a priori*, using dummy variables. The relevant references here are Hamilton's work involving US GNP (1989), the term structure (1988), and (with Engel) exchange rates (1990).[9] One might wish to use this approach if the objective were to model the collapse of a target zone system, for instance.

The second nonlinearity has been dealt with recently in the context of Kalman filtering (Wolff, 1987) and stationary stochastic coefficient models (Schinasi and Swamy, 1989). Such models are consistent with aggregation problems, disturbances to the money demand functions and the impact of regime changes (*i.e.*, the operation of the Lucas Critique), to name but a few issues.

The third sort of nonlinearity is most consistent with the Krugman and Froot and Obstfeld models, and suggests nonparametric and semi-parametric estimation techniques.

II.C. Non-parametric approaches

Given the voluminous literature on non-parametric estimation, no survey will be attempted here. For a brief review of such approaches applied to economic examples, see Diebold and Nason (1990). It will be useful, however, to define some related terms in passing.

Locally weighted regression (LWR) is exactly what it sounds like: fitting a local regression surface to data via multivariate smoothing. Multivariate smoothing is a process wherein the dependent variable is 'smoothed' as a function of the independent variables, not unlike the way one computes a moving average (see Cleveland and Devlin, 1988). The procedure bears a close resemblance to 'nearest-neighbor' (NN) models in which the predicted value is a function of the average of the k-nearest neighbor observations to the ith observation. The difference is that LWR uses the predicted value from a locally fitted regression surface.

The Alternating Conditional Expectations (ACE) technique is due to Breiman and Friedman (1985a). ACE essentially transforms both the independent and dependent variables nonlinearly so as to linearize the relationship between the two transformed variables, using a NN type procedure.

One can see the analogy to least squares by observing that ACE minimizes the sum of squared residuals, where the (normalized) squared residual is:

$$\langle 1 \rangle \qquad e^2(\Theta, \phi_1, \phi_2, \ldots, \phi_p) = \frac{E\{[\Theta(Y) - \sum_{i=1}^{p} \phi_i(X_i)]^2\}}{E\Theta^2(Y)},$$

And the optimal transformations are the *functions* $\Theta^*, \phi_1^*, \ldots, \phi_p^*$ that minimize equation ⟨1⟩ such that ⟨2⟩ obtains:

$$\langle 2 \rangle \qquad e^2(\Theta^*, \phi_1^*, \ldots, \phi_p^*) = \min_{\Theta, \phi} e^2(\Theta, \phi_1, \ldots, \phi_p).$$

The procedure works iteratively, *alternating* between minimizing with respect to one function holding the other constant, and vice versa. Hence the process calculates conditional expectations alternating between the Y and the Xs, until the estimates converge.[10] The exact method of forming these expectations is performed by the Friedman and Stuetzle (1982) 'Super smoother,' which is similar to an equal-weighted LWR.

In a study related to the current one, Meese and Rose (1991) implement ACE as a diagnostic tool, and LWR as a means of comparing forecast performance. Using monthly data for five bilateral exchange rates and the fundamentals (where the fundamentals are relative money stocks, industrial production levels, real interest and inflation rates, and cumulated trade balances) over the floating rate period.[11] The implied transformations they find are highly nonmonotonic. Moreover, they fail to find evidence of cointegration between the linearized variables, thereby casting further doubt upon the validity of current theoretical models of exchange rate determination.

II.D. Some new ACE results

The current study uses quarterly data on the bilateral exchange rates between the USA and Germany and Japan. It differs from the Meese–Rose study in data frequency, variables and model specification. A modified sticky-price monetary model is examined below:[12]

$$\langle 3 \rangle \qquad s_t = \alpha_0 + \alpha_1(m - m^*)_t + \alpha_2(y - y^*)_t + \alpha_3(i - i^*)_t + \alpha_4(\pi - \pi^*)_t + \alpha_5(rw - rw^*)_t + \varepsilon_t + \text{seasonal dummies},$$

where:

s = log spot exchange rate (\$/foreign currency unit),
m = log nominal money,
y = log real GNP,
i = the interest rate,
π = the inflation rate, measured as annual change in log levels of prices,
rw = real net wealth measured as the sum of the cumulated trade balance and the central government debt owned by domestic agents,[13]
* denotes foreign country variables,

and

$$\alpha_1 > 0;\ \alpha_2 < 0;\ \alpha_3 < 0;\ \alpha_4 > 0;\ \alpha_5 < 0.$$

The α_is' signs correspond to implied slopes in Figures 1 and 2. Specifically, these figures are scatterplots of the posited determinants of the exchange rate along the abscissa plotted against the transformed variables along the ordinate. Only two graphs are shown in order to conserve space. Graphs of all the transformations are available from the author in a working paper version of this article. Table 2 presents each transformation's Sigma statistic, which measures the strength with which each independent variable enters into the equation, for both levels and first-difference specifications.[14]

The results of implementing ACE[15] are rather disappointing. For the \$/DM rate, the money to exchange rate relationship implies a generally positively sloped transformation for money (see Figure 1). Since the exchange rate transformation

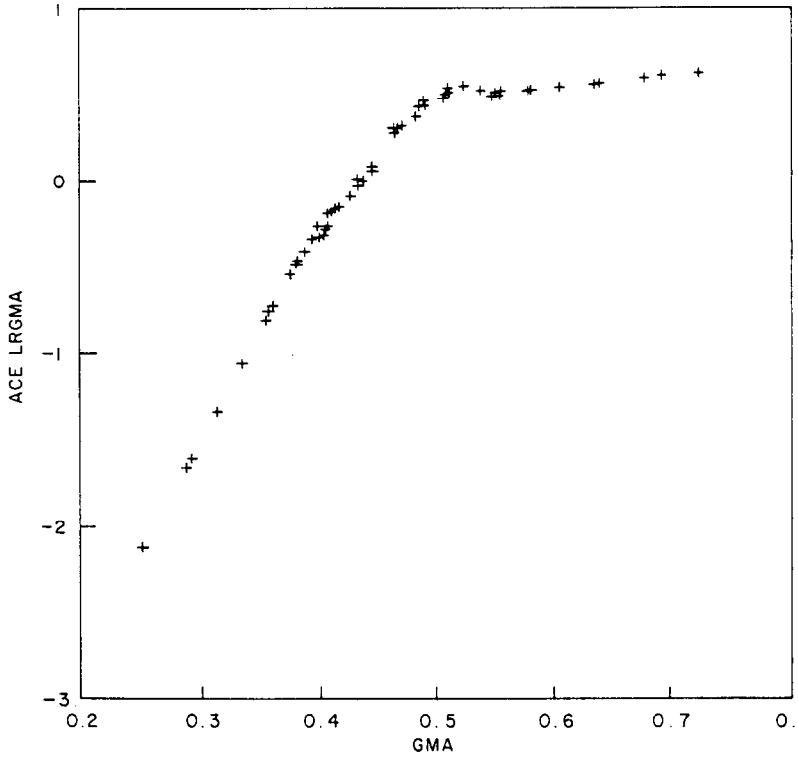

FIGURE 1. $/DM relative money stock transformation.

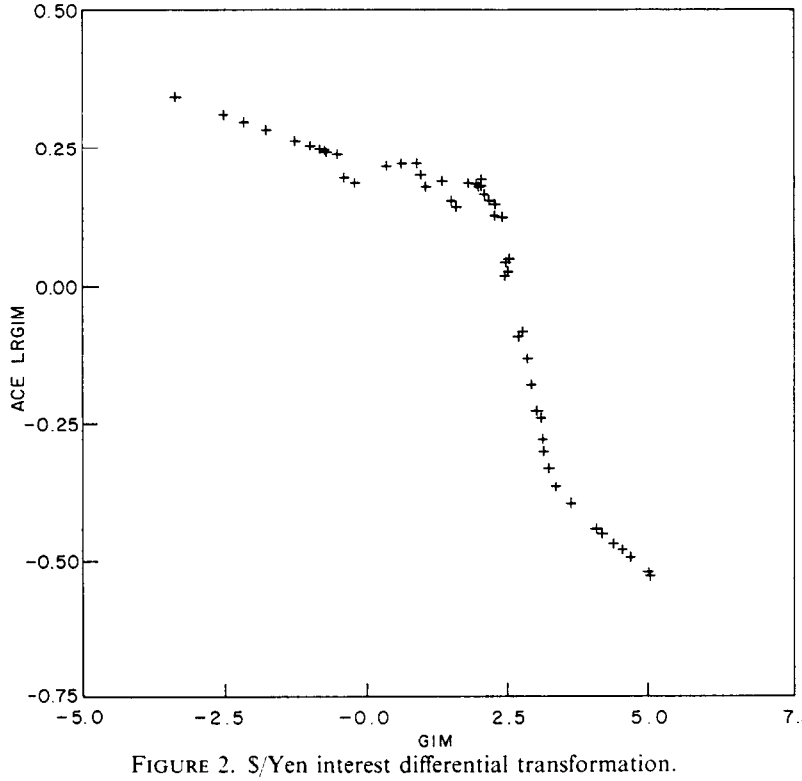

FIGURE 2. $/Yen interest differential transformation.

TABLE 2. Normalized sigma statistics.

Variable (in levels)	\$/DM	\$/¥	DM/¥
Money stocks	0.630	0.424	0.122
Incomes	0.153	0.770	0.824
Interest rates	0.271	0.306	0.358
Inflation rates	0.295	0.214	0.299
Real wealth stocks	1.233	1.491	0.197
Variable (in differences)	**\$/DM**	**\$/¥**	**DM/¥**
Money stocks	0.240	0.260	0.220
Incomes	0.008	0.310	0.173
Interest rates	0.210	0.329	0.258
Inflation rates	0.271	0.176	0.137
Real wealth stocks	0.279	0.324	0.267

Note: The sigma statistic indicates the relative explanatory power (standard deviation) of the transformed variable, where the standard deviation of the dependent variable has been normalized to unity. Sigma statistics are not comparable across equations.

is constrained to be linear, then this implies a positive slope parameter for low US to German money stock ratios, but no effect at high ratios. The income transformation is nonsensical, but is similar to implied transformations found when running ACE on variables with low associations. Note in particular the low Sigma (standard deviation) statistic. The interest rate (Figure 2) and inflation rate variable transformations indicate correctly sloping patterns with 'bumps.' On the other hand the relative wealth variable transformation appears almost linear, in the right direction and with a large Sigma statistic.

For the \$/¥ exchange rate, the money variable transformation is an inverted 'V'-shape. The income variable is monotonically *increasing* with the exchange rate, which also indicates wrong-signedness. The interest rate transformation appears somewhat in accord with conventional theory (downward sloping), but inflation rates are also essentially downward sloping. Relative wealth once again shows up with a linear transformation, strongly in the right direction.

Finally, with respect to the DM/¥ rate, one finds that the money and income transformations are essentially correct, the interest rate and inflation rate transformations are (weakly) in the wrong direction, and the implied wealth relation is a 'U' shape. This last relation also has a weak association as measured by the Sigma statistic of 0.197.

It is difficult to say what one can glean from these transformations. There is no concrete support for the theoretical model cited in Section II.A, since there are obvious sign reversals.[16] However, recall that the ACE procedure does not circumvent problems of simultaneity, and to the extent that there are central bank reaction functions that make monetary policy endogenous with respect to the exchange rate (*e.g.*, McNees, 1986; Hutchison, 1988), these results are not altogether unexpected. Moreover, it is interesting that ACE finds near linear transformations for the relative wealth variable that are in the right direction for two rates, and with the greatest effect, as measured by the Sigma statistic, buttressing arguments that that measure, at least, is cointegrated with the exchange rate.

III. Comparison of forecasting equations

III.A. Overview

The expected payoff to this investment in nonparametric techniques is, hopefully, better prediction and forecasting. The relevant benchmark is whether they can beat a naive random walk out-of-sample (Meese and Rogoff's, 1988, criterion). Thus far, the record has been mixed. In Diebold and Nason's study (1990), using one-step and multi-step-ahead forecasts, their autoregressive LWR forecasts often come close, but do not consistently beat, the naive random walk's.[17] In the Meese and Rose study, the LWR technique is used to implement several tests: First, to evaluate whether the in-sample fit is improved by allowing for nonlinearities; and second, to determine whether out-of-sample forecasts can be improved upon using LWR estimates. Briefly, while out-of-sample fit is marginally improved, the improvement is never significant.[18]

In order to assess the relative merits of the various estimation techniques and specifications *vis-à-vis* ACE, a number of tests will be employed. They fall into three categories:

1. In-sample *ex post* simulations.
2. Out-of-sample *ex post* forecasts.
3. One-step-ahead and four-step-ahead forecasts from rolling regressions.

In each case, the relevant metrics are the mean absolute error (MAE) and root mean square simulation error (RMSE),[19] although the rankings do not change much between each measure.

The various specifications include linear regressions in levels, with either static FITs or dynamic FORECASTs; linear regressions in first differences, with either static FITs or dynamic FORECASTs; nonlinear estimations on levels *or* first differences;[20] and error correction model regressions FITs.[21] The forecasts of the specifications in first differences are 'reintegrated' into levels before the comparisons of the MAEs and RMSEs are made.

A general note is in order here. Since the nonlinear forecasts never include a serial correlation correction, then these comparisons are biased in favor of the linear in levels (and the random walk models), since correction of highly autoregressive errors (rho differencing where rho ≈ 1) approximates inclusion of a lagged dependent variable.

III.B. In-sample regressions

The results of this exercise are shown in Table 3.[22] A caveat is in order here: in-sample comparisons are the weakest test of model specification since, by construction, predictions should match actual values fairly well. An exception to this critique are the cases where *dynamic* forecasts are used (with AR errors), as the errors are then allowed to cumulate over time.

For both the \$/DM and \$/¥ rates, the nonlinear model in levels has the lowest RMSE (excepting the static fit; see below). To a certain degree, this is not very surprising, since ACE performs something like LWR (via the Breiman–Friedman 'Super-smoother'), which is often accused of overfitting the data.[23] For the \$/DM rate, the RMSE is a full 43 per cent smaller than the dynamic forecast on levels. No other dynamic candidate comes close. (The best prediction is a *static* forecast,

TABLE 3. In-sample predictions.

Description	MAE	RMSE	Name	Comments
$ DM exchange rate, 1974.1–88.4 estimation and simulation period				
0. Random walk	0.050545	0.061691	RWG	No drift term
1. Linear FIT in levels	**0.040584**	**0.049692**	LPRDGIFT	AR1 correction
2. Linear F'CAST levels	0.067839	0.089399	LPRDG1FR	AR1 correction
3. Nonlinear FIT levels	0.043812	0.058090	NLPRDG1F	
4. Linear FIT in diffs.	0.089931	0.112539	LPRDG4I	Reintegrated
5. Nonlinear FIT diffs.	0.105206	0.125608	NLPRDG4I	Reintegrated
6. Error correction FIT	0.087490	0.104498	ECMG1I	Reintegrated
$/¥ exchange rate, 1974.1–88.4 estimation and simulation period				
0. Random walk	0.047022	0.059553	RWJ	No drift term
1. Linear FIT in levels	0.043715	0.049612	LPRDJ1FT	AR1, AR2
2. Linear F'CAST levels	0.075541	0.095281	LPRDJ1FR	AR1, AR2
3. Nonlinear FIT levels	**0.039199**	**0.047708**	NLPRDJ1F	
4. Linear FIT diffs.	0.115440	0.125778	LPRDJ4TI	Reintegrated
5. Linear F'CAST diffs.	0.159419	0.169222	LPRDJ4RI	Reintegrated
6. Nonlinear FIT diffs.	0.132484	0.167680	NLPRDJ4I	Reintegrated
7. Error correction FIT	0.076286	0.086481	ECMJ1I	Reintegrated
8. Error correction FIT[a]	0.062458	0.080397	ECMJ1I2	Reintegrated
DM/¥ exchange rate, 1974.1–88.4 estimation and simulation period				
0. Random walk	0.041003	0.053594	RWGJ	No drift term
1. Linear FIT levels	**0.037054**	**0.046819**	LPRDGJ1T	AR1
2. Linear F'CAST levels	0.058972	0.073268	LPRDGJ1R	AR1
3. Nonlinear FIT levels	0.044096	0.057219	NLPDGJ1F	
4. Linear FIT diffs.	0.057039	0.071033	LPRDGJ4I	Reintegrated
6. Nonlinear FIT diffs.	0.062857	0.078922	NLPDGJ4I	Reintegrated
7. Error correction FIT	0.055098	0.073606	ECMGJ1I	Reintegrated

Notes:

[a] Error correction model with error correction term lagged two periods.

The random walk specification is:

$$s_t = s_{t-1} + \varepsilon_t;$$

For the linear regression in levels:

$$s_t = \alpha_0 + \alpha_1(m-m^*)_t + \alpha_2(y-y^*)_t + \alpha_3(i-i^*)_t + \alpha_4(\pi-\pi^*)_t + \alpha_5(rw-rw^*)_t + \varepsilon_t;$$

For the first-difference specification:

$$\triangle s_t = \beta_0 + \beta_1\triangle(m-m^*)_t + \beta_2\triangle(y-y^*)_t + \beta_3\triangle(i-i^*)_t + \beta_4\triangle(\pi-\pi^*)_t + \beta_5\triangle(rw-rw^*)_t + \varepsilon_t.$$

For the error correction model specification:

$$\triangle s_t = \tau_0 + \tau_1\triangle(m-m^*)_t + \tau_2\triangle(y-y^*)_t + \tau_3\triangle(i-i^*)_t + \tau_4\triangle(\pi-\pi^*)_t + \tau_5\triangle(rw-rw^*)_t$$
$$+ \tau_6(m-m^*)_{t-1} + \tau_7(y-y^*)_{t-1} + \tau_8(i-i^*)_{t-1} + \tau_9(\pi-\pi^*)_{t-1} + \tau_{10}(rw-rw^*)_{t-1} + \tau_{11}s_{t-1} + \varepsilon_t.$$

where $\triangle$ denotes the first difference; *i.e.*, $(1-L)$.

The nonlinear in levels specification is:

$$\Theta(s_t) = \phi_1(m-m^*)_t + \phi_2(y-y^*)_t + \phi_3(i-i^*)_t + \phi_4(\pi-\pi^*)_t + \phi_5(rw-rw^*)_t + \varepsilon_t.$$

The nonlinear in differences specification is:

$$\Theta'(\triangle s_t) = \phi'_1\triangle(m-m^*)_t + \phi'_2\triangle(y-y^*)_t + \phi'_3\triangle(i-i^*)_t + \phi'_4\triangle(\pi-\pi^*)_t + \phi'_5\triangle(rw-rw^*)_t + \varepsilon_t.$$

The ACE transformations use a 33 per cent window and a restriction on the predictor transformations to be monotonic.

which benefits by virtue of the AR correction. Since the AR coefficient is near unity, then this specification is similar to including a lagged exchange rate term. Note how the simulated turning points lag the actual by one period.) The improvement is 25 per cent and 73 per cent for the DM/¥ and $/¥ rates respectively, using ACE on levels.

III.C. Out-of-sample forecasts

If the correlations derived from the regressions are spurious in the Granger and Newbold (1986) sense, then the forecasts are likely to go off track in the post-sample period. Hence, out-of-sample forecasts are a more rigorous test of competing specifications. In this exercise, the results of which are shown in Table 4, the forecasting period is 1986.1 to 1988.4.

The results are not consistent. For the $/DM rate, the linear forecasts perform best. The nonlinear in differences fit comes in third. More promising are the results for the $/¥ and DM/¥ rates. In the former, the nonlinear-in-differences forecast narrowly outperforms the linear forecast. In the latter, the nonlinear fit in levels is very narrowly beaten (4 per cent) by the linear in differences. The

TABLE 4. Out-of-sample forecasts.

Forecast description	MAE	RMSE	Variable	Comments
$/DM exchange rate, 1974.1–85.4 estimation period, 1986.1–88.4 simulation period				
0. Random walk	0.269796	0.288723		No drift term
1. Linear F'CAST levels	**0.074364**	**0.082993**	LPRDG6FR	AR1
2. Nonlinear FIT levels	0.307006	0.323509	NLPRDG6F	
3. Linear F'CAST diffs.	0.091133	0.104920	LPRDG8I	Reintegrated
4. Nonlinear FIT diffs.	0.120702	0.135007	NLPRDG7I	Reintegrated
5. Error correction FIT	0.899501	1.165735	ECMG3I	Reintegrated
6. Error correction FIT[a]	0.092606	0.142198	ECMG3I2	Reintegrated
$/¥ Exchange rate, 1974.1–85.4 estimation period, 1986.1–88.4 simulation period				
0. Random walk	0.321509	0.338079		No drift term
1. Linear F'CAST levels	0.234681	0.240838	LPRDJ6FR	AR1, AR2
2. Nonlinear FIT levels	0.404945	0.435748	NLPRDJ6F	
3. Linear F'CAST diffs.	0.239013	0.252651	LPRDJ8RI	AR1, Reint'd
4. Nonlinear FIT diffs.	**0.225805**	**0.233039**	NLPRDJ7I	Reintegrated
5. Error correction FIT[b]	0.453053	0.482588	ECMJ3I	Reintegrated
DM/¥ exchange rate, 1974.1–85.4 estimation period, 1986.1–88.4 simulation period				
0. Random walk	0.066026	0.079883		No drift term
1. Linear F'CAST levels	0.072557	0.083523	LPRDGJ6R	AR1
2. Nonlinear FIT levels	**0.052435**	0.062849	NLPDGJ6F	
3. Linear F'CAST diffs.	0.046314	**0.060453**	LPRDGJ8I	Reintegrated
4. Nonlinear FIT diffs.	0.134181	0.154945	NLPDGJ7I	Reintegrated
5. Error correction FIT	0.146030	0.170179	ECMGJ3I	Reintegrated

[a] Error correction term is at third lag.
[b] Error correction term is at second lag.

error correction specification sometimes yields the *worst* fits, although the forecast quality is very sensitive to the ECM lag structure. This is strange, since this specification nests both the levels and differences specifications (Hendry *et al.*, 1984). In all cases, a nonlinear specification outpredicts a multi-step-ahead random walk.

III.D. Rolling forecasts

The rolling regressions were run over a 12-year period, with the one-quarter-ahead forecasts running from 1986.1 to 1988.4 (Table 5) and with four-quarter-ahead forecasts 1986.4 to 1988.4 (Table 6). They were produced in the following manner (for the one-step-ahead forecasts): Estimates were made over the period 1974.1 to 1985.4. A forecast was made for 1986.1. Then another set of estimates were made for 1974.2 to 1986.1, and another forecast made for 1986.2. This process of estimation, forecast, moving up the sample period by one quarter, and then repeating, continues until the sample period of 1976.4 to 1988.3, and forecast period 1988.4. An analogous procedure was implemented for the four-quarter-ahead forecasts.[24]

The results for one-quarter ahead forecasts are presented in Table 5. For two of the exchange rates (\$/DM, \$/¥), the linear regression in levels does best, beating the random walk. However, it is important to recall that in any of these linear-in-levels regressions, the autoregressive terms are often the only significant coefficients. Hence, whatever explanatory power is in these equations, it is not derived from the estimated relationships of the fundamentals to the exchange rate.

The second best prediction for the \$/DM is provided by the rolling ACE procedure on the levels. This is interesting, as the procedure performs fairly well without any autoregressive correction of any sort. Moreover, the deterioration in RMSE is only marginal. The other specifications show worse performance.

A more formal statistical test of relative performance due to Granger and Newbold (1986) can be implemented. Under the assumption that the forecast errors are white noise, then one can derive a statistic distributed as t:

$$t_{T-2} = \mathrm{rho}(x, y)(T-2)^{1/2}/(1-\mathrm{rho}(x, y)^2)^{1/2},$$

where x is the difference between the benchmark forecast error and the relevant forecast error, y is the sum of the benchmark forecast error and the relevant forecast error, and rho is the correlation coefficient.

For the \$/DM rate, only the nonlinear-in-differences and ECM forecasts are significantly worse than a random walk. The nonlinear-in-levels forecast comes closest to having a zero t-statistic. For the \$/¥ rate, the nonlinear in differences forecast beats the random walk, although not significantly. The linear and nonlinear in differences forecasts have positive t-statistics; however, this is a result of the forecasts having a non-zero sample mean, rather than their being better forecasts than a random walk.

When the forecast horizon is four quarters ahead (see Table 6), the results are slightly more favorable to the nonlinear models. Specifically, the nonlinear in levels forecasts are best for the DM/¥ exchange rate, and the nonlinear in differences forecasts comes a close second to the linear in differences (with autoregressive errors) for the \$/¥ rate. This nonlinear specification also has a smaller mean error. Both of the nonlinear specifications do poorly for the \$/DM rate. Unfortunately, a test for the statistical significance of deviations from the

TABLE 5. Rolling regression forecasts (1974.1–85.4 to 1976.4–88.3, with one-quarter-ahead forecasts).

Forecast description	MAE	RMSE	Variable	Comments
$/DM exchange rate, rolling regressions				
0. Random walk	0.058828	0.069626	RWG	No drift term
1. Linear F'CAST levels	**0.056784**	**0.067997** (−0.49)	ROLLGPRE	AR1 correction
2. Nonlinear FIT levels	0.060427	0.069466 (0.00)	ROLLGACE	
3. Linear F'CAST diffs.	0.087870	0.094526 (−1.55)	ROLZGPRI	Reintegrated
4. Nonlinear FIT diffs.	0.102096	0.114308 (−2.12)**	ROLZGACI	Reintegrated
5. ECM FIT[a]	0.072124	0.060899 (−0.85)	ROLLGECI	Reintegrated
$/¥ exchange rate, rolling regressions				
0. Random walk	0.057778	0.067520	RWJ	No drift term
1. Linear F'CAST levels	0.065592	0.072983 (−0.32)	ROLLJPRE	AR1, AR2
2. Nonlinear FIT levels	0.070848	0.084763 (−1.21)	ROLLJACE	
3. Linear F'CAST diffs.	**0.044259**	**0.050255** (1.42)	ROLZJPRI	Reintegrated
4. Nonlinear FIT diffs.	0.170896	0.176752 (0.67)	ROLZJACI	Reintegrated
5. ECM FIT[b]	0.064561	0.057837 (−1.03)	ROLLJECI	Reintegrated
DM/¥ exchange rate, rolling regressions				
0. Random walk	**0.043625**	0.061930	RWGJ	No drift term
1. Linear F'CAST levels	0.048610	**0.060756** (0.08)	ROLLGJPR	AR1
2. Nonlinear FIT levels	0.059170	0.070700 (−0.63)	ROLGJACE	
3. Linear F'CAST diffs.	0.057485	0.071485 (−0.39)	ROLZGJPI	Reintegrated
4. Nonlinear FIT diffs.	0.092586	0.112293 (−0.67)	ROLZGJAI	Reintegrated
5. ECM FIT	0.076304	0.093826 (−1.50)	ROLGJECI	Reintegrated

Notes: The MAEs and RMSEs are for one-step-ahead forecasts. The numbers in the parentheses are t-statistics for the null hypothesis that the difference between the RMSE from the random walk and the respective forecast is zero. ** indicates significance at the 1 per cent level.

[a] Error correction term is at third lag.

[b] Error correction term is at second lag.

TABLE 6. Rolling regression forecasts (1974.1–85.4 to 1976.1–87.4, with four-quarter-ahead forecasts).

Forecast description	MAE	RMSE	Variable	Comments
$/DM exchange rate, rolling regressions				
0. Random walk	0.133347	0.158141	RWG4	No drift term
1. Linear F'CAST levels	**0.098097**	**0.121392**	RLLGPRE4	AR1 correctior
2. Nonlinear FIT levels	0.152047	0.179839	RLLGACE4	
3. Linear F'CAST diffs.	0.128456	0.151406	RLZGPRI4	Reintegrated
4. Nonlinear FIT diffs.	0.147379	0.172886	RLZGACI4	Reintegrated
5. ECM FIT[a]	0.105997	0.132312	RLLGECI4	Reintegrated
$/¥ exchange rate, rolling regressions				
0. Random walk	0.130831	0.141915	RWJ4	No drift term
1. Linear F'CAST levels	0.170449	0.185202	RLLJPRE4	AR1, AR2
2. Nonlinear FIT levels	0.193811	0.212187	RLLJACE4	
3. Linear F'CAST diffs.	**0.061965**	**0.077397**	RLZJPRI4	Reintegrated
4. Nonlinear FIT diffs.	0.072074	0.094135	RLZJACI4	Reintegrated
5. ECM FIT[b]	0.107191	0.130259	RLLJECI4	Reintegrated
DM/¥ exchange rate, rolling regressions				
0. Random walk	0.081034	0.099217	RWGJ4	No drift term
1. Linear F'CAST levels	0.073862	0.088866	RLLGJPR4	AR1
2. Nonlinear FIT levels	**0.072055**	**0.083512**	RLGJACE4	
3. Linear F'CAST diffs.	0.098714	0.112679	RLZGJPI4	Reintegrated
4. Nonlinear FIT diffs.	0.148980	0.163413	RLZGJAI4	Reintegrated
5. ECM FIT	0.191272	0.216504	RLGJECI4	Reintegrated

Notes: The MAEs and RMSEs are for four-step-ahead forecasts.
[a] Error correction term is at third lag.
[b] Error correction term is at second lag.

random walk performance is not available, since k-step-ahead forecasts exhibit MA forecast errors. Meese and Rogoff (1988) do provide a test analogous to that performed in Table 5, which is valid asymptotically, but for which there are insufficient observations here to implement.

IV. Conclusions

This paper has summarized some of the disappointing results of linear models of exchange rate determination, including those pertaining to the cointegration literature. While these results are not uniformly negative, they are sufficiently inconclusive to suggest alternative theoretical, and hence econometric models.

In applications of ACE, a nonparametric estimation algorithm, it appears that the estimated transformations are often nonlinear, but also sometimes non-monotonic, in contrast to the predictions of recent theoretical models. These nonlinear predictions are only consistently superior in-sample. Out-of-sample, nonlinear models occasionally yield the best forecasts. These two results suggest some overfitting by ACE. In one-quarter-ahead sequential forecasts, the random

walk still prevails, although only marginally so. In four-step-ahead forecasts, nonlinear models outperform random walk specifications, but not always the linear competitors. This result is somewhat encouraging.

There is another empirical regularity apparent in Tables 4 to 6. The best \$/DM and DM/¥ forecasts are always produced by specifications in log-levels. For the \$/¥, it is always a specification in first differences that predicts best. This suggests, but does not confirm, that the linear martingale model is not always the optimal model of the exchange rate.

Appendix

Description of variables

Note: All lowercase variables, except interest and inflation rates, are in log levels.

- m M1 equivalent (not seasonally adjusted), line 34 *International Financial Statistics*; or for the USA, where specified, M1A = (M1 − NOW accounts), *Federal Reserve Bulletin*.
- p Consumer Price Index, 1980 = 1.00, line 64 *International Financial Statistics*.
- y Real GNP (seasonally adjusted), in 1980 domestic currency, line 99a.r, *International Financial Statistics*.
- s End of period exchange rate, in \$/DM or \$/¥, log of inverse of line ae, *International Financial Statistics*.
- i US 3-year T-bills, line 61a, *IFS*; German Central Government bonds (4 years to maturity, max.), OECD; and Japan 7-year government bonds, OECD, *Main Economic Indicators*, and Bank of Japan, *Monthly Financial Statistics*.
- π $(\log(\mathrm{CPI}_t) - \log(\mathrm{CPI}_{t-4})) \times 100\%$.
- rw (real govt. debt + cumulated real current account). Debt is central government debt held by domestic agents (except for Germany, where total debt is used), line 88, *IFS*, deflated by the CPI; and Bank of Japan, *Monthly Financial Statistics*. Cumulated real current account is derived by converting the current account in \$ terms to domestic currency terms using the period average exchange rate (*IFS* line rf), then deflating by the CPI and cumulating on an estimated baseline of \$30.7 (nominal) bn for the USA and DM49.91 (nominal) bn at the end of 1972.4. A similar baseline is used for 1973.4 for Japan (\$169.427 bn = ¥46541.6 bn). These numbers are derived by subtracting off the money base and domestic debt from the baseline financial wealth figures provided in Frankel (1984, p. 258).

Notes

1. Those who believe that maximizing models such as Lucas (1982) are inherently superior should examine their empirical performance in Meese and Rose (1991). Based on their results, this study restricts itself to a variant of the Hooper–Morton model.
2. A vector x_t is said to be cointegrated of order $d-b\,[CI(d-b)$ where $d \geqslant b > 0]$ if all components of x_t are $I(1)$, and if there exists a cointegrating vector α such that

$$\alpha' x_t = z_t \sim I(d-b).$$

Note that α need not be unique.

The way cointegration has usually been interpreted is as a long-run equilibrium between two or more variables. Hence, deviations from equilibrium ('z_t') should be $I(0)$ in order for the concept to be sensible. An error correction model can be written as:

$$A(L)(1-L)x_t = -\tau z_{t-1} + u_t, \quad u_t \sim iid(0, \Omega).$$

Where $A(0) = I_n$ (the identity matrix), $A(1)$ has all finite elements, $\tau \neq 0$, and z_t is as defined before. (L is the backwards lag operator.)

3. Notice that the emphasis here is upon cointegration as a 'diagnostic,' rather than as a long-run equilibrium condition. That is, it is all too likely that we do not know the entire vector of cointegrating variables, if indeed it exists (see Swamy *et al.*, 1989). If an integrated element is omitted from the cointegrating regression, then the residual will appear to be integrated itself. Hence failure to reject the null of no cointegration may not necessarily invalidate the model. See Sims *et al.* (1990) for some general asymptotic results.
4. In a related result, Baillie and Bollerslev (1989) find cointegration among exchange rates, and between spot and forward rates. Francis Diebold has pointed out to me that, although this finding implies an ECM (incorporating the cointegrated variables) should be able to outpredict a random walk, this is not clearly apparent in his empirical investigations.
5. See Chinn (1989) for regression and in-sample forecast results to this effect.
6. The reverse cointegrating regressions were run, with no appreciable differences in the results.
7. Reflecting barriers are target zone boundaries which imply that the government authorities will intervene to change the fundamentals in the appropriate direction when the exchange rate touches a boundary. Absorbing barriers are those which trigger a fixed exchange regime when the exchange rate touches a boundary.
8. More recent work by Bertola and Caballero (1989) has suggested an inverted 'S' relationship in the context of the recurring EMS realignments.
9. Regime switching models go back to Goldfeld and Quandt (1973). Hamilton (1988, 1989) and Engel and Hamilton (1990) introduce a nonlinear filtering method which allows for two 'regimes,' each with its own mean, variance, and autoregressive parameters. Unlike previous switching regressions, the probability of being in a particular regime is a function of past observations and past imputed probabilities of being in a particular state, where the state variable is unobserved. As an aside, Breiman and Friedman (1985b, p. 616) observe that ACE, described below, *can* detect data regime shifts, as long as each regime has a 'good structure.' However, their definition of a regime is slightly different than Hamilton's.
10. For example, suppose there are two right-hand-side variables, X_1 and X_2. Then ACE would first calculate the expectations of Y conditional on X_1; then the expectation of X_1 conditional on Y. Then the process is repeated for X_2. This overall procedure is repeated until the transformation estimates converge. Breiman and Friedman show that although the transformations are estimated pairwise, the estimated transformations will converge to the multivariate transformations. Technically, 'The optimal transformations are characterized as the eigenfunctions of a set of linear integral equations whose kernels involve bivariate distributions' (Breiman and Friedman, 1985a: 581). The ACE procedure is contained in the Berkeley Interactive Statistical System (BLSS) software (Abrahams and Rizzardi, 1988, pp. 201–203).
11. The five-variable specification nests the Lucas model, the Flex-price and Sticky-price monetary models, the Hooper–Morton model, and (with appropriate quadratic terms) the Hodrick model. To accommodate nonstationarity in levels, they implement ACE on the first differences.
12. It is arguable that the cumulated current account and real government debt should enter into the equation separately. The coefficients on these variables in regressions in levels over the 1974.1 to 1985.4 period are not statistically significant, while the coefficients on the aggregate wealth variable are (except for the DM/¥ rate).
13. The standard caveat about Ricardian equivalence holds. In this model, real wealth enters in either because it enters into the money demand equation, or because it enters into consumption.
14. See Breiman and Friedman (1985a, p. 587). Since the standard deviation of the transformed dependent variable is normalized to unity, then each variables' Sigma is a measure of how strongly they enter into the determination of the dependent variable.
15. The ACE results reported here are on data in levels, with a 33 per cent of sample window, to control for small sample problems. Usually, the window size is chosen automatically by local cross-validation. While the ACE technique is developed for stationary time series, the results from data on first differences are not substantially different, in terms of signs and degree of ambiguity.
16. It is unlikely that such problems will be resolved by dealing with temporal ordering. Meese and Rose (1991) find that their results are essentially unchanged after reordering the data.

17. They use weekly data on ten major currencies. Since the data are of such high frequency, the regressors are lagged dependent variables.
18. However, the Hooper–Morton specification, which includes cumulated trade balances, comes close in both situations.
19. The RMSE is the generally accepted statistic, although the Theil U-statistic, the ratio of the model RMSE to a naive model RMSE is often reported instead. An anonymous referee has pointed out that comparisons of residual-variance measures yield correct conclusions about model accuracy only on average. See Theil (1971, pp. 544–545). The MAE statistic provides an appropriate statistic if the exchange rate is drawn from a stable Paretian distribution with infinite variance.
20. Note that the dependent variable transformation must be restricted here to be monotonic in order to obtain predicted values. Since theory predicts such transformations, this restriction is at least partially justified.
21. ECM FORECASTs were also generated, but they are not reported since they do not differ substantially from those in the tables below. The difference between the two is that the FIT uses the predictions of regressions on *actual* levels of the exchange rate. The FORECASTs use the *predicted* levels implied by the forecasted previous changes in the exchange rate. A nonlinear (in differences and levels) version of the ECM was also implemented, but yielded highly implausible transformations.
22. The graphs comparing the forecasts to the actual exchange rates are available from the author upon request.
23. For 60 observations, there are approximately 36 degrees of freedom, since the smoother uses about four degrees per transformation (Owen, 1983, p. 17).
24. An anonymous referee has pointed out that optimal forecasts need not include in the regression equation the most recent data. Moreover, such sequential estimation procedures cloud the distinction between nonlinearities in functional form and time variation in the parameters. Thus one interpretation of these results is that the sequential estimations are proxying for the latter effect.

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