Kiel Institut für Weltwirthschaft
Advanced Studies in International Economic Policy Research
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Final Exam Answers
Exchange Rate Economics

This exam is 1 ½ hours long. Answer all questions. Use equations and figures if helpful. [Because of the omission in Question 1.2, the exam is graded out of 80 points]

1. Portfolio Balance model. [20 minutes] Suppose the risk premium (*rp*) on euro denominated assets is given by:

$$rp = -\beta^{-1}\alpha + \beta^{-1}x$$

where *x* is the share of euro denominated assets in the world (assume there are only two currencies, the US$ and the euro).

1.1 (10 minutes) In the context of this model, what do you think will happen if one's assessment of the size of future ***United States*** Government budget deficits rises (and those of the euro zone governments remain the same)?

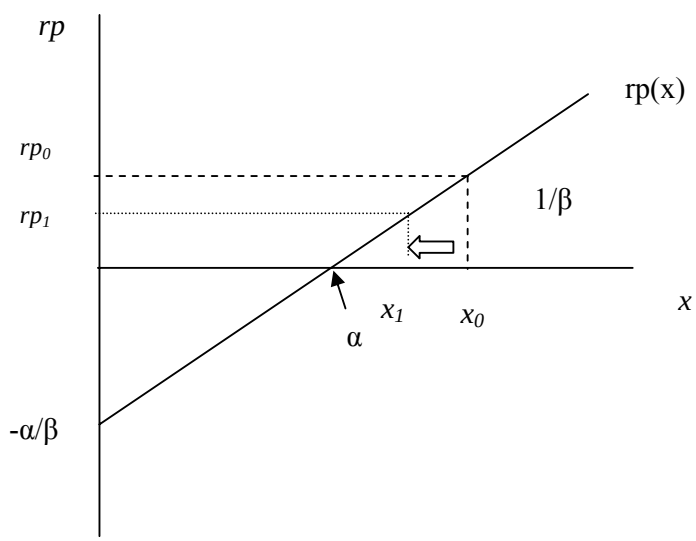
Define the risk premium as the additional return necessary to induce the holding of a particular asset. This means one drops explicitly the uncovered interest rate parity condition. Hence:

$$rp_t \equiv i_t^{Eu} - i_t^{US} - \Delta s_{t+1}^e \qquad (1)$$

where depreciation expected between time t and t+1, based on time t information, is expressed as:

$$\Delta s_{t+1}^e \equiv \varepsilon_t(s_{t+1}) - s_t \qquad (2)$$

One then obtains the following diagram:



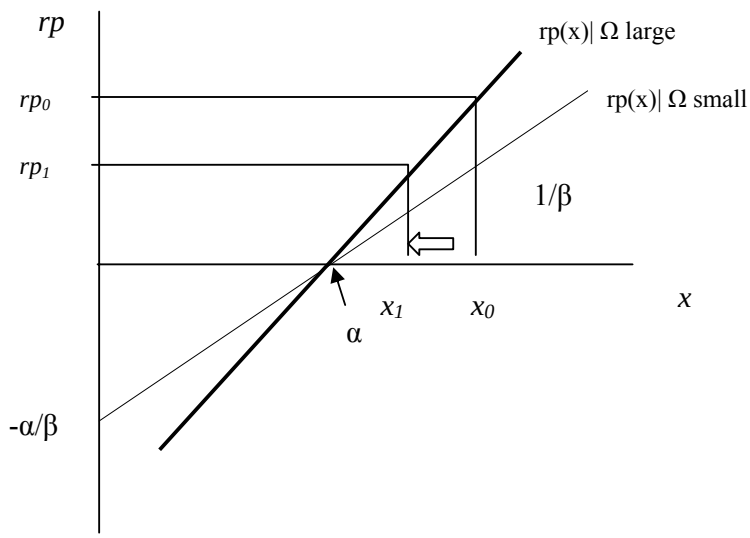
Here *x* is the share of the total portfolio that is euro denominated. If *x*=α, then the risk premium is zero. However, if $x=x_0$, then in some sense, the interest rates on euro assets must be higher in order to

compensate individuals for the fact that they are holding greater amounts of euro assets than they would like (consistent with no risk premium). What does α equal? In a mean-variance framework (familiar to those of you who know the capital asset pricing model, or CAPM, for stock returns), α is a function of the degree of risk aversion, and the extent to which real returns on euro assets are correlated with the dollar returns on US dollar denominated assets.

The question asks about what happens if the budget deficit – and hence the debt denominated in dollars – rises above what is currently expected. Then in the future, the *x* should fall to x_1 (remember, this is all from the Euro area resident's perspective), so the risk premium on euro denominated assets should *fall*. But since the value of a bond depends upon the value it is expected to have in the future, the value today of US bonds should fall, while those of euro area bonds should rise.

1.2 (10 minutes) If the variance of relative returns on the two assets rises, *in US$ terms*, what do you think is the quantitative magnitude of the effect you indicate in your answer to question 1.1? [Since I accidentally omitted "rises" in the question, I made the total number of points allocated to problem 1 equal 10, giving extra credit if anybody answered the question assuming *either* rises or falls.]

In the mean-variance framework $\beta^{-1} = \rho\Omega$. Ω is the variance of returns in euro terms; but it is also the variance of returns in USD terms, so when this rises, the slope is flat, then β^{-1} rises and assets become less substitutable. Graphically, this means the rp(x) curve rotates counter-clockwise.



Notice that now, any change in *x* now causes a larger change in the risk premium.

2. Purchasing power parity. [20 minutes] Define the real exchange rate for the US dollar against the Thai baht as:

$$Q_t \equiv (S_t \times P_t^*) / P_t \qquad (1)$$

where *S* is measured in US$/baht, *P* and *P** are *price indices* (namely, CPI in the US and Thailand, with a base year of 1995=100). One obtains the following picture:

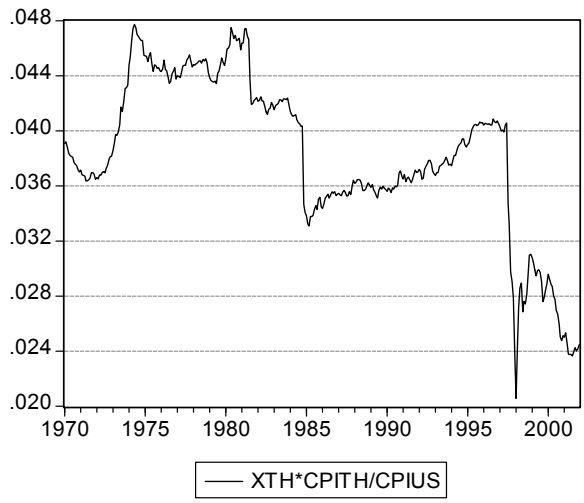


Figure 1: Real US$/Thai baht exchange rate (CPI deflated).
Source: IMF, *International Financial Statistics*, authors calculations.

2.1 (10 minutes) Can one conclude from this picture that the Thai baht was overvalued, in absolute purchasing power parity terms, in at the end of 1996? Explain why or why not.

No, one cannot conclude that, according to absolute PPP, the Thai baht was overvalued in 1996. That is because the price variable is not a price of a bundle of goods, but rather a price *index*, which equals 100 in a particular base year. One could conclude that Q was high relative to the average value of Q over the sample period; but that using that criterion is consistent with the concept of relative PPP.

2.2 (10 minutes) Now consider this diagram which plots the real exchange rate as the number of bundles of goods evaluated at US prices, required to purchase a single bundle of goods evaluated at Thai prices. In other words, *P* and *P** are now measured for identical bundles of goods (i.e., food accounts for the same proportion in each bundle, etc.).

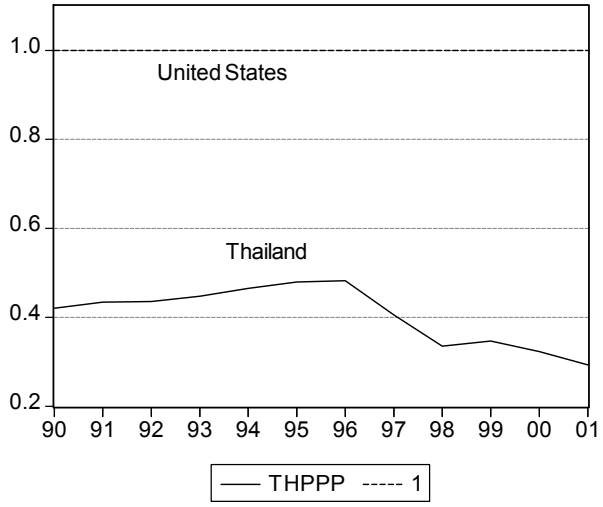


Figure 2: Price levels for bundles of goods in US and Thailand (US=1.0).
Source: World Bank, *World Development Indicators.*

Can one conclude that the Thai baht was overvalued by absolute purchasing power parity in 1996? Why or why not?

Here, the prices are for (theoretically) identical bundles of goods, expressed in a common currency, hence one can make assessments of absolute purchasing power parity. However, now with this data, one can conclude that by this criterion, the Thai baht was undervalued (the dollar was overvalued) in 1996. This counter-intuitive result suggests that absolute PPP may not be very useful for determining exchange rate misalignment for many questions.

3. Competing models of the exchange rate. [20 minutes total] Consider the graph of the (trade-weighted) value of the euro, against a broad basket of currencies.

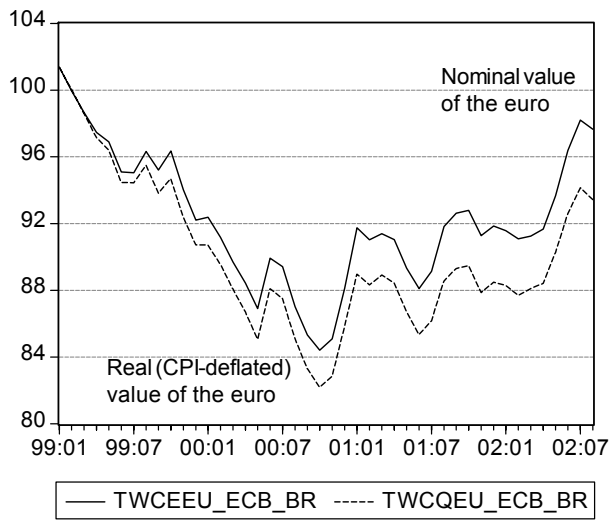


Figure 3: (Broad) Trade weighted effective exchange rates for the euro, nominal and real.
Source: European Central Bank.

What model (or models) of the exchange rate does this figure support? Use equations to support your argument.

This figure supports both models of the exchange rate. In other words, the high correlation between nominal and real exchange rates is consistent with either a real business cycle model of the exchange rate, as in Stockman's approach, or the sticky price monetary models of Dornbusch and Frankel.
The high covariation between the nominal and real exchange rate can arise for (at least) two reasons: (i) monetary shocks in the presence of sticky nominal prices or (ii) real shocks in the context of perfect price flexibility. Consider first the following identity:

$$s = p - p^* \Leftrightarrow q = s - p + p^*$$

If p and p* are fixed in the very short run, but the common currency relative price, q, is not, then definitionally s and q will covary. This sticky price assumption is common in the Dornbusch (1976) and Frankel (1979) models.

On the other hand, consider the Lucas model:

$$e(s, M, N) = \frac{p_x(s, M)\, p_y(s)}{p_y(s, N)} = \frac{M}{N}\frac{\eta}{\xi}\, p_y(s)$$

In this case, the exchange rate e can move because of money shocks or real shocks (the latter include the goods-endowments ζ and η, as well as the $p_y(s)$ which may or may not move depending on these goods-endowments and the utility function). If real shocks are large relative to the nominal shocks (here shocks to M and N), then e will tend to covary with p_y . But p_y is the real exchange rate, although it is assumed that the home country and the foreign produce different goods.

4. Present value models. [30 minutes]

4.1 (15 minutes) Solve for the present value relation of the nominal exchange rate in the flexible price monetary model, assuming no bubbles, and that the fundamentals follow a random walk (without drift) process. Show your work.

$$s = \tilde{M}_t + \lambda(s^e_{t+1}) - \lambda(s_t)$$
$$\textit{where} \quad \tilde{M}_t \equiv (m_t - m_t^*) - \phi(y_t - y_t^*) \tag{9}$$

$$s_t + \lambda s_t \equiv s_t(1+\lambda) = \tilde{M}_t + \lambda s^e_{t+1}$$
$$s_t = \frac{1}{1+\lambda}\tilde{M}_t + \frac{\lambda}{1+\lambda}s^e_{t+1}$$

Imposing rational expectations yields and expression for the future expected spot rate in

period period *t+1*:

$$E_t(s_{t+1}) = \frac{1}{1+\lambda}E_t\tilde{M}_{t+1} + \frac{\lambda}{1+\lambda}E_t s_{t+2} \tag{10}$$

substituting equation (10) into (9) yields:

$$s_t = \frac{1}{(1+\lambda)}\tilde{M}_t + \frac{\lambda}{(1+\lambda)^2}E_t\tilde{M}_{t+1} + \frac{\lambda}{1+\lambda}\frac{\lambda}{1+\lambda}E_t s_{t+2} \tag{11}$$

but consider:

$$E_t(s_{t+2}) = \frac{1}{1+\lambda}E_t\tilde{M}_{t+2} + \frac{\lambda}{1+\lambda}E_t s_{t+3} \tag{12}$$

so that by substituting iteratively, one obtains:

$$s_t = \left(\frac{1}{1+\lambda}\right)\sum_{\tau=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^{\tau}E_t\tilde{M}_{t+\tau} \tag{13}$$

$$\tilde{M}_t = \tilde{M}_{t-1} + \epsilon_t \rightarrow E_t\tilde{M}_{t+1} = \tilde{M}_t \tag{18}$$

$$\begin{aligned} s_t &= \left(\frac{1}{1+\lambda}\right)\sum_{\tau=0}^{\infty}\left(\frac{\lambda}{1+\lambda}\right)^{\tau} E_t\tilde{M}_{t+\tau} \\ &= \left(\frac{1}{1+\lambda}\right)\left[\frac{1}{1 - \lambda/(1+\lambda)}\right]\tilde{M}_t \\ &= \left(\frac{1}{1+\lambda}\right)(1+\lambda)\tilde{M}_t \end{aligned} \tag{20}$$

$$s_t = \tilde{M}_t \tag{19}$$

4.2 (5 minutes) Show that in this case that the change in the exchange rate equals the change in the fundamentals.

Clearly, from equation (19) above, taking the total differential yields:

$$ds_t = d\tilde{M}_t$$

In other words, since the level of the spot exchange rate depends upon the current level of fundamentals, then the change in the spot rate depends upon the change in current fundamentals. Since all *future* fundamentals move up (or down) by exactly the same amount that the current fundamentals changed by.

4.3 (10 minutes) Obtain a general expression for the change in the exchange rate today as a function of the change in the fundamentals today, and the discounted value of fundamentals in the future. Can you explain why the expression you obtain in 4.2 is so simple?

Consider the present value – no bubbles formulation of the spot exchange rate:

$$s_t = \left(\frac{1}{1+\lambda}\right)\sum_0^{\tau}\left(\frac{\lambda}{1+\lambda}\right)^{\tau} E_t\tilde{M}_{t+\tau} \tag{1}$$

$$s_{t+1} = \left(\frac{1}{1+\lambda}\right)\sum_0^{\tau}\left(\frac{\lambda}{1+\lambda}\right)^{\tau} E_{t+1}\tilde{M}_{t+1+\tau} \tag{2}$$

Subtracting (1) from (2) yields:

$$s_{t+1} - s_t = \left(\frac{1}{1+\lambda}\right)\sum_0^{\tau}\left(\frac{\lambda}{1+\lambda}\right)^{\tau} (E_{t+1}\tilde{M}_{t+1+\tau} - E_t\tilde{M}_{t+\tau}) \tag{3}$$

$$s_{t+1} - s_t = \left(\frac{1}{1+\lambda}\right)\left[(\tilde{M}_{t+1} - \tilde{M}_t) + \sum_1^{\tau}\left(\frac{\lambda}{1+\lambda}\right)^{\tau} (E_{t+1}\tilde{M}_{t+1+\tau} - E_t\tilde{M}_{t+\tau})\right] \tag{4}$$

Notice that the change in the spot exchange rate is equal to the present discounted value of differences in expected fundamentals, as well as the current change in the fundamentals. When the fundamentals follow a random walk, then all the terms after the summation operator in (4) sum to zero.