Lecture 3, Part 1 (Bubbles, Portfolio Balance Models)

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1. Rational Bubbles in Theory

In this portion of the lecture, I refer to Obstfeld and Rogoff *Foundations of International Macroeconomics*, Section 8.2, and use their notation. The objective is to show how bubbles show up in the derivation of rational expectations/perfect foresight models. In this case, the variable of interest is the price level; however by PPP, this is trivially different from the exchange rate.

Suppose there is a stable money demand equation:

$$(m_t - p_t)^d = -\eta i_{t+1} \qquad \textbf{(1)}$$

where the absence of *y* indicates that income has been held constant, and its effect subsumed into a constant (suppressed for convenience). Notice further that this notation differs from previous, in that the interest rate for period t is denoted by i_{t+1}. By the Fisher relation, this conventional money demand equation can be re-written as a Cagan money demand equation:

$$m_t^d - p_t = -\eta E_t(p_{t+1} - p_t) \qquad \textbf{(1)}$$

where once again, the r_t term has been subsumed into a constant, and suppressed for notational ease.

Set:

$$m_t^d = m_t \qquad \textbf{(2)}$$

that is, there is money market equilibrium and money supply is set exogenously. Further

assume the process defining money demand is nonstochastic (for the moment):

$$m_t - p_t = -\eta(p_{t+1} - p_t) \qquad \textbf{(3)}$$

Solving for p_t yields:

$$-p_t = -m_t - \eta p_{t+1} + \eta p_t$$
$$(1+\eta)p_t = m_t + \eta p_{t+1}$$

$$p_t = \left(\frac{1}{1+\eta}\right)m_t + \left(\frac{\eta}{1+\eta}\right)p_{t+1} \qquad \textbf{(6)}$$

Notice that this is a recursive expression; it can be rewritten for the price level, lead by one period, viz.

$$p_{t+1} = \left(\frac{1}{1+\eta}\right)m_{t+1} + \left(\frac{\eta}{1+\eta}\right)p_{t+2}$$

Substituting this latter equation into equation (6) yields:

$$p_t = \left(\frac{1}{1+\eta}\right) \times \left[m_t + \left(\frac{\eta}{1+\eta}\right)m_{t+1}\right] + \left(\frac{\eta}{1+\eta}\right)^2 p_{t+2}$$

this leads, via repeated substitution to:

$$p_t = \underbrace{\left(\frac{1}{1+\eta}\right)\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t} m_s}_{\textit{Fundamentals}} + \underbrace{\underset{T\Rightarrow\infty}{limit}\left(\frac{\eta}{1+\eta}\right)^{T} p_{t+T}}_{\textit{Bubble Term}} \qquad \textbf{(7)}$$

Clearly, equation (7) differs from the expression derived in Lecture 4 for the present value form of the flex-price monetary model. Only if the "no-bubble" restriction,

$$\underset{T \Rightarrow \infty}{limit} \left(\frac{\eta}{1+\eta} \right)^T p_{t+T} = 0 \qquad \textbf{(8)}$$

is imposed will the two expressions be the same. The problem is that in general, this restriction cannot be imposed, so that one cannot in turn rule out the following expression for the price level:

$$p_t = \left(\frac{1}{1+\eta} \right) \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} m_s + b_0 \left(\frac{1+\eta}{\eta} \right)^t \qquad \textbf{(11)}$$

where b_0 is an initial deviation of p_0 from the value implied by the fundamentals,

$$b_0 = p_0 - \left(\frac{1}{1+\eta} \right) \sum_{s=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^s m_s$$

2. An Early Test for Price Bubbles (Flood and Garber, 1980)

2.1. A Bubble Model

This derivation is a stochastic version of the above.

Consider the price level, solved for from money demand equations (using Flood and Garber's notation, and equation numbers):

$$\begin{aligned} m_t - p_t &= \gamma + \alpha \pi_t^* + \epsilon_t \\ \pi_t^* &\equiv E p_{t+1} - p_t \\ \alpha &< 0 \end{aligned} \qquad \textbf{(1)}$$

This means the money demand equation can be rewritten as:

$$m_t - p_t = \gamma + \alpha E(\pi_t \mid I_t) + \epsilon_t \qquad \textbf{(3)}$$

where I is the information set. Notice that now an error term is allowed into the money demand equation. Taking the first difference of (3), one obtains:

$$\mu_t - \pi_t = \alpha[E(\pi_{t+1}|I_{t+1}) - E(\pi_t|I_t)] + w_t \quad \textit{where} \quad \mu_t \equiv m_{t+1} - m_t \tag{A1}$$

By the Law of Iterated Expectations, *viz*

$$E(\pi_{t+1}|I_t) = E[E(\pi_{t+1}|I_{t+1})|I_t]$$

equation (A2) holds:

$$E(\mu_t|I_t) - E(\pi_t|I_t) = \alpha[E(\pi_{t+1}|I_t) - E(\pi_t|I_t) + E(w_t|I_t) \tag{A2}$$

Equation (A2) is a first order difference equation. The solution is:

$$E(\pi_t|I_t) = A_t - \left(\frac{\psi^{-1}}{\alpha}\right)\sum_{i=0}^{\infty} E(\mu_{t+i} - w_{t+i}|I_t)\psi^{-i} \quad \textit{where} \quad w_{t+1} \equiv \epsilon_{t+1} - \epsilon_t, \quad \psi \equiv (\alpha-1)/\alpha > 1 \tag{A3}$$

where A_t is an arbitrary constant determined at each period t.
Rational expectations places the following restrictions on these solutions constants:

$$E(A_{t+j}|I_t) = A_t\psi^j, \quad j = 0,1,2... \tag{A4a}$$

$$A_{t+j} = A_t\psi^j + \sum_{i=1}^{j} z_{t+j}\psi^{j-i}, \quad j = 0,1,2... \tag{A4b}$$

and the z's are white noise. Flood and Garber choose a *deterministic* process on the *A*'s such that equation (A4) is fulfilled,

$$A_t = A_0\psi^t \tag{A5}$$

Substituting (A5) into (A3) yields equation (4):

$$E(\pi_t|I_t) = A_0\psi^t - \left(\frac{\psi^{-1}}{\alpha}\right)\sum_{i=0}^{\infty} E(\mu_{t+i} - w_{t+i}|I_t)\psi^{-i} \qquad \textbf{(4)}$$

Now substituting this expression for expected inflation into (3) yields

$$p_t = -\alpha A_0\psi^t + \left[m_t - \gamma + \psi^{-1}\sum_{i=0}^{\infty} E(\mu_{t+i} - w_{t+i}|I_t)\psi^{-i} - \epsilon_t \right] \qquad \textbf{(5)}$$

where the term in square brackets [ ] is the fundamentals term.

2.2. Estimation

Flood and Garber propose several tests. I describe two of them.

2.2.1. Money demand and inflation expectations

The first involves joint estimation of the inflation expectations process and the money demand equation (equations 8 and 9, respectively) following Hansen and Singleton (1982):

$$\pi_t = \delta + \beta_1\mu_{t-1} + \beta_2\mu_{t-2} + .. + \beta_k\mu_{t-K} + A_0\psi^t + v_{1t} \qquad \textbf{(8)}$$

$$m_t - p_t = \gamma + \alpha(\delta + \beta_1\mu_{t-1} + ... + \beta_k\mu_{t-K} + A_0\psi^t) + \epsilon_t \qquad \textbf{(9)}$$

using seemingly unrelated regression estimation (SURE), imposing the nonlinear cross-equation restrictions (recalling the definition of ψ). The results are reported in Table 1; in all cases, the estimates of A_0 are not statistically significant. Hence, they conclude that there is no evidence for a bubble.

2.2.2. Money demand, and exogenous money supply driving variable

Here the Hansen and Sargent (1980) procedure is implemented. Substitute (4) into the money demand equation to obtain:

$$m_t - p_t = \gamma + \alpha[A_0\psi^t - \left(\frac{\psi^{-1}}{\alpha}\right)\sum_{i=0}^{\infty} E(\mu_{t+i} - w_{t+i}|I_t)\psi^{-i}] + \epsilon_t \qquad \textbf{(10)}$$

The money supply process (actually the fundamentals) is rewritten as an AR(K) process;

$$\mu_t = \eta + \phi_1\mu_{t-1} + \phi_2\mu_{t-2} + ... + \phi_k\mu_{t-K} + v_{2t} \qquad \textbf{(11)}$$

Equations (10) and (11) can be jointly estimated, after solving for the terms in the square brackets of (10). In order to solve for this first term, (11) must first be identified; Flood and Garber use the usual Box-Jenkins methods to identify the process in (11) as an AR(2),

$$\mu_t = \eta + (1+\rho)\mu_{t-1} - \rho\mu_{t-2} + v_{2t} \qquad \textbf{(12)}$$

such that the infinite sum in (10) can be solved analytically as:

$$\left(\frac{\psi^{-1}}{\alpha}\right)\sum_{i=0}^{\infty} E(\mu_{t+i} - w_{t+i}|I_t)\psi^{-i} = \mu_{t-1} - \frac{\eta(\alpha-1)}{1-\rho} + \left[\frac{\rho(\alpha-1)}{\alpha-1-\rho\alpha}\right]\left(\mu_{t-1} - \mu_{t-2} - \frac{\eta}{1-\rho}\right) \qquad \textbf{(13)}$$

The results of estimating the system jointly, after first differencing, are reported in Table 4. They point to the same conclusions as in the Singleton procedure: a no-bubble hypothesis cannot be rejected.

3. Meese's Tests for Foreign Exchange Bubbles

3.1. A Joint Test for Bubbles

Once again, assume a money demand equation:

$$m_t - p_t = a_1 y_t - a_2(i_t - i_t^*) \qquad \textbf{(1)}$$

where now *m* and *p* are expressed in *relative* (to foreign country) terms. Assume uncovered interest parity (UIP), which is perfect capital substitutability.

$$i_t - i_t^* = E(s_{t+1}|\Phi_t) - s_t \qquad \textbf{(2)}$$

The object on the right-hand side of the equation is "expected depreciation", which is modeled as the mathematical expectation of the log spot exchange rate at time t, based on time t information set (Φ_t) minus the time t log-spot exchange rate. The next relation indicates that deviations from purchasing power parity (PPP) in log-levels follows a random walk:

$$s_t - p_t = u_t, \quad u_t = u_{t-1} + \epsilon_t \tag{3}$$

where the ϵ is a white noise error term. This is a slightly odd assumption, as it implies the real exchange rate follows a random walk. However, Meese motivates it as an approximation to slow mean reversion to PPP (as would occur in the Dornbusch-Frankel model with very sticky prices).

Substituting (2) and (3) into (1) yields:

$$s_t = m_t - a_1 y_t + a_2[(E(s_{t+1}|\Phi_t) - s_t] + u_t \tag{4}$$

Defining $b = a_2/(1 + a_2)$, $0 < b < 1$, then equation (4) can be rewritten as:

$$s_t = (1-b)(m_t - a_1 y_t) + b[E(s_{t+1}|\Phi_t)] + (1-b)u_t \tag{5}$$

Since there is evidence of unit roots in nominal exchange rates, Meese first-differences

the series being examined.

$$\Delta s_t = (1-b)(\Delta m_t - a_1 \Delta y_t) + b[E(s_{t+1}|\Phi_t) - E(s_t|\Phi_{t-1})] + (1-b)\epsilon_t \tag{6}$$

Let the "fundamentals" be defined as

$$\Delta x_t \equiv (\Delta m_t - \alpha_1 \Delta y_t) = c\Delta x_{t-1} + \delta, \quad |c| < 1 \tag{7}$$

In other words, the fundamentals follow an AR(1) process.

If the no-bubbles, or transversality, condition is imposed, then one obtains the usual present value expression for the exchange rate.

$$\begin{aligned} \Delta s_t^* \equiv \Delta s_t &= \Delta x_t + \frac{bc}{1-bc}(\Delta x_t - \Delta x_{t-1}) + \epsilon_t \\ \Delta x_t &= c\Delta x_{t-1} + \delta \end{aligned} \tag{12}$$

On the other hand, if the transversality condition in (9) is violated then any solution of the form:

$$s_t = s_t^* + d_t, \quad \textit{where} \quad E(d_{t+1}|\Phi_t) = (1/b)d_t \tag{13}$$

will be satisfied.

Meese exploits the fact that McCallum's procedure, as applied to equation (6), yields consistent estimates, while maximum likelihood applied to equation (12) will in general yield inconsistent estimates if the bubble term is correlated with the RHS variables. Then a Hausman specification error test can then be applied, where the test statistic is given by:

$$\frac{T(\hat{b}_{IV} - \hat{b})^2}{\dfrac{b^2(1+c)^2}{c^2} + \sigma_\epsilon^2 \dfrac{(1+c)^3(1-bc)^2[(1-bc)^2+2b^2(1-c)]}{\sigma_\delta^2 2(1-c^2)c^2}} \tag{19}$$

which is distributed χ^2 with one degree of freedom. Such an approach does not impose a specific form on the bubble, and as such is a more general test for bubbles. Another difference between Meese and other tests is that instead of estimating the income elasticity of money demand, he searches over a grid of plausible parameter values.

Note that in order to implement estimation of (12), Δy and Δm must be represented by lag polynomials of the same order, and that Δx must be strongly exogenous with respect to Δs.

Examining the DM/$, £/$ and ¥/$ exchange rates, he in general he finds that the bubble test statistic always rejects the null of identical coefficients. Since his coefficient estimates and the diagnostic statistics for the US-Japan regressions are so poor, he considers these test results weak, and hence does not report them. This omission reflects the joint nature of the test, discussed further below.

3.2. A Cointegration Test for Bubbles

Another test for bubbles is to see if the exchange rate and its posited fundamentals

are cointegrated. Meese uses the state of the art cointegration methodology (at the time of the paper's writing, in 1985 or so), the Engle-Granger regression approach. He finds that he cannot reject the no cointegration null in all cases. However, the Engle Granger test is particularly weak, so it's unsurprising that cointegration is not found.

4. Limitations of Bubble Tests

Both sets of tests, the Flood-Garber and Meese approach, as well as the cointegration approach, have severe limitations. In the Flood-Garber approach, a particular form of bubble is assumed. The test then is a *joint* test of (i) correctly specified and stable specifications of the exchange rate equation (or the underlying money demand equations), (ii) stable autoregressive driving processes for the fundamentals, and (iii) a particular form of the bubble. The first half of the Meese paper relaxes the conditions imposed on the bubble's evolution, but maintains (i) and (ii).

It might appear that the cointegration tests of Meese are able to circumvent such limitations, especially in light of recent work using more powerful cointegration techniques (e.g., MacDonald and Taylor, 1993). However, if the bubbles do not take the form of rational stochastic bubbles that start exogenously, but rather are correlated with the fundamentals, then cointegration tests will have low power to discern such bubbles. Froot and Obstfeld (1991) forward this idea of intrinsic (depending upon fundamentals) versus extrinsic (like the bubbles we've examined), and demonstrate how in principle one might be able to detect such intrinsic bubbles.

The problem is that the methodology is difficult to apply even to stock prices, where one is reasonably confident about the nature of the "fundamentals". In the case of exchange rates, it is much more difficult to apply such tests since we are so unsure about what set of variables belong in the "fundamentals", let alone with what weights. Application of related "variance bounds" tests to exchange rates suffers from similar difficulties (see Frankel and Meese, 1987).

Finally, Flood and Hodrick (1990) have argued that bubbles cannot be distinguished from model misspecification and regime switching.

5. A Simple Portfolio-Balance Model

In this portion of the lecture, we depart from the assumption of uncovered interest rate

parity. Recall the decomposition

$$i - i^* \equiv [i - i^* - (f - s)] + [(f - s) - (s^e - s)] + (s^e - s) \quad \textbf{(20)}$$

The UIP condition holds if

$$s^e - s = f - s \quad \textbf{(21)}$$

or, in other words, the exchange risk premium

$$f - s^e = 0 \Rightarrow f = s^e \quad \textbf{(22)}$$

$$f - s^e \equiv rp \quad \textbf{(23)}$$

In portfpolio balance models, the exchange risk premium, rp, can be expected to be non-zero, and generally time varying.

It is rather artificial to discuss the portfolio balance model without a detailed examination of the exchange risk premium. However, we will reserve this for the future, while providing only a rudimentary discussion at this point, since it will turn out that such models will prove empirically wanting.

That being said, portfolio balance models may still prove to be useful in providing insight. For example, drawing from recent events, such models may shed light on why the dollar seemed to appreciate in response to expectations of smaller US government deficits during 1995, while a decade earlier the opposite was observed.

2.1. Theory and Derivation

In this model (drawn from Frankel, 1983), assume perfect capital mobility (CIP) holds, while perfect capital substitutability does not. That is, investors view domestic and foreign bonds as imperfect substitutes. Then investor j will allocate her holdings in response to expected returns.

$$\frac{B_j}{SF_j} = \underline{\beta}_j(i - i^* - E\Delta s) \quad \textbf{(16)}$$

where B_j (F_j) is the stock of domestic (foreign) denominated bonds held by investor j, and s is the log-exchange rate. $\underline{\beta}_j$ is a positive-valued function which Frankel sets as

$$\underline{\beta}_j(i - i^* - E\Delta s) = \exp[\alpha_j + \beta_j(i - i^* - E\Delta s)]$$

Hence equation (16) implies that a rise in the interest differential or a reduction in expected depreciation will induce investors to shift from foreign denominated bonds to domestic. Aggregating yields:

$$\begin{aligned} \frac{B}{SF} &= \underline{\beta}(i - i^* - E\Delta s) \\ B &\equiv \sum_{j=1}^{n} B_j \\ F &\equiv \sum_{j=1}^{n} F_j \end{aligned} \qquad \textbf{(17)}$$

where B and F are now net supplies of bonds, where it is assumed for now that governments issue debt denominated only in their own currencies.

The difficulty is in determining what EΔs is. For simplicity, assume static expectations (consistent with the exchange rate following a random walk). Then (17) reduces to

$$\frac{B}{SF} = \exp[\alpha + \beta(i - i^*)] \qquad \textbf{(18)}$$

which after taking logs yields

$$s = -\alpha - \beta(i - i^*) + b - f \qquad \textbf{(18)}$$

Notice the equation indicates that as f increases, s falls (appreciates). This means that if the country runs a current account surplus, the stock of foreign assets held by home rises, and the exchange rate appreciates.

In the specification represented by equation (17), it is assumed that all investors have the same portfolio preferences, presumably because they consume the same basket of goods.

There are a variety of assumptions that yield different versions of (17). If the home country is a small country, such that only home residents wish to hold domestically denominated assets, then one can equate capital inflows with increases in the supply of foreign assets in the home market. Thus, one obtains (19) instead of (17):

$$\begin{aligned} \frac{B_H}{SF_H} &= \underline{\beta_H}(i - i^*) \\ B_H = B &\equiv \sum_{j=1}^{n} B_j \\ F_H &\equiv \sum_{j=1}^{n} F_j \end{aligned} \qquad \textbf{(19)}$$

where F_H is the sum of all foreign bonds held by home residents, and is in turn equal to the sum of past current account surpluses under the small country assumption. This results in

$$s = -\alpha_H + \beta_H(i - i^*) + b - f_H \qquad \textbf{(20)}$$

The small country assumption is untenable if the country being investigated is the United States. Then one might want to make the converse assumption, that the foreign country is the small country. Then (20) is instead

$$s = -\alpha_F + \beta_F(i - i^*) + b_F - f \qquad \textbf{(21)}$$

where b_F is defined as the log of domestic bonds held by foreign residents, equal to the sum of past foreign current account surpluses.

Clearly, neither of these versions fit the typical large country. Then what one needs to specify is a separate asset-demand function, such that the two equations for the two countries are

$$\frac{B_H}{SF_H} = \underline{\beta_H}(i - i^*) \qquad \textbf{(19)}$$

$$\frac{B_F}{SF_F} = \underline{\beta_F}(i - i^*) \qquad \textbf{(22)}$$

The usual response of the exchange rates to cumulated trade balances obtains as long as $ß_H \geq ß_F$.

One issue remains to be resolved, regarding definitions. So far, we have discussed matters as if the only way in which *B* and *F* can change is through current account imbalances. In fact,

$$B = \int BuD - MB - \int INT$$

where *BuD* is the budget deficit, *MB* is the money base, assumed to be held solely by domestic residents, and INT is foreign exchange intervention (purchases of domestic currency bonds).

3. A Rational Expectations Model

Rodriguez (1980) model is a portfolio-balance model in the sense that it tries to get the current account back into the model of exchange rates, by focusing on the fact that the current account is (approximately) the accumulation of foreign assets. He forwards a rational expectations (actually perfect foresight) version of a portfolio balance model. Let the desired log ratio of domestic money to foreign be given by:

$$m - s - f = L(\dot{s}^*) \qquad \textbf{(1)}$$

where m is the log domestic money stock, s is the log exchange rate, and f the log foreign money stock. One can always rearrange to solve for s:

$$s = m - f - L(\dot{s}^*) \qquad \textbf{(2)}$$

In order to get the trade balance into the model, one must introduce some type of rigidity, such that at any given instant, one is not at the desired, equilibrium holdings of foreign currency, f^d, so that one obtains, instead of equation (1), the following set of equations:

$$f^d = m - s - L(\dot{s}^*) \tag{3}$$

$$(\dot{f})^d = \lambda(f^d - f) \tag{4}$$

$$\dot{f} = H(.) \tag{5}$$

where (4) indicates the flow of foreign currency in a partial adjustment framework, and (5) the supply of foreign currency (which is left general at the moment). Setting (4) and (5) equal to each other yields:

$$\lambda[m - s - L(\dot{s}^*) - f] = H(.)$$

Hence, the exchange rate depends upon whatever flow aspects are included in the H(.) function. To see this more explicitly in a rational expectations framework, re-write (1) as:

$$m(t) - s(t) - f(t) = a - \frac{1}{b}[ds(t)/dt] \tag{7}$$

Equation (7) is a difference equation; the solution of (7) as the horizon T goes to infinity is (after imposing the appropriate transversality condition):

$$s(t) = b\int_0^\infty [m(t+x) - a]e^{-bx}dx - b\int_0^\infty f(t+x)e^{-bx}dx \tag{9}$$

Equation (9) indicates that the spot exchange rate today is a function of the discounted values of the fundamentals (m and f) into the infinite future. Since we are concerned with shocks other than domestic monetary policy induced ones, set m to be constant at $\bar{m}$ for all time. Then equation (9) becomes:

$$s(t) = \overline{m} - a - b\int_0^\infty f(t+x)e^{-bx}dx \tag{10}$$

Equation (10) simplifies (9) inasmuch that the spot exchange rate now depends only upon the anticipated path of f over time. This in turn suggests imposing some structure on the evolution of f. Rodriguez assumes that the rate of change of the foreign currency stock is equal to the ratio of the trade surplus to actual holdings of foreign currency, viz.:

$$df/dt = T(t)/F(t) \qquad \textbf{(11)}$$

where Rodriguez calls *T/F* the normalized trade balance (*NTB*). Further assume that *NTB* does not depend on s (this is relaxed in the appendix), but that it is negatively related to the stock of foreign money, since the wealth effect raises absorption.

$$df/dt = T_0 - \theta f(t), \quad \theta>0 \qquad \textbf{(12)}$$

Let T_0 be the structural trade balance, while the θf is the endogenous component. Integrating (12) yields:

$$f(t+x) = T_0/\theta + [f(t) - T_0\theta]e^{-\theta x} \qquad \textbf{(13)}$$

Equation (13) has the characteristic that as time goes to infinity, *f(t+x)* converges to its long run value, of

$$\bar{f} = T_0/\theta$$

and furthermore that *NTB* goes to zero.

Substitute (13) into (10) to obtain:

$$s(t) = (\bar{m} - a - T_0/\theta) + \frac{b}{\theta(b+\theta)}[T_0 - \theta f(t)] \qquad \textbf{(14)}$$

Since the term in the square bracket is just the *NTB(t)*, then using (12), one obtains:

$$s(t) = (\bar{m} - a - T_0/\theta) + \frac{b}{\theta(b+\theta)}NTB(t) \qquad \textbf{(15)}$$

The long run value of the exchange rate can be obtained by imposing the $NTB = 0 = T_0 - \theta f(t)$ condition on (15);

$$\bar{s}(t) = (\bar{m} - a - T_0/\theta) \qquad \textbf{(16)}$$

Substituting (16) into (15) yields an expression for the determination of the spot exchange rate:

$$s(t) = \bar{s} + \frac{b}{\theta(b+\theta)} NTB(t) \qquad \textbf{(17)}$$

Equation (17) should look vaguely familiar; it expresses the exchange rate as a function of the long run exchange rate, and a positive function of the *NTB*. The greater θ, the correction parameter, is, the more rapid the convergence of the exchange rate to its long run value. Alternatively, for any given shock to the trade balance, the greater θ is, the smaller the deviation from the long run value of the exchange rate.

To sum up, when there is a shock to the *NTB*, this implies a change in the desired stock of foreign exchange. However, since the stocks of domestic and foreign moneys are held fixed in the short run, the exchange rate must "jump" to such a level to restore portfolio balance.

Notice in this model, interest rates do not come into play. Holdings of assets (which more broadly construed could be bonds) are determined by trade balances. In other models (Dooley and Isard, 1982), interest rates do enter into the equations, and hence fits in more closely with the typical view of a portfolio balance model.

A more complete exposition of the portfolio balance models incorporating both interest rates and rational expectations can be found in Sarno and Taylor (2002), pp. 115-123.

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