Problem Set 3 Answers
Exchange Rate Economics

1. Solve for the present value relation of the nominal exchange rate in the flexible price monetary model, assuming no bubbles, and that the fundamentals follow an AR(1) process. Show your work.

\[ s_t = p_t - p_t^* \]  \hspace{1cm} (9)

Finally, assume stable money demand functions in the two countries:

\[ (m_t - p_t^d)^d = \phi y_t - \lambda l_t \]
\[ (m_t^* - p_t^d)^d = \phi^* y_t^* - \lambda^* l_t^* \]  \hspace{1cm} (10)

where the \(d\) superscripts indicate "demand". Rearranging, assuming money supply equals money demand, and imposing PPP one obtains:

\[ s_t = p_t - p_t^* \]
\[ = (m_t - m_t^*) - \phi (y_t - y_t^*) + \lambda (l_t - l_t^*) \]  \hspace{1cm} (11)

for \( \phi = \phi^* \) and \( \lambda = \lambda^* \)

Note by UIP,

\[ l_t - l_t^* = \Delta s_{t-1} = E(s_{t+1}|\Phi) - s_t \]  \hspace{1cm} (12)

Combining (11) and (12) yields:

\[ s = \tilde{M}_t + \lambda (s_{t+1}) - \lambda (s_t) \]

where \( \tilde{M}_t = (m_t - m_t^*) - \phi (y_t - y_t^*) \)  \hspace{1cm} (13)

By manipulating this expression
Imposing rational expectations yields an expression for the future expected spot rate in period $t+1$:

$$E_r(s_{t+1}) = \frac{1}{1+\lambda} E_r \hat{M}_{t+1} + \frac{\lambda}{1+\lambda} E_r \hat{M}_{t+2}$$  \hspace{1cm} (15)$$

Substituting equation (7) into (6) yields:

$$s_t = \frac{1}{1+\lambda} \hat{M}_t + \frac{\lambda}{1+\lambda} E_r \hat{M}_{t+1} + \frac{\lambda \lambda}{1+\lambda} E_r \hat{M}_{t+2}$$  \hspace{1cm} (16)$$

But consider:

$$E_r(a_{t+2}) = \frac{1}{1+\lambda} E_r \hat{M}_{t+2} + \frac{\lambda}{1+\lambda} E_r \hat{M}_{t+3}$$  \hspace{1cm} (17)$$

So that by substituting iteratively, one obtains:

$$s_t = \left( \frac{1}{1+\lambda} \right) \sum_{n=0}^{\infty} \left( \frac{\lambda}{1+\lambda} \right)^n E_r \hat{M}_{t+n}$$  \hspace{1cm} (18)$$

The expression for the current spot exchange rate can be determined under certain circumstances. If the fundamentals follow an AR(1) process,

$$\hat{M}_t = \rho \hat{M}_{t-1} + \varepsilon_t \rightarrow E_r \hat{M}_{t+1} = \rho \hat{M}_t$$  \hspace{1cm} (19)$$

Then substituting this into (18), one obtains:

$$s_t = \left( \frac{1}{1+\lambda} \right) \left[ \hat{M}_t + \left( \frac{\lambda}{1+\lambda} \right)^1 E_r \hat{M}_{t+1} + \left( \frac{\lambda}{1+\lambda} \right)^2 E_r \hat{M}_{t+2} + \ldots \right]$$

$$= \left( \frac{1}{1+\lambda} \right) \left[ \hat{M}_t + \left( \frac{\lambda}{1+\lambda} \right)^1 \rho \hat{M}_t + \left( \frac{\lambda}{1+\lambda} \right)^2 \rho^2 \hat{M}_t + \ldots \right]$$

$$= \left( \frac{1}{1+\lambda} \right) \left[ \hat{M}_t + \frac{1}{1-(\lambda \rho)}/(1+\lambda) \right] \hat{M}_t$$

$$= \left( \frac{1}{1+\lambda} \right) \left[ \frac{1}{1-\lambda \rho/1+\lambda} \right] \hat{M}_t$$

$$s_t = \frac{1}{1+\lambda-\lambda \rho} \hat{M}_t$$  \hspace{1cm} (20)$$

Equation (20) expresses the current exchange rate as a function of the current fundamentals, because all the expected future fundamentals can be expressed as a simple function of current fundamentals.
2. Briefly describe how, assuming rational expectations, you would implement an efficient estimation method of this present value/flex-price monetary model.

3. Frankel’s simplified model, assuming representative investors, assumes:

\[
\log \left( \frac{B_t}{S_t F_t} \right) = \alpha + \beta(i - i^* - s_{t+1} - s_t)
\]

Show algebraically how expectations of future budget deficits might affect the current value of a currency. Explain the intuition of your result.

\[
\begin{align*}
\sigma_t &= \left( \frac{1}{1+\beta} \right) (b_t - r_t) - \alpha - \beta(i_t - i_t^*) + \beta \sigma_{t+1}^* \\
\sigma_{t+1}^* &= \left( \frac{1}{1+\beta} \right) (b_{t+1}^* - r_{t+1}^*) - \alpha - \beta(i_{t+1}^* - i_{t+1}^*) + \beta \sigma_{t+1}^* \\
\end{align*}
\]

Substituting (23) into (22) yields:

\[
\sigma_t = \nu \left( (b_t - r_t) - \alpha - \beta(i_t - i_t^*) \right) + \beta \nu \left( (b_{t+1}^* - r_{t+1}^*) - \alpha - \beta(i_{t+1}^* - i_{t+1}^*) + \beta \sigma_{t+1}^* \right)
\]

Hence, it is clear that not only next period’s expected bond supplies (and interest rates) determine the current exchange rate, but indeed the entire path of expected bond supplies and interest rates into the infinite future. Since bond supplies are determined by budget deficits (and the conduct of monetary policy), future budget deficits obviously determine the current exchange rate in this model.