1. Suppose the economy is described by the following equations (so we are looking at a closed economy):

**Real Sector**

1. \( Y = Z \) Output equals aggregate demand, an equilibrium condition
2. \( Z = C + I + G \) Definition of aggregate demand
3. \( C = c_o + c_1 Y_D \) Consumption fn, \( c_1 \) is the marginal propensity to consume
4. \( Y_D = Y - T + T_r \) Definition of disposable income
5. \( T = t_1 Y \) Tax function; \( t_1 \) is marginal tax rate.
6. \( T_r = T R_0 \) Transfer payments; \( T R_0 \) is lump sum transfers.
7. \( I = b_0 + b_1 Y - b_2 i \) Investment function
8. \( G = G O_0 \) Government spending on goods and services, exogenous

**Asset Sector**

9. \( \frac{M^d}{P} = \frac{M^s}{P} \) Equilibrium condition
10. \( \frac{M^s}{P} = \frac{M_0}{P} \) Real money supply
11. \( \frac{M^d}{P} = \mu_0 + Y - hi \) Real money demand

1.1 Solve for the LM curve \((i \text{ as a function of } Y)\).

\[
\frac{M_0}{P_0} = \frac{M^s}{P} = \frac{M^d}{P} = \mu_0 + Y - hi
\]

Solving for the interest rate, \( i \), yields the LM curve:

\[
i = \frac{\mu_0}{h} - \left( \frac{1}{h} \left( \frac{M_0}{P_0} \right) + \frac{1}{h} Y \right)
\]

1.2 Solve for the IS curve \((Y \text{ as a function of } i)\).

\[
Y = Z = C + I + G
\]
\[
Y = c_o + c_1 Y_D + b_0 + b_1 Y - b_2 i + G O_0 \quad \text{substitute in for } C, I, G
\]
\[
Y = a_0 + c_1 (Y - T + T_r) + b_0 + b_1 Y - b_2 i + G O_0 \quad \text{substitute in for tax, transfers functions}
\]
\[ Y = a_0 + c_1 (Y - t_1 Y + TR_0) + b_0 + b_1 Y - b_2 i + GO_0 \]

bring the "Y" terms to left hand side.

\[ Y - b(Y - t_1 Y) = Y(1-b(1-t)) = c_0 + c_1 TR_0 + b_0 + GO_0 - b_2 i \]

divide both sides by \((1- c_1 (1-t_1)-b_1)\) and let \(\Lambda = c_0 + c_1 TR_0 + b_0 + GO_0\)

\[ Y_0 = \hat{\varphi}(\Lambda_0 - b_2 i) \]

let \(\hat{\varphi} = \frac{1}{[1 - c_1 (1-t_1) - b_1]}\)

1.3 What is the channel (or variable) by which factors in the monetary or asset sector affect the real goods sector in this model?

Monetary policy influences (in part) interest rates. Interest rates in turn affect investment, and via the simple Keynesian multiplier (\(\hat{\varphi}\)) affects the entire real sector.

1.4 Solve for the equilibrium value of \(Y\).

To solve for the equilibrium value of income, substitute the LM into the IS equation from 1.1:

\[ Y = \left( \frac{1}{1 - c_1 (1-t_1) - b_1} \right) \times \left[ \Lambda_0 - b_2 \left( \frac{\mu_0}{h} - \frac{1}{h} \frac{M_0}{P_0} + \frac{1}{h} Y \right) \right] \]

Move the term in parentheses (.,) and the \((b_2 h)Y\) term to the LHS; factoring out the \(Y\)s on the LHS yields:

Dividing both sides by the term in the parentheses yields:

\[ Y_0 = \hat{\varphi}[\Lambda_0 - b_2 \mu_0 + \left( \frac{b_2}{h} \right) \left( \frac{M_0}{P_0} \right) ] \]

where \(\hat{\varphi} \equiv \frac{1}{1 - c_1 (1-t_1) - b_1 + b_2 / h}\)

1.5 Graph the IS and LM curves on one diagram. Clearly indicate the intercepts and the slopes. Label the equilibrium income and interest rate \(Y_0\) and \(i_0\).
2.1 Assume $G$ decreases by $\Delta GO$, and is completely bond financed (no portfolio effects here). Calculate the government spending multiplier.

Take the total differential of your answer to 1.4.

$$\Delta Y = \hat{\gamma} \left[ \Delta A - \frac{b_2 \mu_0}{h} + \left( \frac{b_2}{h} \right) \Delta \left( \frac{M}{P} \right) \right]$$

To find the government spending multiplier, set the changes in real money to zero and the money constant, and divide both sides by $\Delta GO$:

$$\frac{\Delta Y}{\Delta GO} = \frac{\Delta Y}{\hat{\gamma} \Delta GO} \Rightarrow \hat{\gamma} \equiv \frac{1}{1 - c_1(1-t_1) - b_1 + b_2/h}$$

2.2 Suppose instead $Tr$ decreases by $\Delta TR$. Calculate the government transfers multiplier.

Take the total differential again:

$$\Delta Y = \hat{\gamma} \left[ \Delta \Lambda - \frac{b_2 \mu_0}{h} + \left( \frac{b_2}{h} \right) \Delta \left( \frac{M}{P} \right) \right]$$

To find the government transfers multiplier, set the changes in real money to zero and the money constant, set the change in $\Delta \Lambda$ to equal $c_1 \Delta TR$, and divide both sides by $\Delta TR$:

$$\frac{\Delta Y}{\Delta TR} = \hat{\gamma} c_1 \Rightarrow \hat{\gamma} c_1 \equiv \frac{c_1}{1 - c_1(1-t_1) + b_2/h}$$
2.3 Redraw your answer to 1.5. Then in the same graph, show what happens to the equilibrium income and interest rate if government spending on goods and services is decreased by $\Delta GO$. Include in your graph the level of income that would be achieved if somehow the interest rate stayed constant (label this point $Y_A$).

2.4 At the new equilibrium, do we know if investment is higher or lower than the level it started out with? Do we know if it is higher or lower than at $Y_A$?

Recall the investment function is given by:

$$I = b_0 + b_1 Y - b_2 i$$

So the change in investment is given by:

$$\Delta I = \Delta b_0 + b_1 \Delta Y - b_2 \Delta i$$

Notice that at the new equilibrium, income is lower ($Y_1$), but the interest rate is lower as well ($i_1$). Hence, there are offsetting effects on investment, and the end results could be higher or lower, depending on the magnitudes of the changes in income and interest rates and the parameter values ($b_1$, $-b_2$).

Regarding the second question, if one were at $Y_A$, and interest rate $i_0$, then investment would unambiguously be lower.

2.5 Suppose the Fed targets the interest rate at $i_0$ (call this $i_{\text{target}}$). Returning to 2.3, show graphically what happens if government is decreased. What happens to the level of investment?
When the Fed targets the interest rate, and the target interest rate remains constant, then the LM is now the Effective LM. A drop in government spending, reducing autonomous spending, induces an unambiguous reduction in investment, since income is lower, but interest rates are unchanged.

Note that the answer is the same if we are in a liquidity trap.

3. Consider the Aggregate Demand—Aggregate Supply framework. Suppose government spending is increased when we are in a liquidity trap, and the Fed does NOT target the interest rate. You can assume for simplicity expected inflation is always zero.

3.1 Show what happens in an IS-LM and AD-AS graph in the period the government spending increase occurs, and output ends up below potential GDP, and remains in a liquidity trap.

3.2 Show what happens over time to output, the price level, and the interest rate.

3.3 Show what happens in an IS-LM and AD-AS graph in the period the government spending increase occurs, and output ends up above potential GDP, and out of a liquidity trap.

3.2 Show what happens over time to output, the price level, and the interest rate.