Midterm Exam

You have 75 minutes to complete this 70 minute exam. Be sure to “box in” your answers. Show your work (so that partial credit can be granted if the final answer is incorrect).

1. [20 minutes] Suppose one is examining the term structure of a 3 year discount bond, and the expectations hypothesis of the term structure holds.

\[ i_{3t} = \frac{\left( i_t + i_{t+1}^e + i_{t+2}^e \right)}{3} \]  

Suppose yesterday,
\[ i_{3t} = 0.15 \]
\[ i_t = 0.05 \]

1.1 (5 minutes) Calculate the average value of \( i_{t+1}^e \) and \( i_{t+2}^e \).

1.2 (5 minutes) Returning to equation (1), suppose today the yield to maturity on the 3 year discount bond has increased by \( \Delta i_{3t} \), while the yield to maturity on a one year bond is unchanged from yesterday. Derive the algebraic expression for the implied change in the expected one year yield, assuming that the short term expected yield on the one year bond in period \( t+2 \) is unchanged.

1.3. (5 minutes) Assume the 3 year bond yield is given by:

\[ i_{3t} = \frac{\left( i_t + i_{t+1}^e + i_{t+2}^e \right)}{3} + rp_{3t} \]  

And the yield on the 3 year discount bond has increased by \( \Delta i_{3t} \). Calculate the implied increase in the liquidity premium for the three year bond, if it is assumed that all the expectations of future one year interest rates are unchanged. Can you draw a picture of how the yield curve has changed (you can assume the liquidity premium on one year interest rates are always zero)?

1.4 (5 minutes) Assume \( rp_{nt} \) is always zero, so the pure expectations hypothesis of the term structure holds. Can the central bank affect the long term interest rate by purchasing additional amounts of long term bonds? Explain your answer in words and/or equations.

2. [20 minutes] Suppose the stock price is given by:

\[ P_t = \frac{D_{t+1}}{1 + rp + rf} + \frac{E_t P_{t+1}}{1 + rp + rf} \]  

2.1 (5 minutes) Show how to derive the current stock price as a function of stock prices at time \( t+3 \).

2.2 (7 minutes) Derive the expression for the stock price as a function of only the present discounted value of expected dividends. Explain what assumption(s) you need to make in order to obtain this answer.
2.3 (8 minutes) Assume you can take the log of equation (3), so that \( \log(E(X)) = E(\log(X))\), assuming no dividends are paid out. Derive a mathematical expression that describes the evolution of \(\log\) stock prices over time. Describe in words the behavior of \(\log\) stock prices.

3. [15 minutes] Consider a Bank that has the following balance sheet:

3.1 (8 minutes) Suppose the bank has the following structure:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves</td>
<td>$50 million</td>
</tr>
<tr>
<td>Securities</td>
<td>$25 million</td>
</tr>
<tr>
<td>Govt Sec.</td>
<td>$25 million</td>
</tr>
<tr>
<td>Loans</td>
<td>$150 million</td>
</tr>
<tr>
<td>Checkable deposits</td>
<td>$230 million</td>
</tr>
<tr>
<td>Bank capital</td>
<td>$20 million</td>
</tr>
</tbody>
</table>

Under the Basel II guidelines, reserves and government securities would have zero weight in assets (while securities and loans have weight of one); calculate the capital ratio for this bank. Show your work.

3.2 (7 minutes) Suppose the government securities are actually as risky as non-government securities. Calculate the true capital-asset ratio.

5. [15 minutes] Consider a Taylor rule of the following form:

\[
i_t^{FedFunds} = \pi_t + 0.5 \times (y_t - y^*_t) + 0.5 \times (\pi_t - \pi^*_t) + r_t^*
\]  

5.1 (5 minutes) Suppose the output gap is -6.0\%, and the inflation rate is 1.0\%. Calculate the implied Fed funds rate, assuming the equilibrium real rate of interest is 2\%, and target inflation rate is 2\%. Show your work.

5.2 (5 minutes) Suppose instead of the standard Taylor rule in (4), one places a coefficient of one on the inflation gap variable. Calculate the target interest rate in this case.

5.3 (5 minutes) Suppose in (4) the equilibrium real rate of interest increased from 2\% to 3\%. In words, explain what the Fed would do, and how it would accomplish it with the policy instruments it has at its disposal.