Economics 442
Macroeconomic Policy

Lecture 11

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$$\max_{C_{t,}^{y}, C_{t+1}^{m}, C_{t+2}^{o}} \mathbb{E}_t \left\{ \log\left(C_t^y\right) + \beta \log\left(C_{t+1}^m\right) + \beta^2 \log\left(C_{t+2}^o\right) \right\}$$

where C_t^y is the consumption of the household when young, C_{t+1}^m its consumption when middle aged, and C_{t+2}^o its consumption while old. We assume that borrowing and lending take place via a one period risk-free bond denoted B_t^i where $i = y, m, o$ at an interest rate r_t. Given this structure, we can write the budget constraints facing households of the generation born at time t at in each period as:

$$C_t^y = B_t^y \tag{1}$$

$$C_{t+1}^m = Y_{t+1}^m - (1 + r_t) B_t^y + B_{t+1}^m \tag{2}$$

$$C_{t+2}^o = Y_{t+2}^o - (1 + r_{t+1}) B_{t+1}^m \tag{3}$$

$$(1 + r_t) B_t^i \leq D_t \tag{4}$$

$$C_t^y = B_t^y = \frac{D_t}{1 + r_t} \tag{5}$$

Summary

- Young borrow to consume
- Middle age save for retirement
- Old consume and dis-save

- Key: Exogenous borrowing constraint on young

The old at any time t will consume all their income so that:

$$C_t^o = Y_t^o - (1 + r_{t-1}) B_{t-1}^m \tag{6}$$

The middle aged, however, are at an interior solution and satisfy a consumption Euler equation given by:

$$\frac{1}{C_t^m} = \beta \mathbb{E}_t \frac{1 + r_t}{C_{t+1}^o} \tag{7}$$

We assume that the size of each generation is given by N_t. Let us define the growth rate of births by $1 + g_t = \frac{N_t}{N_{t-1}}$. Equilibrium in the bond market requires that borrowing of the young equals the savings of the middle aged so that $N_t B_t^y = -N_{t-1} B_t^m$ or:

$$(1 + g_t) B_t^y = -B_t^m \tag{8}$$

Dt, gt exogenous

To analyze equilibrium determination, let us focus on equilibrium in the market for savings and loans given by equation (8) using the notation L_t^s and L_t^d; the left hand side of (8) denotes the demand for loans, L_t^d, and the right hand side its supply, L_t^s. Hence the demand for loans (using (5)) can be written as:

$$L_t^d = \frac{1+g_t}{1+r_t} D_t \tag{9}$$

while the supply for savings - assuming perfect foresight for now - can be solved for by substituting out for C_t^m in (2), using (3) and (7), and for B_t^y by using (5). Then solving for B_t^m, we obtain the supply of loans given by:

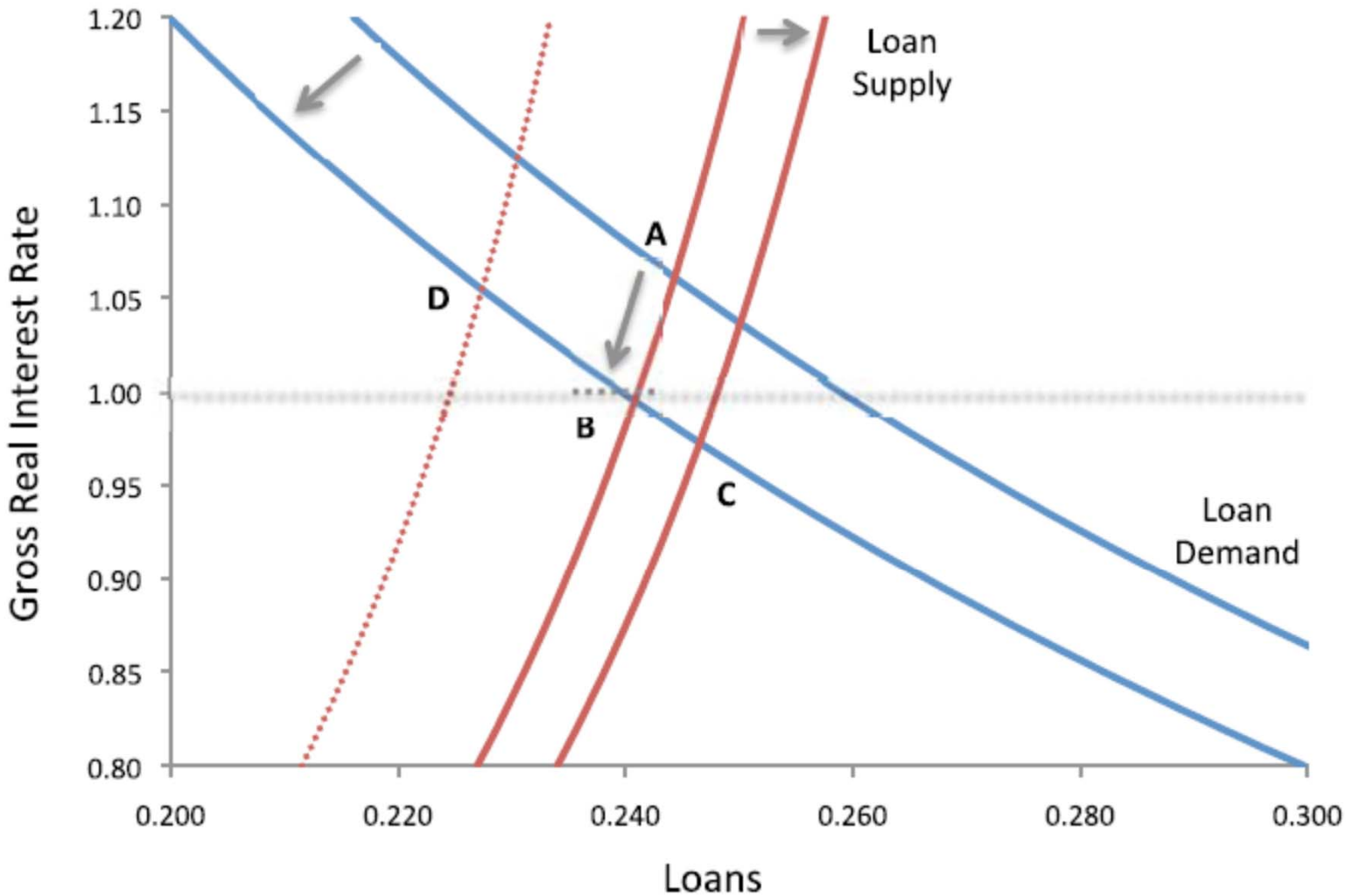
$$L_t^s = \frac{\beta}{1+\beta}\left(Y_t^m - D_{t-1}\right) - \frac{1}{1+\beta}\frac{Y_{t+1}^o}{1+r_t} \tag{10}$$

An equilibrium, depicted in Figure 1, is then determined by the intersection of the demand (L_t^d) and supply (L_t^s) for loans at the equilibrium level of real interest rates given by:

$$1+r_t = \frac{1+\beta}{\beta}\frac{(1+g_t)\,D_t}{Y_t^m - D_{t-1}} + \frac{1}{\beta}\frac{Y_{t+1}^o}{Y_t^m - D_{t-1}} \tag{11}$$

Observe that the real interest rate will in general depend on the relative income between the middle aged and the young as well as on the debt limit, population growth, and the discount factor.

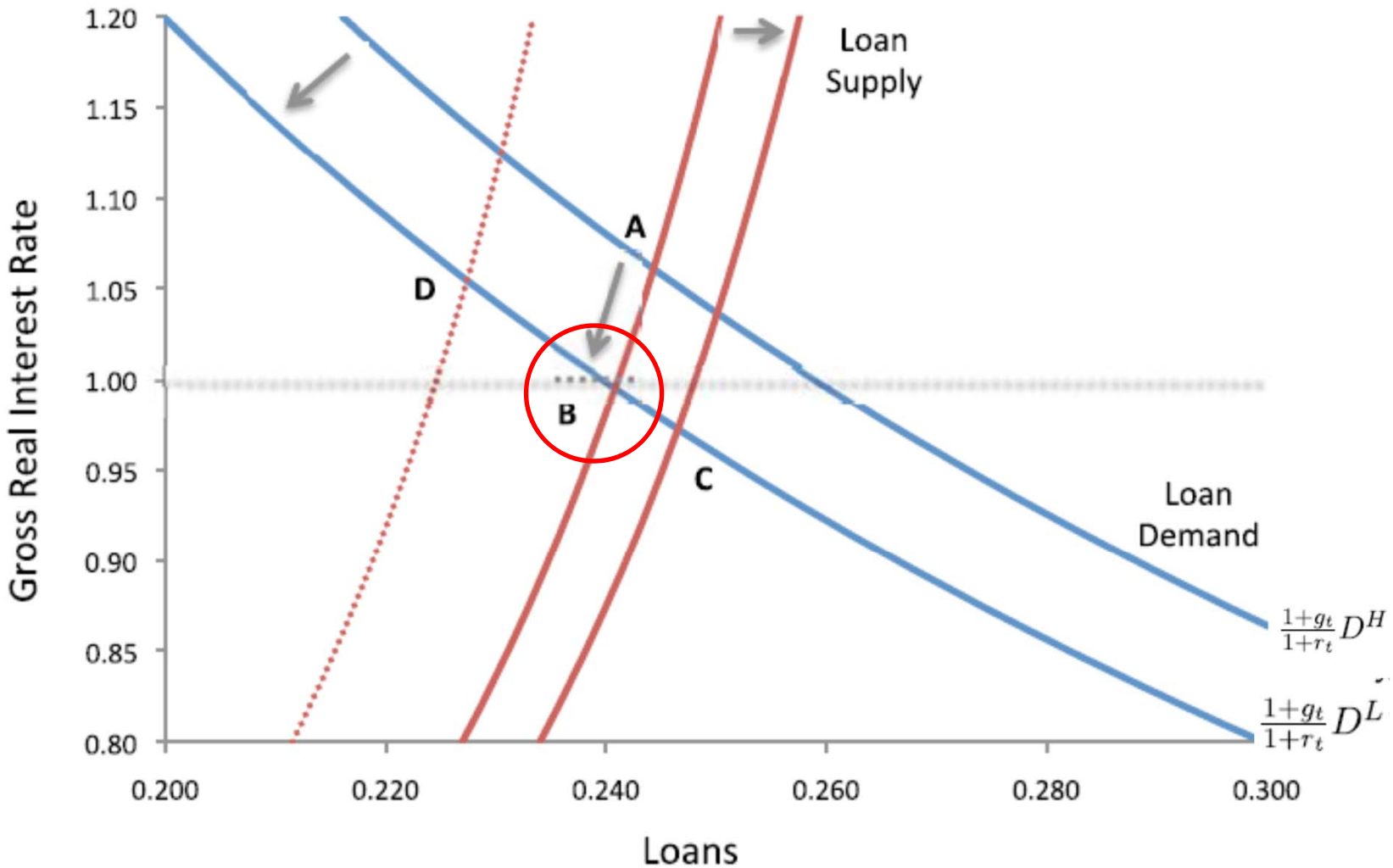
Figure 1: Equilibrium in the asset market



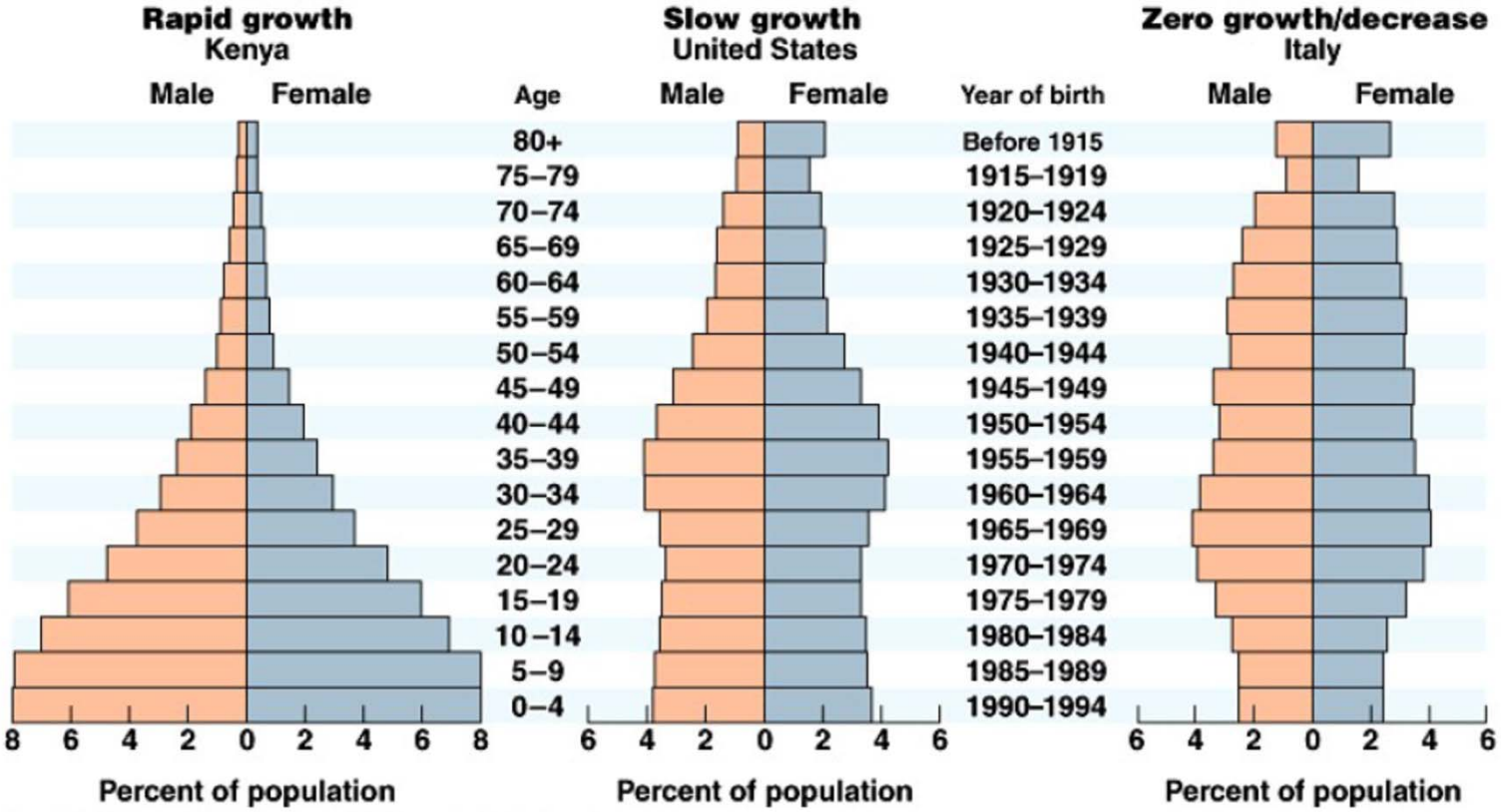
Deleveraging Shock

- A sudden and permanent reduction in D.
- The Young generation borrows less than before.
- When they become the Middle aged generation, they have less debt, and have more to save.
- The resulting lower interest rate loosens constraint for young, induces more spending for Middle

Figure 1: Equilibrium in the asset market



An Aging Society

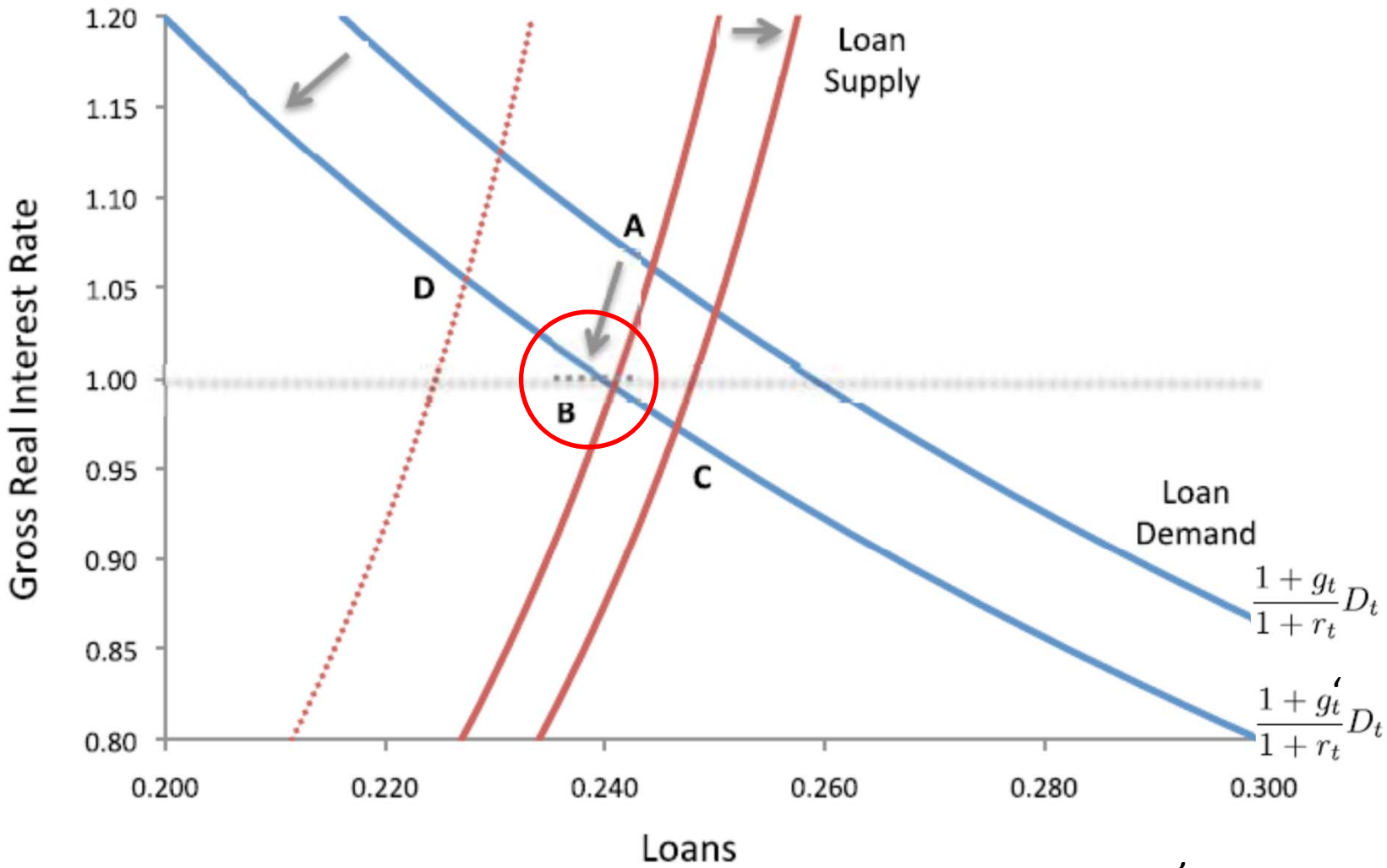


An Aging Society

- Aging society is represented by reduction in parameter *g*
- Recall loan demand equation has *g* parameter in it

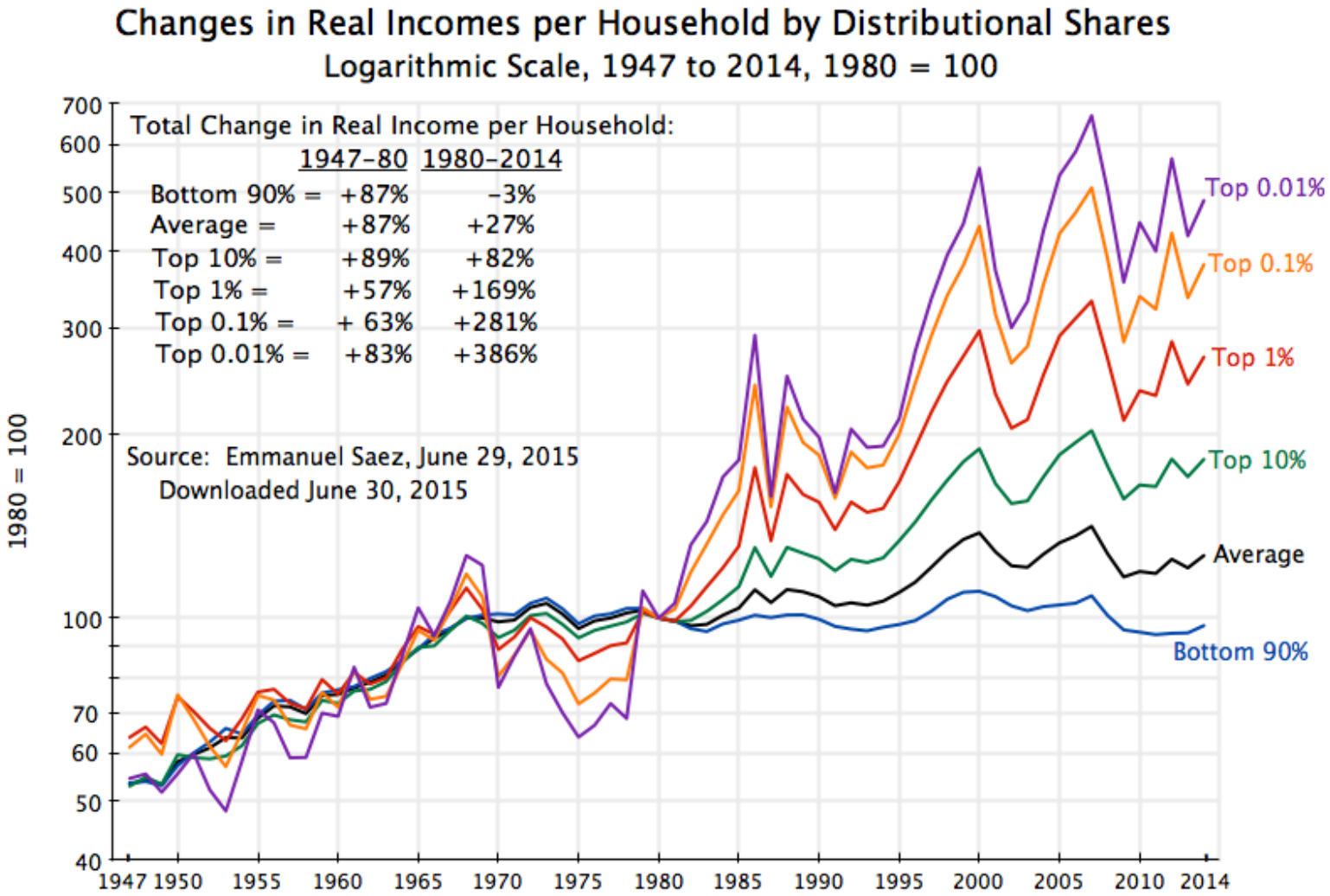
$$L_t^d = \frac{1 + g_t}{1 + r_t} D_t \tag{9}$$

Figure 1: Equilibrium in the asset market



$g' < g$

Income Inequality



Income Inequality

Again let us assume that only middle aged and the old generation receive an income endowment. Now, however, suppose that some fraction of households receive a larger endowment in their middle generation (i.e. high-income households) while the remaining households (i.e. low-income households) receive a very small endowment in the middle period of life. For simplicity, all households receive the same income endowment in old age (this could be thought of as some sort of state-provided pension like Social Security). For sufficiently low levels of the middle-period endowment and a sufficiently tight credit constraint, low-income households will remain credit constrained in the middle period of life. These households will rollover their debt in the middle generation and only repay their debts in old age, consuming any remaining endowment. In this situation, only the high-income households will save in the middle period and will, therefore, supply savings to both credit-constrained middle-generation households and the youngest generation.

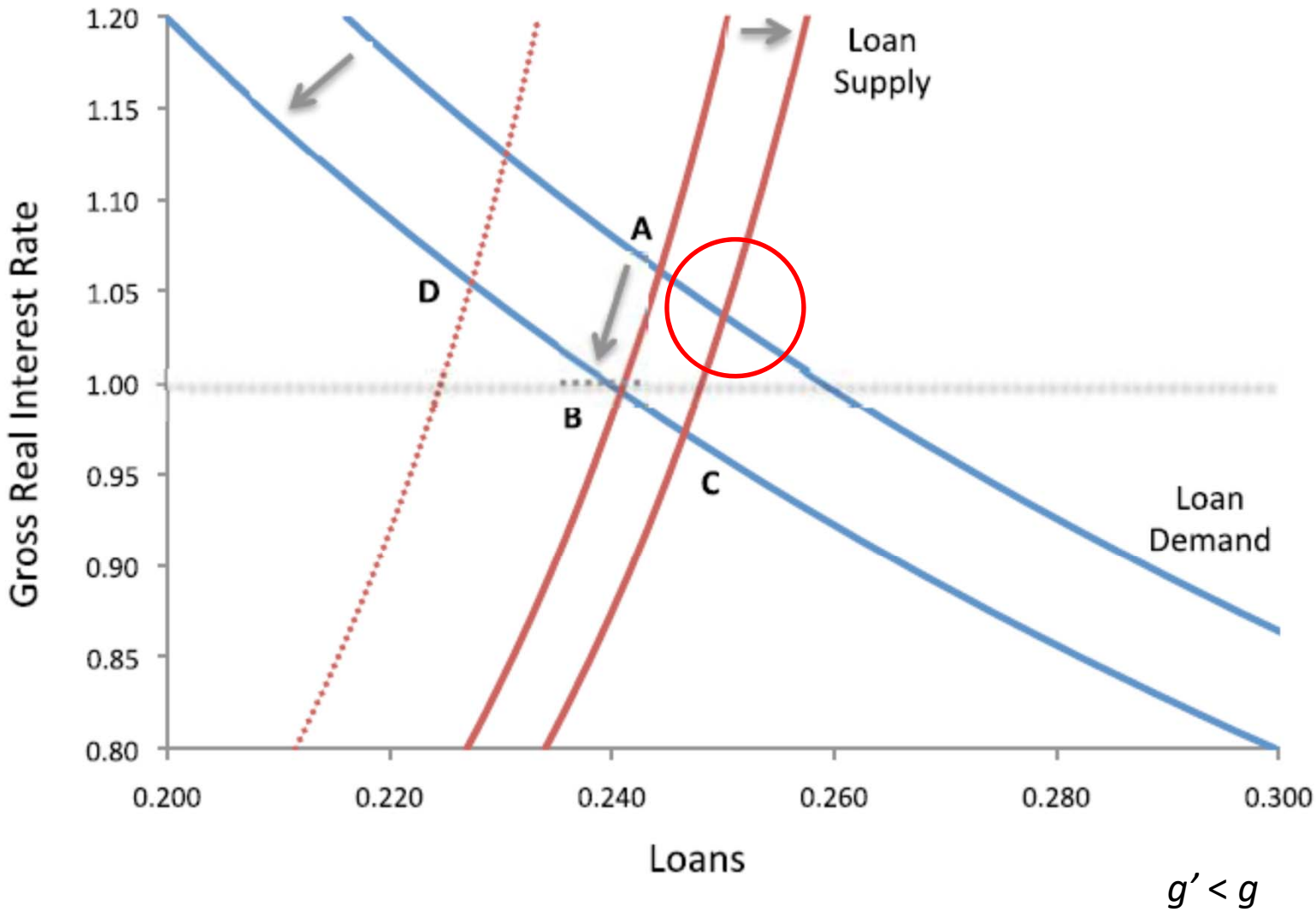
Income Inequality

$$1 + r_t = \frac{1+\beta}{\beta} \frac{(1 + g_t + \eta_s) D_t}{(1-\eta_s)\left(Y_t^{m,h} - D_{t-1}\right)} + \frac{1}{\beta} \frac{Y_{t+1}^o}{\left(Y_t^{m,h} - D_{t-1}\right)} \tag{12}$$

where η_s is the fraction of low-income households, $Y_t^{m,h}$ is the income of the high-income middle-generation and Y_{t+1}^o is the income of these households in the next period (i.e. the pension income received by all households). If $\eta_s = 0$ we recover the expression for the real interest rate derived in (11).

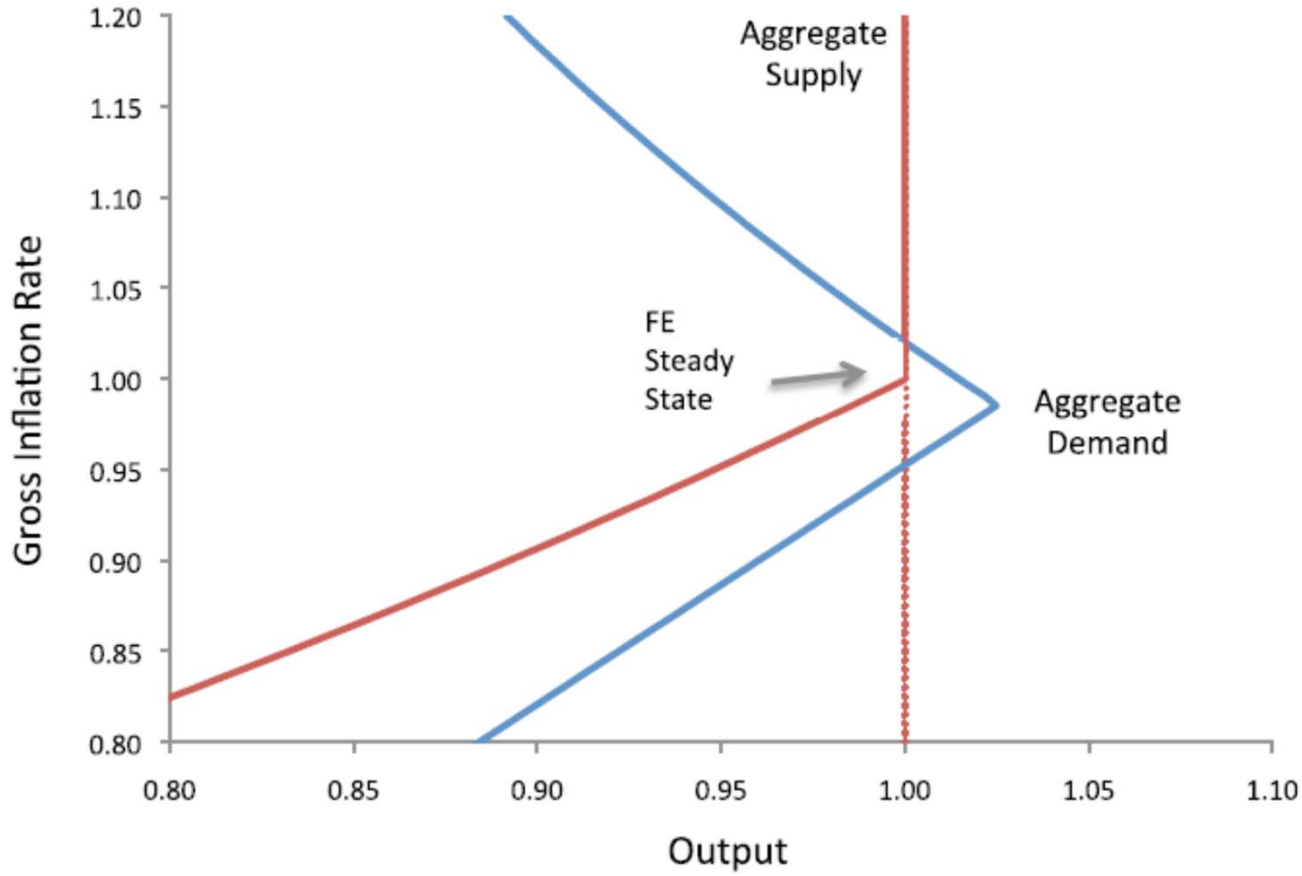
Total income for the middle generation is a weighted average of high and low income workers: $Y_t^m = \eta_s Y_t^{m,l} + (1-\eta_s) Y_t^{m,h}$. Let us then define an increase in inequality as a redistribution of middle-generation income from low to high-income workers, without any change in Y_t^m. While this redistribution keeps total income for the middle generation constant by definition, it must necessarily lower the real interest rate by increasing the supply of savings which is only determined by the income of the wealthy. This can be seen in equation (12) where the real interest rate is decreasing in $Y_t^{m,h}$ without any offsetting effect via $Y_t^{m,l}$.[6]

Figure 1: Equilibrium in the asset market



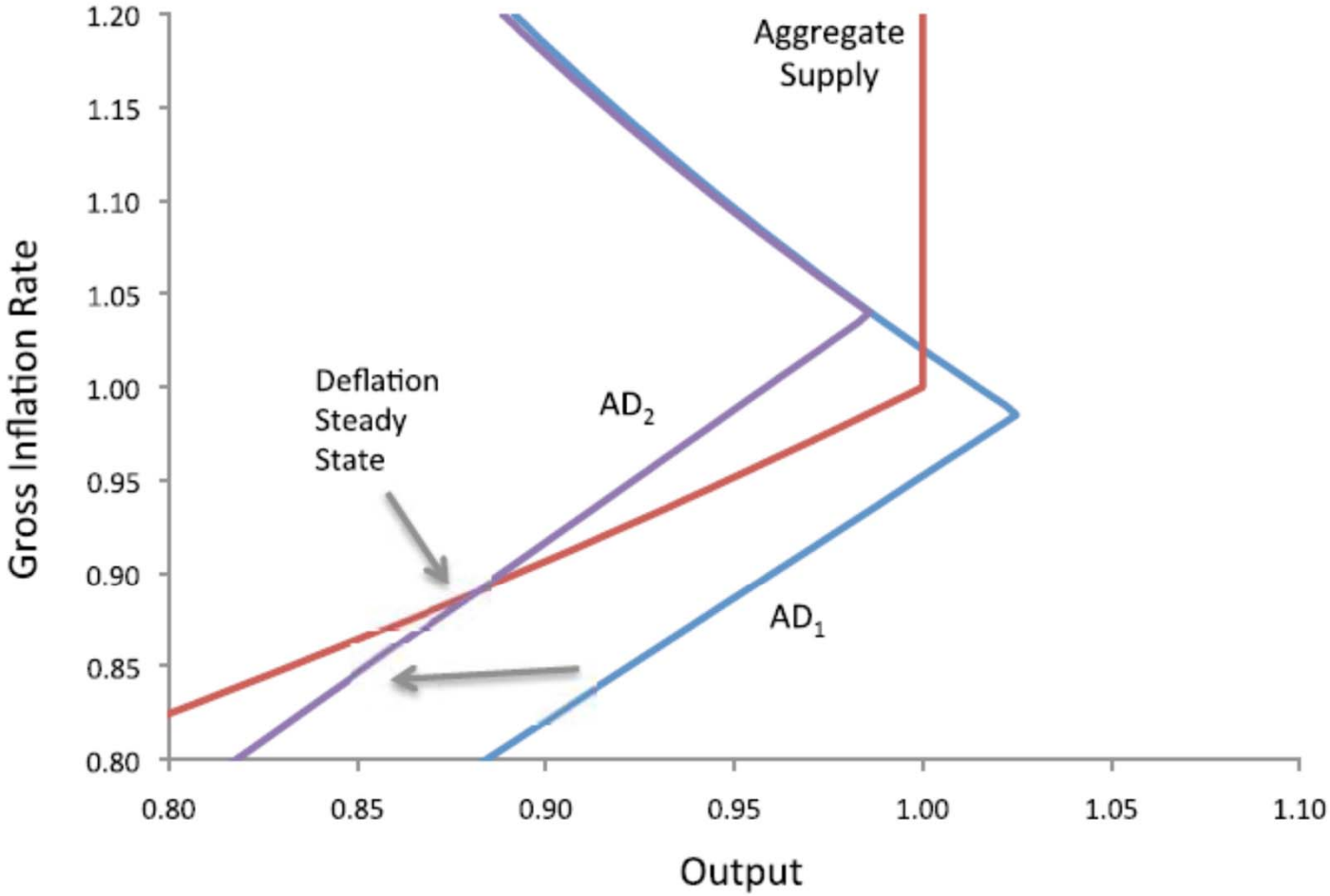
Endogenous Output, Permanent Inflation, ZLB

Figure 2: Steady state aggregate demand and aggregate supply curves



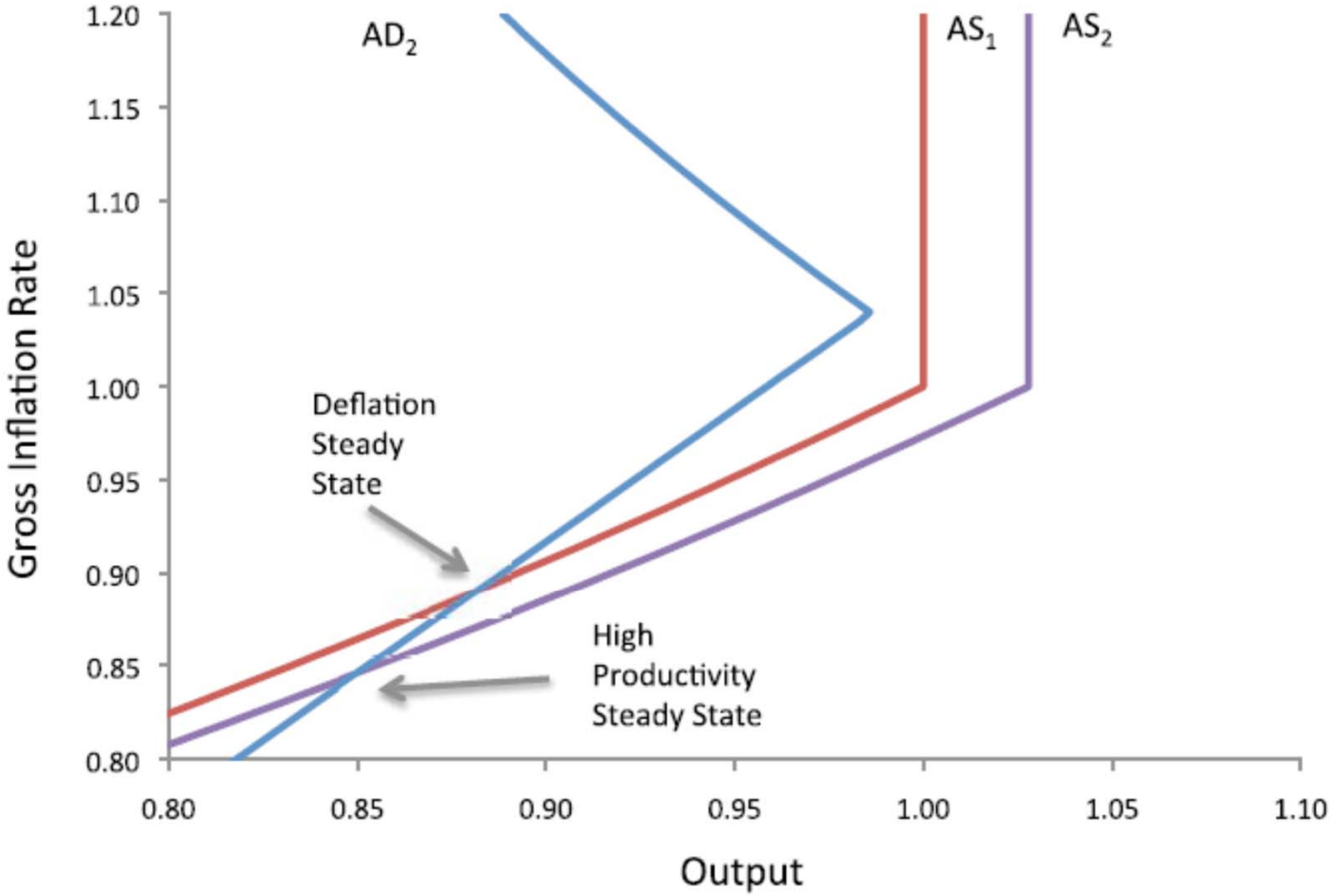
AD Shift

Figure 3: Steady state aggregate demand and aggregate supply curves



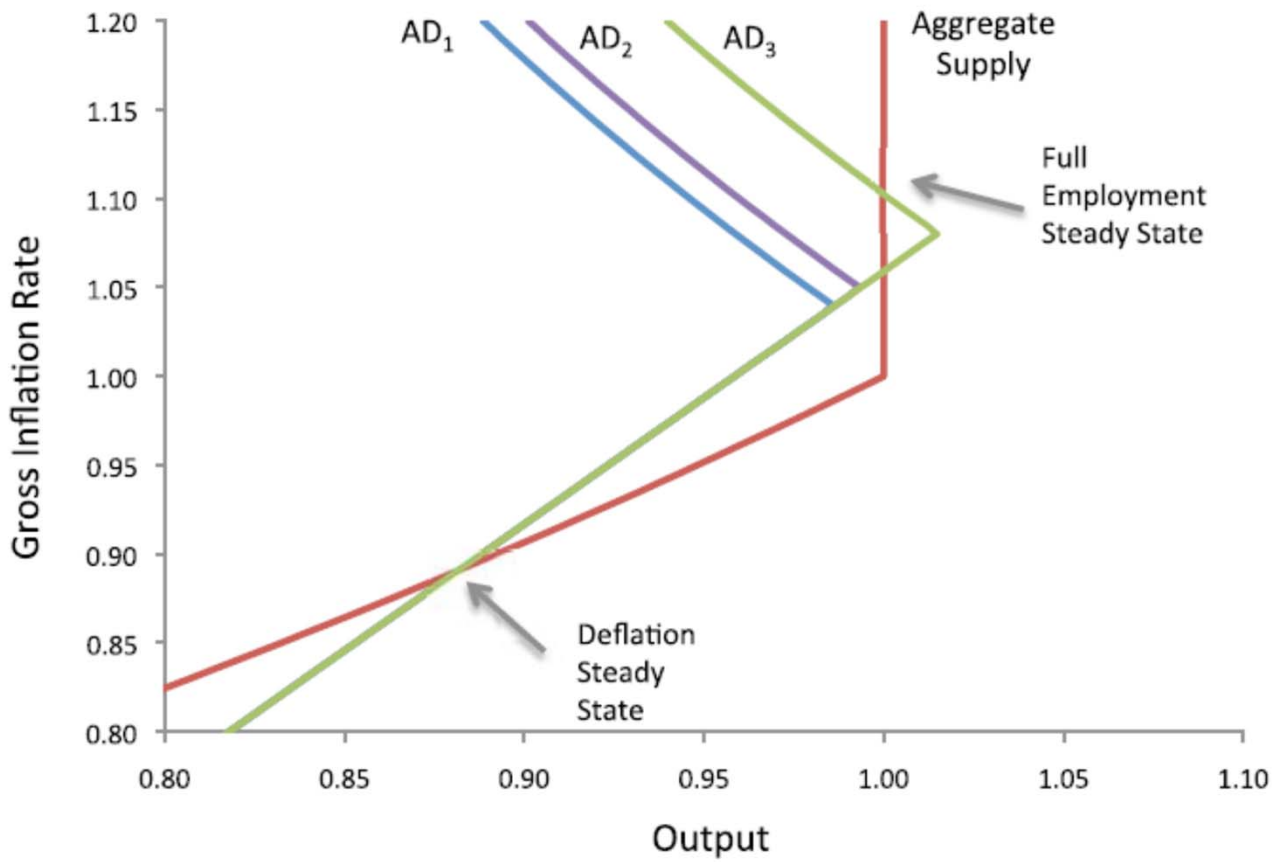
Supply Side Economics?

Figure 4: Paradox of toil



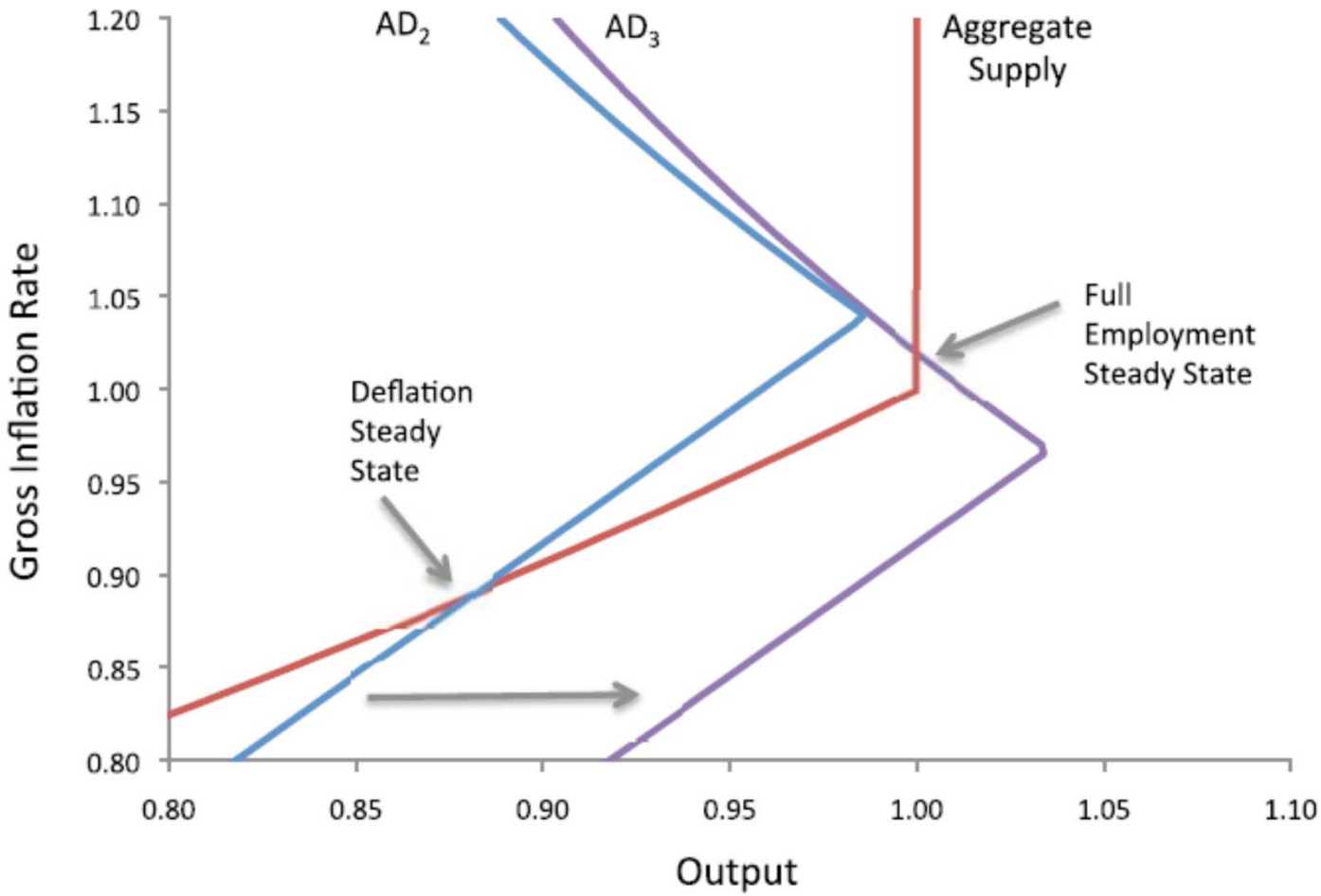
Inflation Targeting

Figure 6: Effect of raising the inflation target



Fiscal Policy

Figure 7: Effect of an increase in government spending



Capital Goods Price

Figure 8: Effect of decrease in relative price of capital goods