Problem Set 1 Answers

Due in lecture on Thursday, October 2. Be sure to put your name on your problem set. Put “boxes” around your answers to the algebraic questions.

1. Suppose the economy is described by the following equations (so we are looking at a closed economy):

   • Real Sector
     
     (1) \( Y = Z \)  
         Output equals aggregate demand, an equilibrium condition
     
     (2) \( Z = C + I + G \)  
         Definition of aggregate demand
     
     (3) \( C = c_o + c_1Y_D \)  
         Consumption fn, \( c_1 \) is the marginal propensity to consume
     
     (4) \( Y_D = Y - T + Tr \)  
         Definition of disposable income
     
     (5) \( T = t_1Y \)  
         Tax function; \( t_1 \) is marginal tax rate.
     
     (6) \( Tr = TR_0 \)  
         Transfer payments; \( TR_0 \) is lump sum transfers.
     
     (7) \( I = b_0 + b_1Y - b_2i \)  
         Investment function
     
     (8) \( G = GO_0 \)  
         Government spending on goods and services, exogenous

   • Asset Sector
     
     (9) \( \frac{M^d}{P} = \frac{M^*}{P} \)  
         Equilibrium condition
     
     (10) \( \frac{M^*}{P} = \frac{M_0}{P} \)  
         Real money supply
     
     (11) \( \frac{M^d}{P} = \mu_0 + Y - hi \)  
         Real money demand

1.1 Solve for the IS curve (\( Y \) as a function of \( i \)).

\[
Y = Z = C + I + G  \\
Y = c_o + c_1Y_D + b_0 + b_1Y - b_2i + GO_0  \\
Y = a_0 + c_1 (Y - T + Tr) + b_0 + b_1Y - b_2i + GO_0  \\
Y = a_0 + c_1 (Y - t_1Y + TR_0) + b_0 + b_1Y - b_2i + GO_0
\]

substitute in for \( C, I, G \)  
substitute in for \( Y_d \)  
substitute in for tax, transfers functions  
bring the "\( Y \)" terms to left hand side.

\[
Y - b(Y - t_1Y - \lambda Y) = Y(1-b(1-t-\lambda)) = c_o + c_1 TR_0 + b_0 + GO_0 - b_2i
\]

divide both sides by \((1-c_1(1-t_1))\) and let \( A_o = c_o + c_1 TR_0 + b_0 + GO_0 \)

\[
Y_o = \bar{\lambda}A_o  \\
\bar{\lambda} = \frac{1}{[1 - c_1(1-t_1) - b_1]}
\]

1.2 Solve for the LM curve (\( i \) as a function of \( Y \)). What is the channel by which monetary influences affect the real goods sector in this model?

\[
\frac{M_0}{P_o} = \frac{M^*}{P} = \frac{M^d}{P} = \mu_0 + Y - hi
\]
Solving for the interest rate, \( i \), yields the LM curve:

\[
i = \frac{\mu_0}{h} - \frac{1}{h} \left( \frac{M_0}{P_0} \right) + \frac{1}{h} Y
\]

Monetary policy influences (in part) interest rates. Interest rates in turn affect investment, and via the simple Keynesian multiplier (\( \bar{\gamma} \)) affects the entire real sector.

1.3 Solve for the equilibrium values of \( Y \).

To solve for the equilibrium value of income, substitute the LM into the IS equation from 1.1:

\[
Y = \frac{1}{1-c_1(1-t_1) - b_1} \times \left[ \Lambda_0 - b_2 \left( \frac{\mu_0}{h} - \frac{1}{h} \frac{M_0}{P_0} + \frac{1}{h} Y \right) \right]
\]

Move the term in parentheses (.) and the \((b2/h)Y\) term to the LHS; factoring out the \(Y\)'s on the LHS yields:

Dividing both sides by the term in the parentheses yields:

\[
Y_0 = \hat{\gamma} [\Lambda_0 - b_2 \frac{\mu_0}{h} + \left( \frac{b_2}{h} \frac{M_0}{P_0} \right)] \quad \text{where} \quad \hat{\gamma} \equiv \frac{1}{1-c_1(1-t_1) - b_1 + b_2 / h}
\]

1.4 Graph the IS and LM curves on one diagram. Clearly indicate the intercepts and the slopes.

2.1 Assume \( G \) decreases by \( \Delta GO \), and is completely bond financed (no portfolio effects here). Calculate the government spending multiplier.
Take the total differential of your answer to 1.3.

\[ \Delta Y = \hat{\gamma} \left[ \Delta A - \frac{b_2 \mu_0}{h} + \left( \frac{b_2}{h} \right) \Delta \left( \frac{M}{P} \right) \right] \]

To find the government spending multiplier, set the changes in real money to zero and the money constant, and divide both sides by \( \Delta GO \):

\[ \Delta Y = \hat{\gamma} \Delta GO \Rightarrow \frac{\Delta Y}{\Delta GO} = \hat{\gamma} \equiv \frac{1}{1 - c_1 (1 - t_1) - b_1 + b_2 / h} \]

2.2 Suppose instead \( Tr \) decreases by \( \Delta TR \). Calculate the government transfers multiplier.

Take the total differential again:

\[ \Delta Y = \hat{\gamma} \left[ \Delta \Lambda - \frac{b_2 \mu_0}{h} + \left( \frac{b_2}{h} \right) \Delta \left( \frac{M}{P} \right) \right] \]

To find the government transfers multiplier, set the changes in real money to zero and the money constant, set the change in \( \Delta \Lambda \) to equal \( c_1 \Delta TR \), and divide both sides by \( \Delta TR \):

\[ \Delta Y = \hat{\gamma} c_1 \Delta TR \Rightarrow \frac{\Delta Y}{\Delta TR} = \hat{\gamma} c_1 \equiv \frac{c_1}{1 - c_1 (1 - t_1) + b_2 / h} \]

2.3 Redraw your answer to 1.4. Then in the same graph, show what happens to the equilibrium income and interest rate if government spending on goods and services is decreased by \( \Delta GO \). Include in your graph the level of income that would be achieved if somehow the interest rate stayed constant (label this point \( Y_A \)).
2.4 At the new equilibrium, do we know if investment is higher or lower than the level it started out with? Do we know if it is higher or lower than at $Y_A$?

Recall the investment function is given by:

$$I = b_0 + b_1 Y - b_2 i$$

So the change in investment is given by:

$$\Delta I = \Delta b_0 + b_1 \Delta Y - b_2 \Delta i$$

Notice that at the new equilibrium, income is lower ($Y_1$), but the interest rate is lower as well ($i_1$). Hence, there are offsetting effects on investment, and the end results could be higher or lower, depending on the magnitudes of the changes in income and interest rates and the parameter values ($b_1, -b_2$).

Regarding the second question, if one were at $Y_A$, and interest rate $i_0$, then investment would unambiguously be lower.

2.5 Suppose the Fed targets the interest rate at $i_0$ (call this $i_{\text{target}}$). Returning to 2.3, show graphically what happens if government is decreased. What happens to the level of investment?
When the Fed targets the interest rate, and the target interest rate remains constant, then the LM is now the Effective LM. A drop in government spending, reducing autonomous spending, induces an unambiguous reduction in investment, since income is lower, but interest rates are unchanged.

Note that the answer is the same if we are in a liquidity trap.

3. Consider the Aggregate Demand-Aggregate Supply framework. Suppose government spending is reduced when we are not in a liquidity trap (and do not end up in a liquidity trap), and the Fed does NOT target the interest rate. You can assume for simplicity expected inflation is always zero.

3.1 Show what happens in an IS-LM and AD-AS graph in the period the government spending reduction occurs.

3.2 Show what happens over time to output, the price level, and the interest rate.

I will answer questions 3.1 and 3.2 starting at full employment, to simplify the exposition. The reduction in government spending shifts in the IS, AD curves (black arrows). Interest rates fall from $i_1$ to $i_2$; income falls from $Y_1$ to $Y_2$. The price level remains fixed at $P_1$. In period 3, the price level falls, driving up the real money stock, dropping interest rates to $i_3$, spurring investment, so that income rises to $Y_3$. (gray arrows). Over time (white arrows), the price level continues to fall, so that $M/P$ continues to rise, driving the interest rate down to $i_{\text{Final}}$, and output back to $Y^*$.
3.3 Show what the IS-LM and AD-AS graphs look like if initially, the economy is not in a liquidity trap, but interest rates are very close to zero, and output is below potential GDP/full employment. Re-answer 3.1, assuming the economy does end up in a liquidity trap.
First, let’s determine the shape of the AD curve. Consider the LM curve for a given money stock, but different price levels ($P_A$, $P_B$, $P_C$, and $P_D$), and a given IS curve, thus deriving the AD curve.

Notice that when the price level falls sufficiently, the interest rate no longer falls (we are in a liquidity trap). The resulting Aggregate Demand curve is kinked at $Y_C$ (where the economy hits the liquidity trap).
Second, consider the question. When G declines, the IS and AD shift in as shown (gray arrows). Output declines. Notice that over time, the predetermined price line would fall, driving down the price level. However, the declining price level would not shift out the LM curve over the relevant portion, so output would not increase over time. There is no natural self-correcting mechanism in operation here.
4.1. Look up on Bloomberg or elsewhere the yield on a one year T-bill. Calculate the price as if the bond were to mature one year from now (specify the date you looked up the data). Show your calculations.

Here are yields as of 1 October 2014, from Bloomberg.

**US TREASURY YIELDS**

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<th>Coupon</th>
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<th>1 Month</th>
<th>1 Year</th>
<th>Time</th>
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Using the formula, \( P = \frac{100}{1+i} = \frac{100}{1.00085} \), one can solve for \( P = 99.915 \).

4.2 Once again, look up the one year and two year yields. Assuming the expectations hypothesis of the term structure holds, what is the expected interest one year interest rate, one year from today. Show your calculations.

According to the expectations hypothesis of the term structure, for a two year bond, \( i_{t+1}^e = 2i_{2t} - i_t \).
Substituting in 0.26 and 0.17 for the two and one year bond yields results in:

\[ e^{-t_i} = 2 \times 0.0052 - 0.0009 = 0.0095 \]

Or

\[ 0.95\% \]

4.3 Draw the yield curve, for 3 months to 30 years.

Your graph should look like this (or the graph above the table in the answer to 4.1).

![U.S. Treasury Yield Curve](http://finance.yahoo.com)

4.4 Assuming the expectations hypothesis of the term structure holds (i.e., there is no liquidity premium), do you expect the US economy to go into recession in the next year? Why or why not?

As of 10/1, the ten year-three month (ten year-two year) spread is 2.38% (1.87%); in other words the yield curve is upward sloping. Moreover, there has been no inversion since the end of the last recession. Historically, in the United States, this has meant that recession is unlikely over the next six months or year.