Lecture 4 (Revised)
1. Mean Variance Optimization and the Risk Premium
2. Empirical Implementation (Frankel & Engel, 1984)
3. Some Intuition on the Consumption Based Approach
4. General Equilibrium Model: Stockman
5. Some Comments on the Redux Model

1. Mean Variance Optimization

1.1 A Simple *ad hoc* Two Country Model

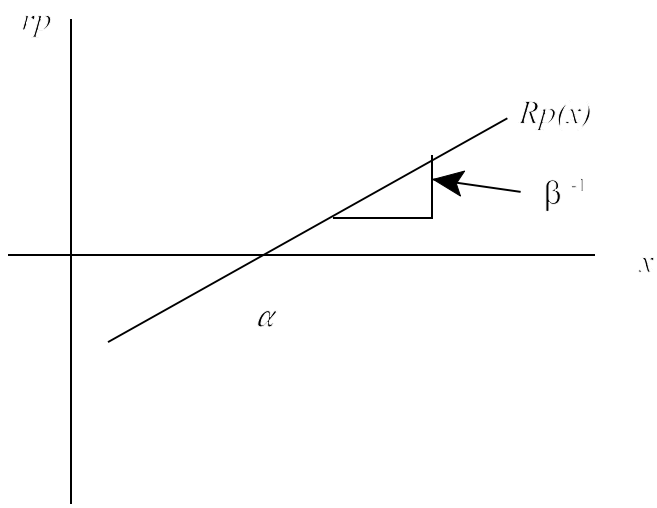
Arbitrarily define x as the share of wealth allocated to $ assets.

$$x = \alpha + \beta rp \qquad \textbf{(1)}$$

Rearranging and assuming asset supply equals asset demand,

$$rp = -\beta^{-1}\alpha + \beta^{-1}x \qquad \textbf{(2)}$$

One then obtains the following diagram:



Notice the similarity to the portfolio balance model of the exchange rate discussed earlier. Then one could say that if $\beta^{-1} = 0$, then $\beta = \infty$, and assets are perfectly substitutable. Graphically, this means that with perfect substitutability, there is a zero slope and intercept to this curve, and hence always a zero risk premium.

The problem is that this functional form is completely *ad hoc*. How does one obtain a rationale for this equation?

1.2 A Theory of Optimal Portfolio Diversification

Consider the one-period mean-variance optimization problem. The representative agent decides at the beginning of the period how to allocate his wealth among assets so as to maximize end of period wealth. For simplicity, consider the two country case (US and Germany):

$$\begin{aligned} W_{t+1} &= W(x(1+r^{\$}_{t+1}) + (1-x)(1+r^{DM}_{t+1})) \\ &= W(x(r^{\$}_{t+1} - r^{DM}_{t+1}) + 1 + r^{DM}_{t+1}) \end{aligned} \qquad \textbf{(3)}$$

where $r^{\$}$ is dollar returns on dollar denominated assets, and r^{DM} is dollar returns on DM denominated assets. Define relative returns, $rr^{\$}$, as

$$rr^{\$} \equiv r^{\$} - r^{DM}$$

Rearranging and taking the expectation,

$$E(W_{t+1}) = W(xE_t(rr^{\$}_{t+1}) + 1 + E_t r^{DM}_{t+1}) \qquad \textbf{(13)}$$

Calculating the variance yields

$$\begin{aligned} V(W_{t+1}) &= W^2(x\Omega x + Var_t(r^{DM}_{t+1}) + 2xCov(rr^{\$}_{t+1}, r^{DM}_{t+1})) \\ &\quad where \quad \Omega \equiv Var_t(rr^{\$}_{t+1}) \end{aligned} \qquad \textbf{(14)}$$

Assume the agent wishes to maximize

$$U(E_t W_{t+1}, Var_t(W_{t+1}))$$

This makes sense if the agent has quadratic utility, or the distribution of returns is log Normally distributed, so that only the first two moments matter.

Take the partial and set to zero,

$$\frac{\partial U}{\partial x} = 0 = U_1 \left(\frac{\partial E_t(W)}{\partial x} \right) + U_2 \left(\frac{\partial Var_t(W)}{\partial x} \right) \qquad \textbf{(15)}$$

Substituting yields

$$0 = U_1 W E_t(rr_{t+1}^{\$}) + U_2 W^2 (2x\Omega + 2Cov_t(rr_{t+1}^{\$}, r_{t+1}^{DM})) \qquad \textbf{(16)}$$

Recalling that the risk premium, rp, is

$$rp_t = E_t(rr_{t+1}^{\$})$$

means that by rearranging,

$$rp_t = -\frac{U_2}{U_1} W2(x\Omega + Cov_t(rr_{t+1}^{\$}, r_{t+1}^{DM}) \qquad \textbf{(17)}$$

or by rearranging,

$$rp_t = \rho\Omega x + \rho Cov_t(rr_{t+1}^{\$}, r_{t+1}^{DM})$$
$$where\ \rho \equiv -\frac{U_2}{U_1} W2 \qquad \textbf{(18)}$$

ρ is called the Arrow-Pratt coefficient of Relative Risk Aversion. Compare with the above equation (11), reproduced below:

$$rp = -\beta^{-1}\alpha + \beta^{-1}x \qquad \textbf{(11)}$$

Hence, $\beta^{-1} = \rho\Omega$, and depends upon the coefficient of relative risk aversion *and* the variance of relative returns. Thus if risk is high, *or* risk aversion is high, then substitutability is low and the slope in the diagram is steep.

2. Empirical Implementation (Frankel and Engel, 1984)

2.1 Specification

Let

$$x_t' = [x_t^{DM}, x_t^{L}, x_t^{Y}, x_t^{F}, x_t^{C\$}] \qquad \textbf{(0)}$$

(using the equation numbering system from the paper). Hence $(1 - x'_t\iota)$ is allocated to dollar assets. Writing out in vector notation the asset shares

$$x_t = \alpha + \beta(E_t r_{t+1} - \iota E_t r_{t+1}^{\$}) \qquad \textbf{(1)}$$

where the term in the parentheses is the expected real relative returns. Solving for expected returns:

$$E_t r_{t+1} - \iota E_t r_{t+1}^{\$} = -\beta^{-1}\alpha + \beta^{-1} x_t \qquad \textbf{(2)}$$

Impose rational expectations, since the left hand side of (2) is unobservable. Recalling

$$r_{t+1} - \iota r_{t+1}^{\$} = E_t r_{t+1} - \iota E_t r_{t+1}^{\$} + \epsilon_{t+1} \quad E(\epsilon_{t+1} | I_t) = 0 \qquad \textbf{(3)}$$

then one obtains:

$$r_{t+1} - \iota r_{t+1}^{\$} = -\beta^{-1}\alpha + \beta^{-1} x_t + \epsilon_{t+1} \qquad \textbf{(4)}$$

Now derive x_t in a manner analogous to before, but using matrix notation.

$$\begin{aligned} W_{t+1} &= W_t\,[x_t' z_{t+1} + 1 + r_{t+1}^{\$}] \\ \textit{where } z_{t+1} &\equiv r_{t+1} - \iota r_{t+1}^{\$} \end{aligned} \qquad \textbf{(5)}$$

$$E_t W_{t+1} = W_t\,[x_t' E_t z_{t+1} + 1 + E_t r_{t+1}^{\$}]$$

and

$$Var_t W_{t+1} = W_t^2[x_t' \Omega x_t + Var_t r_{t+1} r_{t+1}^{\$} + 2x_t' Cov_t(z_{t+1}, r_{t+1}^{\$})]$$
$$where \quad \Omega \equiv E_t(z_{t+1} - E_t z_{t+1})(z_{t+1} - E_t z_{t+1})'$$

Hence Ω is the variance covariance matrix of relative returns. Assume agents maximize

$$F[E_t(W_{t+1}),\ Var_t(W_{t+1})]$$

one obtains,

$$\frac{\partial F}{\partial x_t} = F_1 \frac{\partial E_t W_{t+1}}{\partial x_t} + F_2 \frac{\partial Var_t W_{t+1}}{\partial x_t}$$

Rearranging and substituting in

$$0 = F_1 W_t\ [E_t z_{t+1}] + F_2 W_t^2\ [2\Omega x_t + 2Cov_t(z_{t+1}, r_{t+1}^{\$})]$$

Again, define $\rho = -2W_t F_2/F_1$. Solving for $E_t z_{t+1}$:

$$E_t z_{t+1} = \rho Cov_t(z_{t+1}, r_{t+1}) + \rho\Omega x_t \qquad \textbf{(6)}$$

Rearranging

$$x_t = -\Omega^{-1} cov_t(z_{t+1}, r_{t+1}^{\$}) + (\rho\Omega)^{-1} E_t z_{t+1}$$

Compare this with equation (1).

$$x_t = \alpha + \beta(E_t r_{t+1} - \iota E_t r_{t+1}^{\$}) \qquad \textbf{(1)}$$

so:

$$\alpha \equiv -\Omega^{-1} Cov_t(z_{t+1}, r^{\$}_{t+1}) \quad \textit{min. variance portfolio}$$
$$\beta E_t z_{t+1} \equiv (\rho\Omega)^{-1} E_t z_{t+1} \quad \textit{speculative portfolio}$$

Notice that Ω enters in the coefficients, but is also in the variance covariance matrix of the error term ϵ.

2.2 Estimation and Results

A maximum likelihood estimation technique is used to impose the constraint between coefficients and *variances.* One could also impose the $\rho = 2.0$ (Samuelson's presumption), to get efficient estimates of parameters, but Frankel and Engel are more concerned with testing the joint hypothesis of mean variance optimization and rational expectations.

They reject the null hypothesis, using a likelihood ratio test comparing the constrained and unconstrained models.

It is important to understand what this result means. The rejection is one of the composite null hypothesis of rational expectations *and* that investors maximize over mean and variance. Thus, one rules out either more "naive" behavior (due to say, liquidity constraints), as well as more sophisticated behavior, such as that implicit in the consumption capital asset pricing model.

3. Some Intuition on the Consumption Based Approach

3.1. Motivation

Previously we discussed some of the reasons for the failure of the Mean-Variance optimization approach to the determination of the risk premium. One important possibility is that the form of the optimization is much more complicated, and is intertemporal in nature. The intertemporal approach yields the Euler equations familiar to us from domestic macroeconomics (e.g., Hall's consumption function as a random walk) and finance (the domestic consumption CAPM).

This approach assumes rational expectations and intertemporally optimizing

agents, with time-separable utility functions, and no liquidity constraints. The result is that asset returns should be correlated with the consumption beta, because a higher return is needed to compensate agents for the higher risk of assets which covary with consumption changes. In this sense, this approach is related to the Engel paper, since covariation with consumption is central.

2.2. Derivation

The following discussion is derived from Obstfeld (1990). Define the risk premium as:

$$(1+i*_t)\frac{E_t(S_{t+1})}{S_t} - (1+i_t) = -rp_t \tag{8}$$

Suppose agents maximize an intertemporal utility function:

$$Max\sum_i \beta^i u(c_{t+i}) \ \ s.t.\ \ wealth\ \ constraint\ \ w/\ \ risky\ \ assets \tag{9}$$

(See Blanchard and Fischer,1989: 507-508 for details).The resulting efficiency condition is (see Hodrick,1987: Chapter 2) for a derivation:

$$E_t(S_{t+1}/S_t) - \frac{(1+i_t)}{(1+i_t^*)} = -\frac{Cov_t(Q_{t+1},S_{t+1}/S_t)}{E_t(Q_{t+1})} \tag{10}$$

Where Q_{t+1} = [ß$u'(c_{t+1})/P_{t+1}$]/[$u'(c_t)/P_t$] ; and Cov(.) is a conditional covariance. The term on the right hand side of (10) is proportional to the risk premium defined in (8). If this conditional covariance is zero, then the risk premium is zero, and (10) collapses to uncovered interest parity:

$$\frac{(1+i_t)}{(1+i_t^*)} = E_t(S_{t+1}/S_t) \tag{11}$$

Equation (11) is easily tested using OLS regression (substituting the expected spot rate

with the actual), with the null hypothesis of a zero constant and a coefficient of unity on the interest differential. Obstfeld (1990) provides some recent estimates of this equation, which provides little evidence in support of UIP.

One can rewrite (10), by observing that $E_t(Q_{t+1}) = (1+i_t)^{-1}$ (since in equilibrium, the adjusted ratio of marginal utilities of consumption should equal the ratio of the discount and interest factors). To see this, assume prices are nonstochastic (!). Then $E_t(Q_{t+1}) = [E_t u'(c_{t+1})/u'(c_t)]ß(1+\pi)^{-1}$. We know from FOCs that the term in the square brackets [.] should equal $[ß(1+r_t)]^{-1}$, where r is the real interest rate. Hence, one obtains (12),

$$\frac{(1+i_t)}{(1+i_t^*)} = E_t[(S_{t+1}/S_t)Q_{t+1}(1+i_t)] \tag{12}$$

This equation indicates that the interest differential (which equals the forward discount under perfect capital mobility) will equal the expected depreciation, adjusted by a factor of covariance of consumption with returns -- the greater the correlation of depreciation with consumption reductions, the "riskier" the asset ($s here).

2.3. Empirical Implementation

Obstfeld (1990) tests equation (12), by assuming rational expectations (i.e., replacing the E[.] term with the ex post value), assuming an iso-elastic utility function with elasticity equaling 2, and ß equaling 0.985 on a per quarter basis.

He finds only slightly more evidence in favor of this hypothesis than in the case of UIP. However, it is far from overwhelming, and inspection of the data will show why. Prices move much too little to account for depreciation. Consumption can theoretically explain the variation in the exchange rate, but only with implausibly high estimates of the coefficient of relative risk aversion, since consumption moves very little relative to exchange rates. (The correct consumption variable is the service flow from consumer goods. However this is difficult to measure, so one has to resort to assuming separability

across durables, nondurables and services, and estimating the functions using each of the latter two separately.)

Here one has used "good" theory to motivate an empirical test: there is a representative agent that optimizes intertemporally. Yet this model does not appear to explain the risk premium very well at all.

One should not be surprised at the result. A large number of studies have shown that the Consumption CAPM does not do very well, and the same reason that explain the excess volatility of consumption may explain this result: liquidity constraints. Using more exotic utility functions does not necessarily results in better results, although some aspects of the risk premium are better explained (e.g., variability).

3. General Equilibrium Models: Overview

We now move to examine general equilibrium models of the exchange rate. The first, the Stockman (1987) model, is a stripped-down version of the "real" models presented in his more theoretical work. The basic idea is that money is a veil. Then the real exchange rate, defined as the relative price of home versus foreign goods, determines most of the movement in nominal exchange rates. This reverses the causality implicit in sticky-price monetary models represented by Dornbusch (1976) and Frankel (1979) approaches, wherein nominal exchange rate fluctuation, combined with sticky prices, induces real exchange rate variability.

The Stockman approach, which relies on underlying preferences and technologies to define real exchange rate levels, also has the advantage of being able to show how the real exchange rate evolves in response to various shocks, in terms understandable to the typical microeconomist.

The Lucas (1982) model, which is a more rigorous exposition of this approach, adds a stochastic element to the modeling of exchange rates, and asset prices more generally. This is particularly important for issues of asset pricing, such as equities and the forward exchange rate. Of course, it is this latter variable that is of interest to us.

Since the forward rate may deviate from the expected future spot rate for reasons usually associated with risk, one must have a framework in which to assess risk; this in turn requires a description of the stochastic processes governing the economies. So far, this description has been missing from our discussion of asset pricing.

The basic Stockman model is a static general equilibrium model. It assumes: (A0) one period of time; (A1) two countries identical except where indicated; (A2) two perishable goods, X (home) and Y (foreign), inelastically supplied, in perfect competition; (A3) no barriers to trade; households have the same tastes, and gain utility from the consumption of both goods; (A4) both country's households are equally wealthy.

(A4) produces the result that world supplies of X and Y can be divided by world population to obtain per capita supplies x^s and y^s. (A3) and (A4) imply that households have the same preferences regarding x and y, consuming the same amounts. The equilibrium price (but not quantities) is then determined by preferences. The equilibrium relative price of good Y in term of good X, π_y, is given by the usual maximizing condition, and is called the *real exchange rate* in this model. Note that this is different than the real exchange rate conventionally defined in international finance. That is, usually we compare identical baskets of goods; here the two goods are explicitly different.

We need to make some statements about the monetary sector in order to be able to discuss nominal exchange rates. Hence, (A5) The nominal money supplies of domestic and foreign moneys ($ and £) are denoted by M^S and M^{*S} and are exogenously set by the respective central banks. (A6) defines the money demand equations as:

$$\begin{aligned} M^d/p_x &= \alpha \\ M^{*d}/p_y^* &= \alpha^* \end{aligned} \tag{8}$$

where p_x is the nominal dollar price of good X, p_y is the nominal Pound price of good Y, and α (α^*) are real money demand in terms of the respective goods X(Y), and are treated as constants. While (1) looks like *ad hoc* money demand equations, they can represent

cash-in-advance money demand functions. An interesting aspect of the money demand equations in (1) is that the real quantities demanded are independent of the income level.

Solving (1), setting money supplies equal to money demands, yields:

$$\begin{aligned} p_x &= M^s/\alpha \\ p_y^* &= M^{*s}/\alpha^* \end{aligned} \tag{9}$$

The nominal exchange rate enters in because of the relative price of *X* and *Y*. Since *X* and *Y* are in terms of Dollars and Pounds, respectively, they must be converted to a common currency in order to have a meaning of a relative price:

$$\pi_y = ep_y^*/p_x \tag{10}$$

where *e* is the nominal spot exchange rate, in terms of \$/£. Substituting (2) into (3) provides an equation for the exchange rate:

$$e = \frac{M^s\alpha^*}{M^{*s}\alpha}\pi_y \tag{11}$$

Taking logs of this expression, one obtains

$$\begin{aligned} s &= (m - m^*) - \log(\alpha/\alpha^*) + \log(\pi_y) \\ s &\equiv \log(e) \end{aligned}$$

where lowercase *m*'s stand for the log(*M*)'s. Due to the nature of the money demand equation (and the implicit cash-in-advance constraint), there is no interest rate variable in the equation, nor is there an income (scale) variable. On the other hand, there is a shift variable [$\log(\pi_y)$] , which in a conventional monetary model would be interpreted as a deviation from PPP, and would not show up in a flex-price monetary model.

To obtain a role for income, assume (5):

$$M^d/p_x = \alpha x^s$$
$$M^{*d}/p_y^* = \alpha^* y^s \qquad (5)$$

Then revising equation (4) accordingly one obtains

$$e = \frac{M^s \alpha^* y^s}{M^{*s} \alpha x^s} \pi_y \qquad (4")$$

which in log terms looks like:

$$s = -\log(\alpha/\alpha^*) + (m - m^*) - (\tilde{y} - \tilde{y}^*) + \log(\pi_y)$$
$$s \equiv \log(e)$$

where the y tilde's denote GDPs. Notice that as x^s rises, π_y also rises as before, since more of the home good, holding constant foreign, should induce a price change. The more inelastic demand for the home good (the poorer a substitute the home good is for the foreign) the bigger the price drop. Stockman terms this the "relative price effect". On the other hand, as x^s in equation (4") rises (or y-tilde rises in the log version), the "money demand effect" on the exchange rate is negative. Hence there are two countervailing effects.

What is the net effect? From equation (5), one knows that the income elasticity of money demand is unity; however, $\partial\ln(\pi_y)/\partial\ln(x)$ is unknown. If this object is greater than unity because of low substitutability between x and y, then the overall effect of higher GDP is a *weaker* currency. When $\partial\ln(\pi_y)/\partial\ln(x) < 1$ (x and y are good substitutes) the standard effect of higher income leading to a stronger currency, familiar from the flex-price monetary model, holds. However, only when $\partial\ln(\pi_y)/\partial\ln(x) = 0$ (perfect substitutability) will the exact *quantitative* result from this model hold.

The key distinction in this model vis a vis a sticky-price monetary model is that the key driving variable in determining real exchange rates is the π_y variable. To see this, consider the Dornbusch model. In that framework, a nominal exchange rate movement

gets translated into a deviation from PPP because prices are sticky. In the Stockman model, prices are perfectly flexible, so that movements in real exchange rates get translated into movements into the nominal exchange rate. Movements in the nominal magnitudes (such as *M*) have an impact on the nominal exchange rate, but not the real. Hence, this model possesses the Classical dichotomy between nominal and real variables.

The next portion of the paper is involved with a discussion of how π_y variable changes with changes in wealth holdings. Wealth reallocations have an effect in this simple model if households do not have homothetic utilities (i.e., as wealth rises, they prefer to consume *X* and *Y* in different proportions).

The rest of the paper discusses various complications of the model that can explain a greater than unity elasticity of the nominal exchange rate with respect to the real rate, and also the incorporation of intertemporal aspects.

1.2. The Intertemporal Dimension

In order to discuss the model in the static context, as Stockman did in the first three sections of the paper, he had to make special assumptions regarding wealth. In particular, (A7) assumed that foreigners held exactly as much as domestic residents of each and every firm. Hence, as endowments changed, wealth changed exactly equally for domestic and foreign residents.

If (A7) is dropped, a different assumption is required to replace it. Stockman proposes one that is perhaps more realistic, although it is hard to explain why households would voluntarily choose to allow it: (A8): (i) Firms in each country are owned entirey by households in that country. (ii) the utility function is homothetic, i.e., the relative amounts of good *X* and *Y* consumed depend only on the relative price of *X* and *Y*. Using this assumption, the relative price of *X* and *Y* do not change as one country's wealth increases relative to the other's. Then if the home country gets a productivity increase (or an endowment increase), of amount Δs, home country's consumption rises in both goods (although proportionately less in *Y*) while the foreign country's consumption of *Y* falls and *X* rises somewhat. Relative prices have changed only because of the change in

relative endowments of *X* and *Y*, not because of wealth reallocations. Notice further that the real exchange rate depreciates for the home country.

Additional complications arise if one respecifies the money demand equation as:

$$\begin{aligned} M^d/p_x &= f(x^s) \\ M^{*d}/p_y^* &= f^*(y^s) \end{aligned} \qquad \textbf{(2')}$$

so that money demand depends upon GDP. The α's are then inverses of velocity. Rearranging as before and solving for *e* (but reinstating (A7)), one obtains:

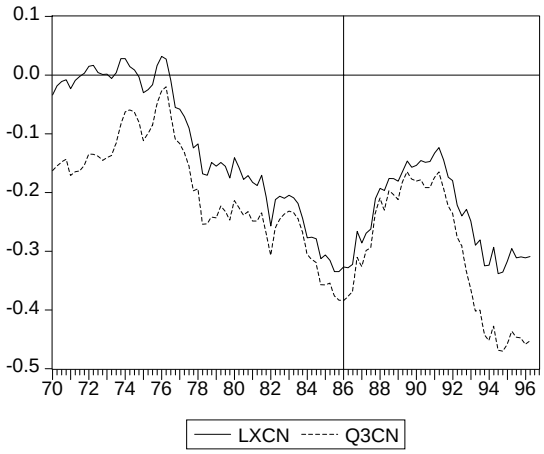
$$e = \frac{M^s f^*(y^s)}{M^{*s} f(x^s)} \pi_y \qquad \textbf{(4')}$$

In the case of (4'), a increase in home good endowment or supplies has two countervailing effects on the exchange rate. The increase in the supply of *X* depreciates the home real exchange rate, as before. However, because money demand depends positively upon home GDP (Xs) then this effect tends to appreciate the home currency. Stockman calls the first effect the "relative price effect", and the second, the "money-demand effect".

1.3. Evidence (Updated)

Stockman presents a number of graphs showing the high correlation between real and nominal exchange rates. He argues that since the changes in both real and nominal exchange rates appear to be nearly permanent (near unit root processes, in the modern lexicon) it must be the case that the underlying shocks are real, and not monetary, since monetary shocks should die out rather quickly.

Below, I provide updated graphs of the nominal and real (GDP deflator adjusted) exchange rates for the Canadian, German and UK bilateral exchange rates against the US. I also provide graphs for the French, Italian and Japanese rates.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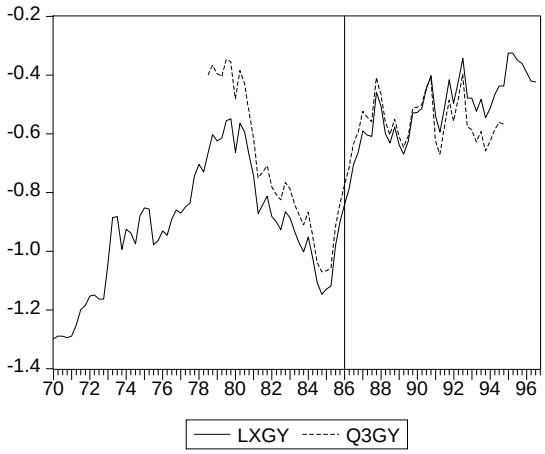
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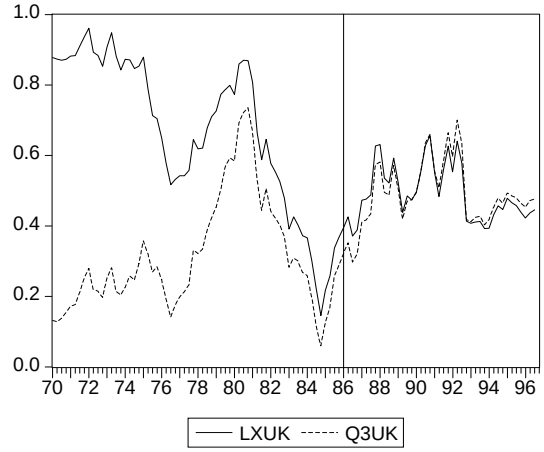
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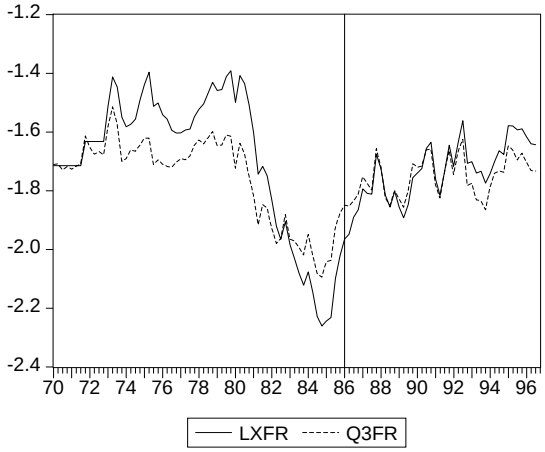
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Q1JP

It is interesting to note that Stockman broke each of the series into two graphs. Doing so makes the deviations of the real from nominal less obvious. Furthermore, the choice of series to plot is of some interest. The French series, in particular, look like it contains a substantial transitory component in the nominal exchange rate.

5. Some Comments on the Redux Model (notation from 1995 *JPE* article)

We've examined two broad sets of models that fall under the general rubric of monetary models of the exchange rate. The first set includes the rational expectations models derived during the mid- to late 1970s that relied upon *ad hoc* money demand formulations (Frenkel, 1978; Dornbusch, 1976). The second encompasses the general equilibrium models the 1980s, which incorporate, to varying degrees, microfoundations for holding money. The Lucas (1982) model directly motivated money's role through a cash-in-advance motivation, while the Stockman paper alluded to it. While money can be introduced through a variety of means besides the cash-in-advance motivation (the money-in-utility function is another popular approach), the theoretical results are not usually sensitive to this aspect. Money typically acts as a veil, although the monetary neutrality result does not necessarily hold in the money-in-utility function approach even when prices are perfectly flexible.

A key distinction between the *ad hoc* rational expectations models of the 1970s and the optimizing models of the 1980s is that the latter typically eschewed "sticky prices". The Obstfeld-Rogoff model can be viewed as a synthesis of the rigor of the intertemporal optimizing approach, with microfoundations for money, and assumed price

stickiness of a particular nature. The motivation for this synthesis is obvious. In the monetary approach, the *ad hoc* money demand equations precluded investigation of aspects of intertemporal optimization (and hence welfare analysis), while in the general equilibrium approach, the absence of price stickiness provided a wealth of counterfactual implications. The theoretical framework will be familiar, as it is essentially an open economy version of the Blanchard-Kiyotaki (1987) model.

Sticky prices are introduced by making $p(h)$ and $p^*(f)$ predetermined, *i.e.*, they are set one period in advance but can be adjusted fully after one period. Output is now demand determined for sufficiently small shocks. The reasoning is as follows: monopolistically competitive firms set price above marginal cost. When unanticipated demand occurs, each additional unit sold will provide positive profit, when sold at a preset price.

where the linearized PPP relationship in (23), which holds in the short and long run The exchange rate equation in this model is:

$$\hat{E} = (\hat{M}-\hat{M}^*) - \frac{1}{\epsilon}(\hat{C}-\hat{C}^*) + \frac{\beta}{(1-\beta)\epsilon}(\bar{\hat{E}}-\hat{E})$$

where consumption differentials stand in for income differentials, and looks very similar to a flex-price monetary model. The key difference to remember is that the consumption differentials are *endogenous* in this model, due to the presence of predetermined prices.

In this model:

$$\hat{E} = (\hat{M}-\hat{M}^*) - \frac{1}{\epsilon}(\hat{C}-\hat{C}^*) \tag{52}$$

since for a permanent money shock current money equals steady state money.

Thus, in this model, the exchange rate jumps to its immediate long run value, even though prices are fixed. The intuition is as follows: *If consumption differentials are expected to be constant, then agents must expect a constant exchange rate*. All

adjustment takes place after one period, because prices are only predetermined for one period. They are free to adjust thereafter, to obtain the long run solution.

The general solution for the exchange rate is:

$$\hat{E}_t = \left(\frac{(1-\beta)\epsilon}{\beta+(1-\beta)\epsilon}\right) \sum_{s=t}^{\infty} \left(\frac{(1-\beta)\epsilon}{\beta+(1-\beta)\epsilon}\right)^{s-t} (\hat{M}_s - \hat{M}_s^*) - \frac{1}{\sigma\epsilon}(\hat{C} - \hat{C}^*) \qquad \textbf{(53)}$$

So far we only have half the story. We need an expression that determines the consumption differential:

$$\overline{\hat{C}} - \overline{\hat{C}^*} = \frac{\bar{r}(1+\theta)}{2\theta}[(\hat{y} - \hat{y}^*) - (\hat{C} - \hat{C}^*) - \hat{E}]$$

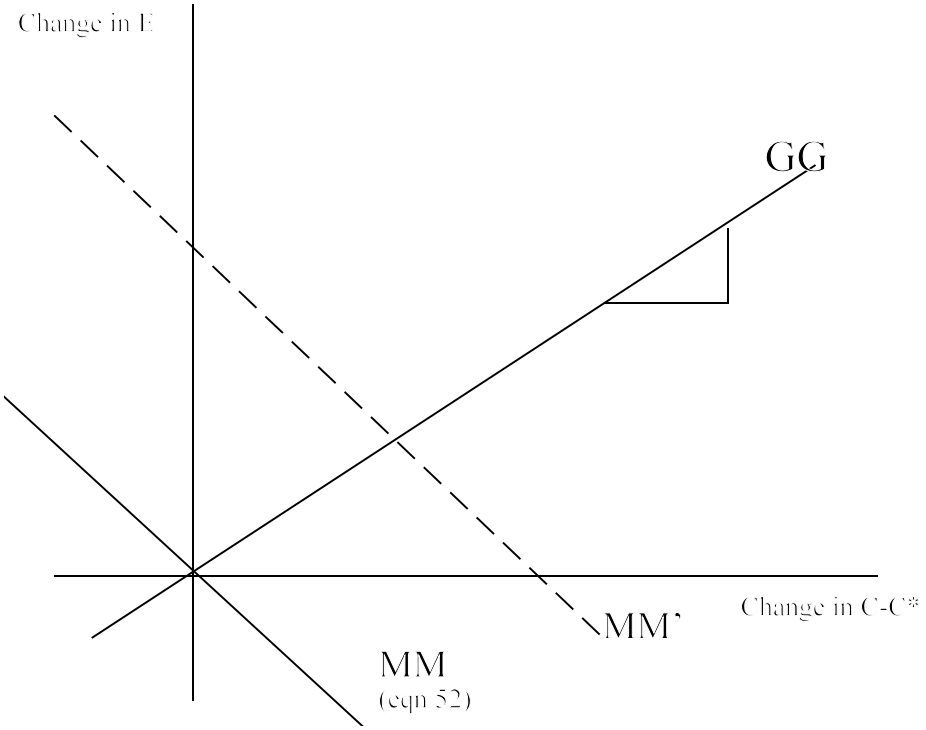
With sticky prices, the output differential is given by:

$$\hat{y} - \hat{y}^* = \theta\hat{E}$$

Combining these last two equations with equations, the two-country consumption Euler equation,

$$\hat{E} = \frac{\bar{r}(1+\theta)+2\theta}{\bar{r}(\theta^2-1)}(\hat{C} - \hat{C}^*) \qquad \textbf{(54)}$$

Equation (54) shows that the currency depreciation necessary for home output to rise enough to justify a permanent consumption differential. An upward slope results since higher output is associated with higher depreciation. Equations (52) and (54) are graphed in Figure 1.



Short run equilibrium is given by the intersection of the MM and GG curves. A given monetary increase induces a less than proportionate depreciation, in both the short and long run (we know this because the exchange rate is constant after the initial movement).

The nominal depreciation, coupled with sticky prices in the first period, causes a change in relative prices of individual goods (even though PPP holds in the aggregate). This causes an increase in domestic output resulting from the switch of production toward domestic goods. The higher income causes a current account surplus as agents attempt to smooth their consumption.

A special case arises when $\theta \to \infty$. Then by inspection of equation (54), one can see that the GG curve becomes horizontal. An increase in money causes no change in $\hat{E}$. With domestic and foreign goods perfect substitutes (that's what $\theta \to \infty$ means), ***and*** prices predetermined, the monetary expansion's effects are manifested completely in increases in domestic output. The exchange rate need not adjust.

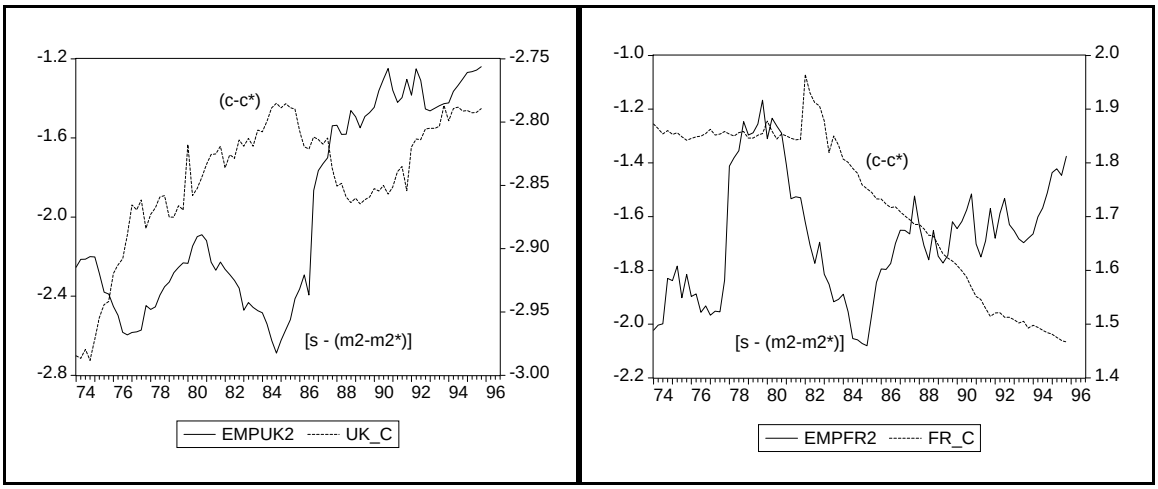
This model has the following implications.

- First, there is no overshooting, or undershooting, of the nominal exchange rate, despite the presence of predetermined (sticky) prices.
- Second, consumption is jointly determined with the nominal exchange rate.
- Third, the exchange rate will not respond at all if domestic and foreign goods are perfect substitutes (LOP holds). The *reduced form* expressions (endogenous variables expressed as a function of exogenous variables) for the exchange rate and the consumption differential are given by the following two equations:

$$\hat{E} = \frac{(\bar{r}(1+\theta)+2\theta)\epsilon}{\bar{r}(1+\theta)(\theta-1)+(\bar{r}(1+\theta)+2\theta)\epsilon}(\hat{M}-\hat{M}^*)$$

$$\hat{C} - \hat{C}^* = \frac{(\bar{r}(1+\theta)+2\theta)\epsilon}{\bar{r}(1+\theta)(\theta-1)+(\bar{r}(1+\theta)+2\theta)\epsilon}(\hat{M}-\hat{M}^*)$$

In principle, one can estimate the relationships in these two reduced form equations, assuming all monetary shocks are permanent. However, a more fruitful course is to examine the relationship in equation (52). A cursory examination of the empirical evidence (Figures 4-6) indicates that there is a lot of variation in nominal exchange rates not captured by this model. However, the chief virtues of this model should not be considered its empirical implications. Rather, it is that it combines microfoundations and intertemporal optimization, with a realistic feature of the world -- sticky prices. In doing so, it also contains implications for other macro variables including the current account.



US-UK US-France

Table 1: DOLS Estimates of Consumption Coefficients

Coeff	CN	FR	UK[b]
c-c*	-0.107	-0.403**	-0.494†
	(0.456)	(0.169)	(0.366)
R^2	.184	.131	.907
N	86	86	27
Smpl	74.1-95.2	74.1-95.2	74.1-95.2

Notes: Regressand is log exchange rate minus log(M2) ratio. Coefficients from Dynamic OLS (Stock and Watson, 1993, DOLS1 estimator), with lags from +2 to -2 of first differences of consumption, money. †{*}(**)[***] indicates significance at the 20%{10%}(5%)[1%] MSL. N is the effective number of observations included in the second stage regression.
a/ Regression includes seasonal dummies.
b/ Regression includes dummy for M2 redefinition.

One key attribute that has not been stressed in this discussion is that of welfare analysis. Typical monetary models of the exchange rate, which rely upon ad hoc money demand equations, are not amenable to conducting welfare analysis. That is because the utility function does not appear in these models. However, because the Obstfeld-Rogoff model is built upon a utility function, welfare analyses *can* be conducted.

Key examples of work building upon this framework include Obstfeld and Rogoff (1998), Corsetti, Pesenti, Roubini and Tille (1998), and Devereux and Engel (1998). In the first case, Obstfeld and Rogoff use a stochastic version of the model to evaluate the determinants of the risk premium. The second paper evaluates the implications of competitive devaluations under a variety of assumptions (Law of One Price vs. Pricing to Market). The final paper evaluates the optimal exchange rate regime (fixed or flexible) under differing pricing regimes.

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Lecture4
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