Economics 435
Fall 2021
University of Wisconsin-Madison

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Social Sciences 7418

Problem Set 1 Answers

Due via Canvas on Tuesday, September 28, 11pm Central time. Be sure to put your name on your problem set. Put "boxes" around your answers to the algebraic questions, or otherwise highlight. (You can type out answers and draw in the graphs in the boxes , or write out answers and scan the document).

1. Suppose the economy is described by the following equations (so we are looking at a closed economy):

• Real Sector

(1)	$Y = Z$	Output equals aggregate demand, an equilibrium condition
(2)	$Z = C + I + G$	Definition of aggregate demand
(3)	$C = c_o + c_1 Y_D$	Consumption fn, c_1 is the marginal propensity to consume
(4)	$Y_D \equiv Y - T + Tr$	Definition of disposable income
(5)	$T = t_1 Y$	Tax function; t_1 is marginal tax rate.
(6)	$Tr = TR_0$	Transfer payments; TR_0 is lump sum transfers.
(7)	$I = b_0 + b_1 Y - b_2 i$	Investment function
(8)	$G = GO_0$	Government spending on goods and services, exogenous

• Asset Sector

(9)	$\frac{M^d}{P} = \frac{M^s}{P}$	Equilibrium condition
(10)	$\frac{M^s}{P} = \frac{M_0}{P}$	Real money supply
(11)	$\frac{M^d}{P} = \mu_0 + Y - hi$	Real money demand

1.1 Solve for the IS curve (Y as a function of i).

$Y = Z = C + I + G$ substitute in for C, I, G
$Y = c_0 + c_1 Y_D + b_0 + b_1 Y - b_2 i + GO_0$ substitute in for Y_d
$Y = a_0 + c_1 (Y - T + Tr) + b_0 + b_1 Y - b_2 i + GO_0$ substitute in for tax, transfers functions
$Y = a_0 + c_1 (Y - t_1 Y + TR_0) + b_0 + b_1 Y - b_2 i + GO_0$ bring the "Y" terms to left hand side.

$Y - b(Y - t_1 Y - \lambda Y) = Y(1-b(1-t-\lambda)) = c_0 + c_1 TR_0 + b_0 + GO_0 - b_2 i$

divide both sides by ***(1- c_1 (1-t_1)-b_1)*** and let ***$\Lambda_0 \equiv c_0 + c_1 TR_0 + b_0 + GO_0$***

$Y_0 = \bar{\gamma}[\Lambda_0 - b_2 i]$ let $\bar{\gamma} = \frac{1}{[1 - c_1(1-t_1) - b_1]}$

1.2 Solve for the LM curve (*i* as a function of *Y*). What is the channel by which monetary *policy* influences ~~affect~~ the real goods sector in this model?

$$\frac{M_0}{P_0} = \frac{M^s}{P} = \frac{M^d}{P} = \mu_0 + Y - hi$$

Solving for the interest rate, i, yields the LM curve:

$$\boxed{i = \frac{\mu_0}{h} - \left(\frac{1}{h}\right)\left(\frac{M_0}{P_0}\right) + \left(\frac{1}{h}\right)Y}$$

Monetary policy influences (in part) interest rates. Interest rates in turn affect investment, and via the simple Keynesian multiplier ($\bar{\gamma}$) affects the entire real sector.

1.3 Solve for the equilibrium value of *Y*.

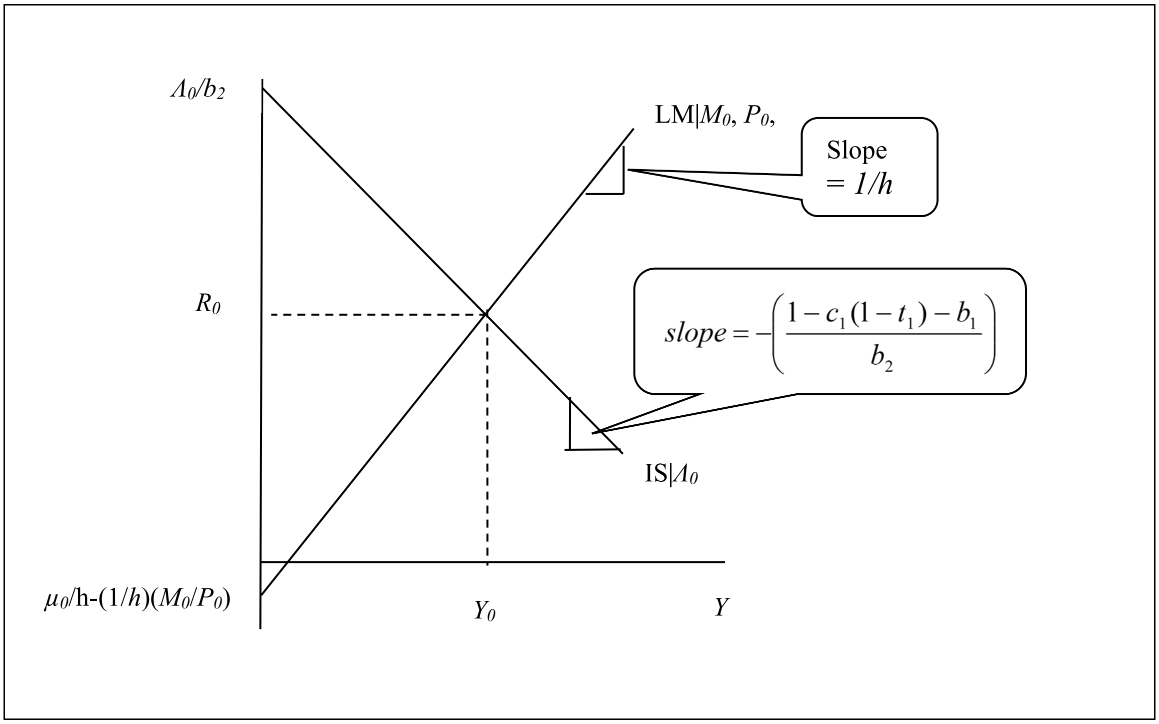
To solve for the equilibrium value of income, substitute the LM into the IS equation from 1.1:

$$Y = \left(\frac{1}{1 - c_1(1-t_1) - b_1}\right) \times \left[\Lambda_0 - b_2\left\langle \frac{\mu_0}{h} - \frac{1}{h}\frac{M_0}{P_0} + \frac{1}{h}Y \right\rangle\right]$$

Move the term in parentheses (.) and the ***(b₂/h)Y*** term to the LHS; factoring out the *Y*'s on the LHS yields, dividing both sides by the term in the parentheses yields:

$$\boxed{Y_0 = \hat{\gamma}[\Lambda_0 - \frac{b_2\mu_0}{h} + \left(\frac{b_2}{h}\right)\left(\frac{M_0}{P_0}\right)]}$$ where $$\hat{\gamma} \equiv \frac{1}{1 - c_1(1-t_1) - b_1 + b_2/h}$$

1.4 Graph the IS and LM curves on one diagram. Clearly indicate the intercepts and the slopes. Label the equilibrium income and interest rate Y_0 and i_0.



2.1 Assume *G* increases by Δ*GO*, and is completely bond financed (assume no portfolio effects here). Calculate the government spending multiplier.

Take the total differential of your answer to 1.3.

$$\Delta Y = \hat{\gamma}\left[\Delta\Lambda - \left(\frac{b_2\Delta\mu}{h}\right) + \left(\frac{b_2}{h}\right)\left(\frac{M}{P}\right)\right]$$

To find the government spending multiplier, set the changes in real money to zero and the money constant, and divide both sides by Δ*GO*:

$$\Delta Y = \hat{\gamma}\Delta GO \Rightarrow \frac{\Delta Y}{\Delta GO} = \hat{\gamma} \equiv \frac{1}{1-c_1(1-t_1)-b_1+b_2/h}$$

2.2 Suppose instead *Tr* increases by Δ*TR*. Calculate the government transfers multiplier.

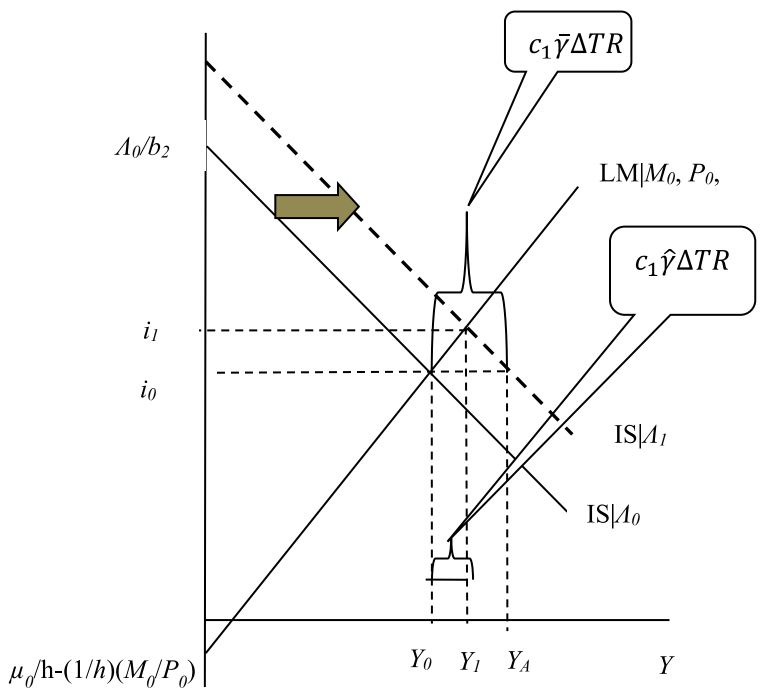
Take the total differential again:

$$\Delta Y = \hat{\gamma}\left[\Delta\Lambda - \frac{b_2\mu_0}{h} + \left(\frac{b_2}{h}\right)\Delta\left(\frac{M}{P}\right)\right]$$

To find the government *transfers* multiplier, set the changes in real money to zero and the money constant, set the change in ΔΛ to equal c_1 ΔTR, and divide both sides by ΔTR:

$$\Delta Y = \hat{\gamma} c_1 \Delta TR \Rightarrow \frac{\Delta Y}{\Delta TR} = \hat{\gamma} c_1 \equiv \frac{c_1}{1 - c_1(1 - t_1) + b_2 / h}$$

2.3 Redraw your answer to 1.4. Then in the same graph, show what happens to the equilibrium income and interest rate if government transfers is increased by ΔTR. Include in your graph the level of income that would be achieved if somehow the interest rate stayed constant (label this point Y_A).



2.4 At the new equilibrium, do we know if investment is higher or lower than the level it started out with? Do we know if it is higher or lower than at Y_A?

Recall the investment function is given by:

$$I = b_0 + b_1 Y - b_2 i$$

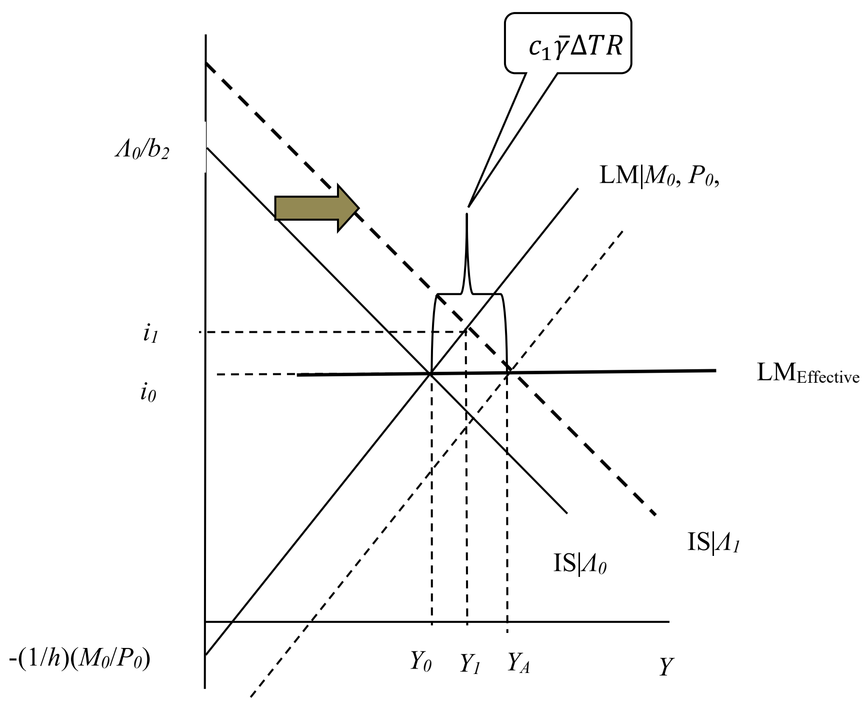
So the change in investment is given by:

$$\Delta I = \Delta b_0 + b_1 \Delta Y - b_2 \Delta i$$

Notice that at the new equilibrium, income is higher (Y_1), but the interest rate is higher as well (i_1). Hence, there are offsetting effects on investment, and the end results could be higher or lower, depending on the magnitudes of the changes in income and interest rates and the parameter values (b_1, $-b_2$).

Regarding the second question, if one were at Y_A, *and* interest rate i_0, then investment would unambiguously be higher.

2.5 Suppose the Fed targets the interest rate at i_0 (call this i_{target}). Returning to 2.3, show graphically what happens if government is increased. What happens to the level of investment?

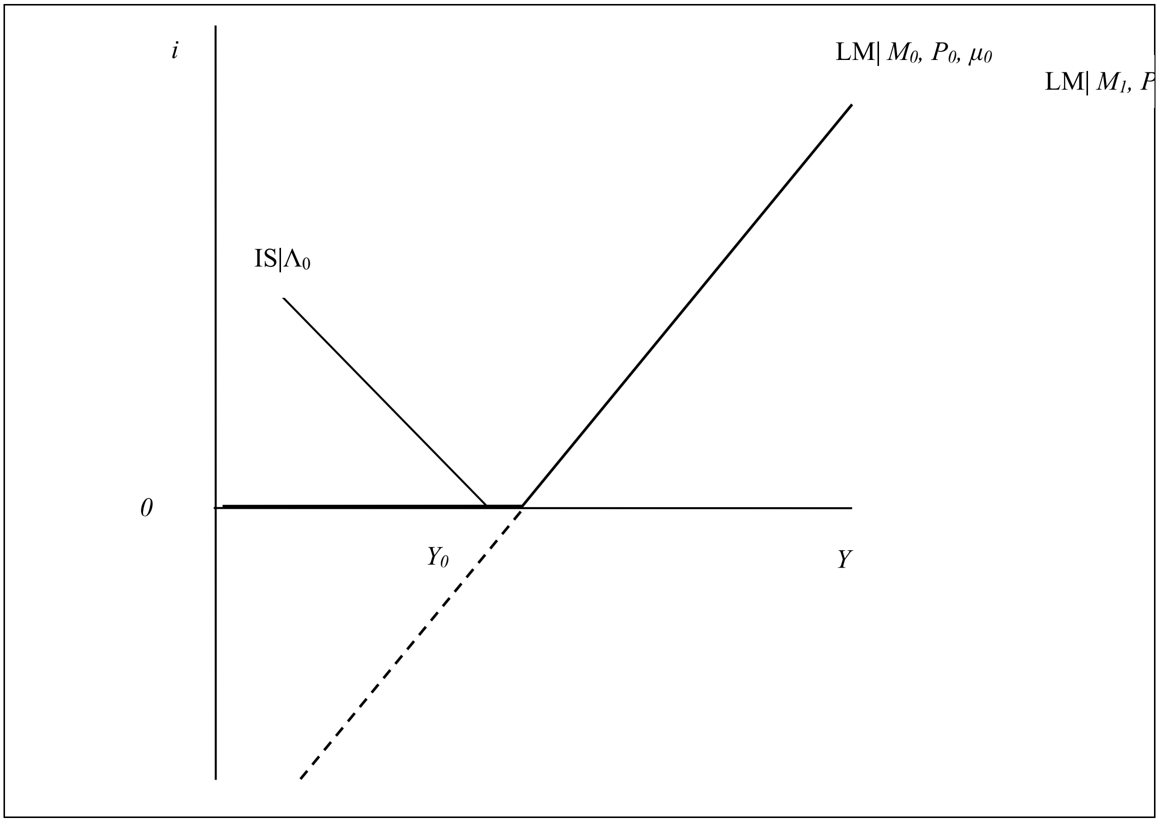


When the Fed targets the interest rate, and the target interest rate remains constant, then the LM is now the Effective LM (flat). An increase in government transfers, increasing autonomous spending, induces an unambiguous increase in investment, since income is higher, but interest rates are unchanged.

Note that the answer is the same if we are in a liquidity trap.

3. Consider the case where the economy is in a liquidity trap.

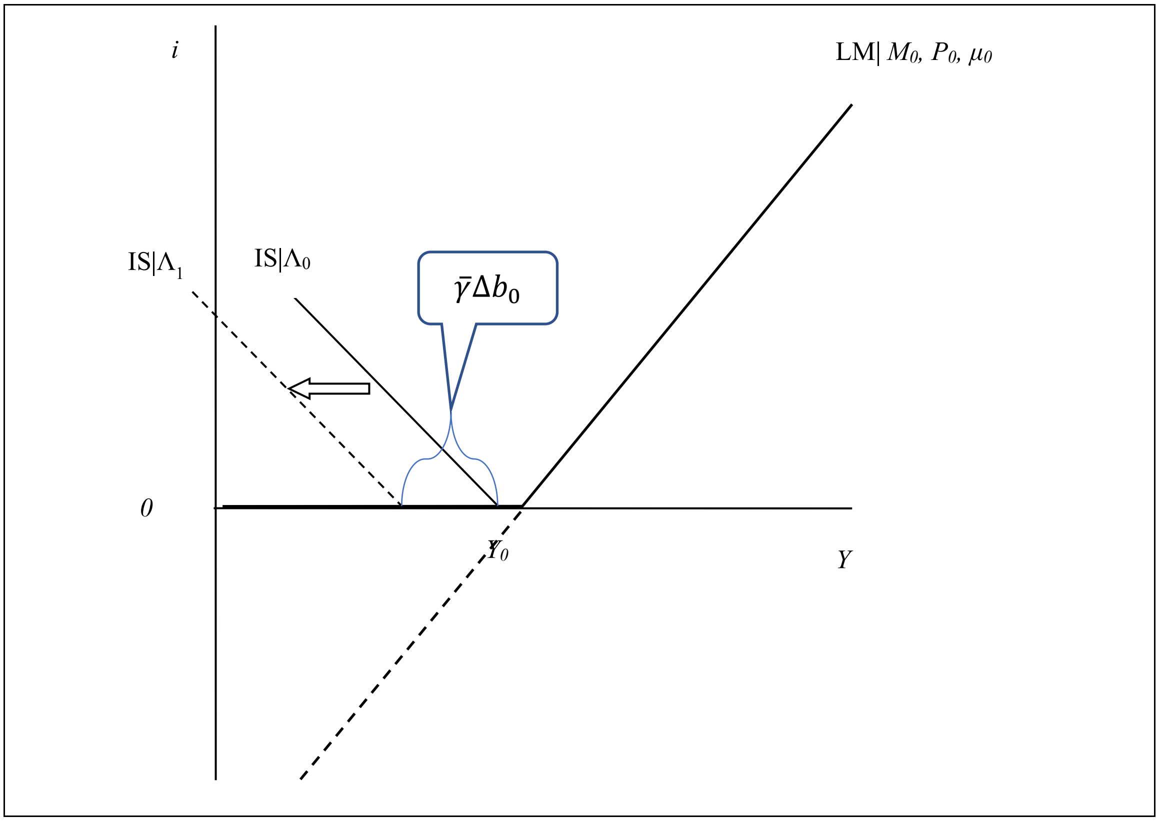
3.1 Draw an IS-LM graph corresponding to this case.



3.2 Show what happens if business owners become more pessimistic, such that the autonomous component of investment declines.

IS shifts in, where:

$$\bar{\gamma} = \frac{1}{[1 - c_1(1 - t_1) - b_1]}$$



3.3 What happens to tax revenues in 3.2? If the budget were initially in balance, what happens to the budget deficit?

Tax revenues are given as:

(5) $T = t_1 Y$ Tax function; t_1 is marginal tax rate.

So, as income falls, tax revenues fall (holding lump sum taxes constant):

$\Delta T = t_1 \Delta Y = t_1 \bar{\gamma} \Delta b_0$

Where

$\Delta b_0 < 0$

The budget balance is given by:

$BuS \equiv T - G = t_1 Y_0 - GO_0$

$\Delta BuS = t_1 \Delta Y - \Delta GO$

$\Delta BuS = t_1 \bar{\gamma} \Delta b_0 < 0$

So the deficit will increase.

3.4 Suppose policymakers wish to improve the budget balance. Would an increase in lump sum taxes accomplish that aim? What would happen to GDP?

Adding in a lump sum tax increase is like making our tax equation like that in the IS-LM handout. Then the budget equation looks like:

$BuS \equiv T - G = t_0 + t_1 Y_0 - GO_0$

$\Delta BuS = \Delta t_0 + t_1 \Delta Y - \Delta GO$

$\Delta Y = -c_1 \bar{\gamma} \Delta t_0$

$\Delta BuS = \Delta t_0 - t_1 c_1 \bar{\gamma} \Delta t_0 = \Delta t_0 (1 - t_1 c_1 \bar{\gamma})$
This looks ambiguous, but in this model, tax revenues *will* rise.

4. Consider the modified IS-LM model as outlined in the "Transactions and Portfolio Crowding Out" handout.
4.1 Suppose the initial budget balance (BuS) is less than zero in period 1. Show graphically what happens in period 2 if nothing is changed with respect to taxes or government spending. Is output higher or lower in period 2?

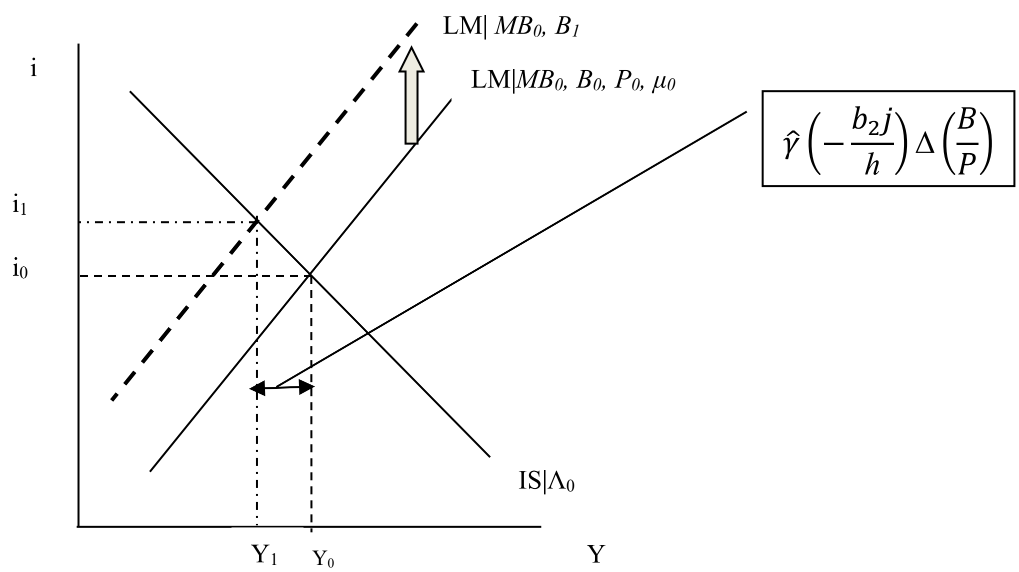
In period 1, there's a deficit, so new bonds have to be issued – i.e., Δ(B/P) > 0; that means the LM curve shifts up (j/h)Δ(B/P) (light gray arrow), even though the IS curve doesn't move (recall there is no change in fiscal policy). Recall:

$$-BuS \equiv BuD \equiv G - T = GO_0 - t_0 = \Delta(B/P)$$

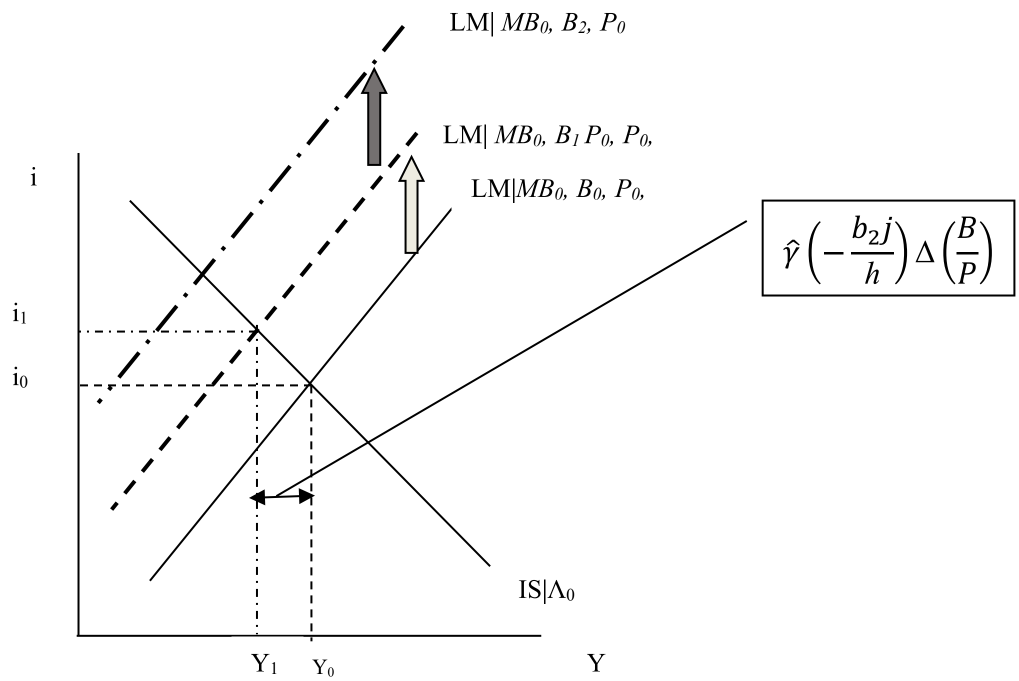
$$\Delta Y = \hat{\gamma}\left[\Delta \Lambda_0 + \frac{b_2 m}{h}\Delta\left(\frac{MB}{P}\right) - \frac{b_2 j}{h}\Delta\left(\frac{MB}{P} + \frac{B}{P}\right) - \frac{b_2 \Delta\mu}{h}\right]$$

$$\Delta Y = \hat{\gamma}\left[-\frac{b_2 j}{h}\Delta\left(\frac{B}{P}\right)\right]$$

Output will be unambiguously lower in period 1.



In period 2, the stock of bonds will increase again if no fiscal action is undertaken, so the LM curve will again shift up by (j/h)Δ(B/P) (dark gray arrow). Output will decrease again.



4.2 Answer 4.1, assuming in period 2 government spending is *reduced* so that the budget is balanced in period 2.

In period 2, with the budget surplus at zero, the stock of bonds will not increase, so the LM curve will not shift up, but the IS curve moves in (dark gray arrow) (recall government spending declines).

$$-BuS \equiv BuD \equiv G - T = GO_0 - t_0 = \Delta\left(\frac{B}{P}\right) = 0$$

$$\Delta Y = \hat{\gamma}[\Delta \mathrm{GO}]$$

Output will be unambiguously lower than in period 1.

4.3 In your answer to 4.2, do you know definitively whether output in period 2 is higher or lower than in period 1? Why or why not?

We know that output is unambiguously lower in this example.

(Whether it would be lower **in period 2** if government spending weren't reduced vs. if it was kept the same is unclear – it depends on the parameters *j*, *h*, and all the other parameters. The smaller *j* is, the more likely cutting government spending is to reduce GDP relative to having kept government spending constant.)

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