Economics 435
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University of Wisconsin-Madison

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Social Sciences 7418

## Midterm Exam 1 Answers

You have 65 minutes to complete this 60 minute exam. All your answers go in the bluebook. Be sure to "box in" your algebraic or numerical answers. Show your work (so that partial credit can be granted if the final answer is incorrect).

1. [10 minutes] Consider the Aggregate Demand-Aggregate Supply framework. Assume oil prices suddenly increase in period 1, and then stay permanently higher. You can assume that the economy starts at both short run and long run equilibrium.
1.1 (4 minutes) Show what happens in AD-AS graph in the period the shock occurs; assume expected inflation is zero to begin with.

1.2 (6 minutes) Reproduce your answer to Question 1.1. Using that graph, suppose in period 1, the government responds to the price shock by increasing spending so as to restore output to initial levels. Further assume:

$$
\pi_{t}^{e}=\pi_{t-1}
$$

Show what happens in periods 1,2 .

2. [25 minutes] Suppose the IS curve is given by:

$$
\begin{gathered}
Y=\left(\frac{1}{1-c_{1}}\right)\left[\Lambda_{0}-b_{2} i\right] \\
\Lambda_{0} \equiv c_{0}+b_{0}+G O_{0}
\end{gathered}
$$

And the LM curve is given by the following (because money demand also depends on wealth):

$$
i=\frac{\mu_{0}}{h}-\left(\frac{1}{h}\right)\left(\frac{m M B_{0}}{P_{0}}\right)+\left(\frac{j}{h}\right)\left(\frac{M B_{0}+B_{0}}{P_{0}}\right)+\left(\frac{1}{h}\right) Y
$$

Where $m$ is the money multiplier (the ratio of money stock to money base, which is assumed to be a constant), and $0<j<1$.
2.1 ( 8 minutes) Solve for the equilibrium interest rate.

Substitute the IS into the LM curve:

$$
\begin{gathered}
i=\frac{\mu_{0}}{h}-\left(\frac{1}{h}\right)\left(\frac{m M B_{0}}{P_{0}}\right)+\left(\frac{j}{h}\right)\left(\frac{M B_{0}+B_{0}}{P_{0}}\right)+\left(\frac{1}{h}\right)\left(\frac{1}{1-c_{1}}\right)\left[\Lambda_{0}-b_{2} i\right] \\
h i=\mu_{0}-\left(\frac{m M B_{0}}{P_{0}}\right)+j\left(\frac{M B_{0}+B_{0}}{P_{0}}\right)+\left(\frac{1}{1-c_{1}}\right) \Lambda_{0}-\left(\frac{b_{2}}{1-c_{1}}\right) i \\
h i+\left(\frac{b_{2}}{1-c_{1}}\right) i=\mu_{0}-\left(\frac{m M B_{0}}{P_{0}}\right)+j\left(\frac{M B_{0}+B_{0}}{P_{0}}\right)+\left(\frac{1}{1-c_{1}}\right) \Lambda_{0} \\
\frac{h\left(1-c_{1}\right)}{\left(1-c_{1}\right)} i+\left(\frac{b_{2}}{1-c_{1}}\right) i=\mu_{0}-\left(\frac{m M B_{0}}{P_{0}}\right)+j\left(\frac{M B_{0}+B_{0}}{P_{0}}\right)+\left(\frac{1}{1-c_{1}}\right) \Lambda_{0} \\
\frac{h\left(1-c_{1}\right)+b_{2}}{\left(1-c_{1}\right)} i=\mu_{0}-\left(\frac{m M B_{0}}{P_{0}}\right)+j\left(\frac{M B_{0}+B_{0}}{P_{0}}\right)+\left(\frac{1}{1-c_{1}}\right) \Lambda_{0} \\
i=\frac{\left(1-c_{1}\right)}{h\left(1-c_{1}\right)+b_{2}}\left[\mu_{0}-\left(\frac{m M B_{0}}{P_{0}}\right)+j\left(\frac{M B_{0}+B_{0}}{P_{0}}\right)+\left(\frac{1}{1-c_{1}}\right) \Lambda_{0}\right] \\
i_{0}=\left(\frac{\left(1-c_{1}\right)}{h\left(1-c_{1}\right)+b_{2}}\right)\left[\mu_{0}-\left(\frac{m M B_{0}}{P_{0}}\right)+j\left(\frac{M B_{0}+B_{0}}{P_{0}}\right)\right]+\left(\frac{1}{h\left(1-c_{1}\right)+b_{2}}\right) \Lambda_{0}
\end{gathered}
$$

2.2 ( 7 minutes) Calculate the change in the interest rate is for a change in government spending. Assume the budget is initially in balance. Show your work.

Take the total differential to your solution to 2.1:

$$
\begin{gathered}
\Delta i=\left(\frac{\left(1-c_{1}\right)}{h\left(1-c_{1}\right)+b_{2}}\right)\left[\Delta \mu-m \Delta\left(\frac{M B}{P}\right)+j \Delta\left(\frac{M B}{P}\right)+j \Delta\left(\frac{B}{P}\right)\right] \\
+\left(\frac{1}{h\left(1-c_{1}\right)+b_{2}}\right) \Delta \Lambda
\end{gathered}
$$

The constant in the money demand equation $\mu$ doesn't change; for a pure fiscal policy, money base doesn't change either. The change in autonomous spending is due solely to the change in government spending on goods and services. Hence, one gets:

$$
\Delta i=\left(\frac{\left(1-c_{1}\right)}{h\left(1-c_{1}\right)+b_{2}}\right)\left[j \Delta\left(\frac{B}{P}\right)\right]+\left(\frac{1}{h\left(1-c_{1}\right)+b_{2}}\right) \Delta G O
$$

If one is starting from initial budget balance, then an increase in government spending (with the marginal tax rate set to zero) is accompanied by an increase of real bonds of equal amount, viz: $\Delta G O=\Delta\left(\frac{B}{P}\right)$, leading to:

$$
\Delta i=\left(\frac{\left(1-c_{1}\right)}{h\left(1-c_{1}\right)+b_{2}}\right)[j(\Delta G O)]+\left(\frac{1}{h\left(1-c_{1}\right)+b_{2}}\right) \Delta G O
$$

Or simplifying:

$$
\Delta i=\left(\frac{\left(1-c_{1}\right) j+1}{h\left(1-c_{1}\right)+b_{2}}\right) \Delta G O>0
$$

2.3 (5 minutes) If one increases the money base, does the interest rate fall or rise, for $m>1$ ? If possible, show your work.

Return to the total differential obtained in 2.2:

$$
\begin{gathered}
\Delta i=\left(\frac{\left(1-c_{1}\right)}{h\left(1-c_{1}\right)+b_{2}}\right)\left[\Delta \mu-m \Delta\left(\frac{M B}{P}\right)+j \Delta\left(\frac{M B}{P}\right)+j \Delta\left(\frac{B}{P}\right)\right] \\
+\left(\frac{1}{h\left(1-c_{1}\right)+b_{2}}\right) \Delta \Lambda
\end{gathered}
$$

The constant in the money demand equation $\mu$ doesn't change; for a pure monetary policy, government spending and the stock of bonds don't change either. Hence, one gets:

$$
\Delta i=\left(\frac{\left(1-c_{1}\right)}{h\left(1-c_{1}\right)+b_{2}}\right)\left[-m \Delta\left(\frac{M B}{P}\right)+j \Delta\left(\frac{M B}{P}\right)\right]
$$

Or simplifying:

$$
\Delta i=\left(\frac{\left(1-c_{1}\right)}{h\left(1-c_{1}\right)+b_{2}}\right)(j-m) \Delta\left(\frac{M B}{P}\right)<0 \text { for } m>1,0<j<1
$$

2.4 (5 minutes) Show, either graphically or algebraically what is the impact of increasing government spending (starting from initial budget balance) on the interest rate if the Fed targets the interest rate.

To show this graphically, consider the IS-LM diagram:


When the Fed targets the interest rate, and the target interest rate remains constant, then the LM is flat; the increase in GDP is equal to $\bar{\gamma} \Delta G O$ where $\bar{\gamma} \equiv \frac{1}{1-c_{1}}>\frac{\left(1-\frac{b_{2} j}{h}\right)}{1-c_{1}+\frac{b_{2}}{h}}$.

To see this algebraically, note the solution for equilibrium income in this model is obtained by substituting the given LM into the given IS curve, and solving for Y :

$$
Y_{0}=\hat{\gamma}\left[\Lambda_{0}+\frac{b_{2}}{h}\left(\frac{m M B_{0}}{P_{0}}\right)-\frac{b_{2} j}{h}\left(\frac{M B_{0}}{P_{0}}+\frac{B_{0}}{P_{0}}\right)-\frac{b_{2} \mu_{0}}{h}\right]
$$

Where $\hat{\gamma} \equiv \frac{\left(1-\frac{b_{2} j}{h}\right)}{1-c_{1}+\frac{b_{2}}{h}}$

Taking the total differential, and starting from initial budget balance (as in 2.1), one obtains:

$$
\Delta Y=\hat{\gamma}\left[\Delta G O-\frac{b_{2} j}{h} \Delta G O\right]=\hat{\gamma}\left(1-\frac{b_{2} j}{h}\right) \Delta G O
$$

Where the change in government spending equals the change in real debt. Note that when the Fed targets the interest rate, this is the same as $h=\infty$. Then $\hat{\gamma} \equiv \frac{\left(1-\frac{b_{2} j}{h}\right)}{1-c_{1}+\frac{b_{2}}{h}}=\frac{1}{1-c_{1}}$
3. [15 minutes] Suppose one is examining the term structure of a 7 year bond, and a 5 year bond, and the expectations hypothesis of the term structure holds.
$i_{7 t}=\frac{\left(i_{1 t}+i_{1 t+1}^{e}+i_{1 t+2}^{e}+i_{1 t+3}^{e}+i_{1 t+4}^{e}+i_{1 t+5}^{e}+i_{1 t+6}^{e}\right)}{7}$
$i_{5 t}=\frac{\left(i_{1 t}+i_{1 t+1}^{e}+i_{1 t+2}^{e}+i_{1 t+3}^{e}+i_{1 t+4}^{e}\right)}{5}$
3.1 ( 7 minutes) Solve for the average of 1 year interest rates in starting in year 5 , and starting in year 6, i.e., solve for the average value of $i_{1 t+5}^{e}$ and $i_{1 t+6}^{e}$.?

Take the expression for the 7 year bond, multiply by 7 :
$7 \times i_{7 t}=\left(i_{1 t}+i_{1 t+1}^{e}+i_{1 t+2}^{e}+i_{1 t+3}^{e}+i_{1 t+4}^{e}+i_{1 t+5}^{e}+i_{1 t+6}^{e}\right)$
Notice the first five terms in the parentheses equals 5 times the 5 year bond:
$7 \times i_{7 t}=\left(5 \times i_{5 t}+i_{1 t+5}^{e}+i_{1 t+6}^{e}\right)$
$7 \times i_{7 t}-5 \times i_{5 t}=\left(i_{1 t+5}^{e}+i_{1 t+6}^{e}\right)$
$\frac{7 \times i_{7 t}-5 \times i_{5 t}}{2}=\frac{\left(i_{1 t+5}^{e}+i_{1 t+6}^{e}\right)}{2}$

$$
\frac{7 \times i_{7 t}-5 \times i_{5 t}}{2}=\frac{\left(i_{1 t+5}^{e}+i_{1 t+6}^{e}\right)}{2}
$$

3.2. (3 minutes) Suppose the values are:,

$$
\begin{aligned}
& i_{7 t}=0.10 \\
& i_{5 t}=0.08
\end{aligned}
$$

Calculate the average value of $i_{t+1}^{e}$ and $-i_{t+2}^{e} i_{1 t+5}^{e}$ and $i_{1 t+6}^{e}$.

$$
\begin{aligned}
& \frac{7 \times i_{7 t}-5 \times i_{5 t}}{2}=\frac{\left(i_{1 t+5}^{e}+i_{1 t+6}^{e}\right)}{2} \\
& \frac{7 \times .10-5 \times .08}{2}=\frac{(.30)}{2}=0.15
\end{aligned}
$$

3.3. ( 5 minutes) Assume the 3 year bond yield is given by:
$i_{7 t}=\frac{\left(i_{1 t}+i_{1 t+1}^{e}+\cdots+i_{1 t+6}^{e}\right)}{3}+r p_{7 t}$
And going from one day to the next day the yield to maturity on the 7 year bond has increased by $\Delta i_{7}$. Can one say whether the increase is due to change in expected future rates, or due to a change in the risk premium? Why or why not?
$i_{7 t}=\frac{\left(i_{1 t}+i_{1 t+1}^{e}+\cdots+i_{1 t+6}^{e}\right)}{3}+r p_{7 t}$
Take total differential
$\Delta i_{7 t}=\frac{\left(\Delta i_{1 t}+\Delta i_{1 t+1}^{e}+\cdots+\Delta i_{1 t+6}^{e}\right)}{3}+\Delta r p_{7 t}$
So without further information, the change in the 7 year yield could be due to changes in expected future short rates, or a change in the risk premium associated with the 7 year bond.
4. [10 minutes] Suppose the stock price is given by:
$P_{t}=\frac{D_{t+1}}{1+i^{\text {equity }}}+\frac{E_{t} P_{t+1}}{1+i^{\text {equity }}}$
Where $i^{\text {equity }}$ the nominal interest rate investors require in order to hold equities.
4.1 (4 minutes) Derive the current stock price as a function of stock prices at time $\mathrm{t}+3$.

Recursively substitute in for P . Note that the price in $\mathrm{t}+1$ is given by:

$$
\begin{equation*}
P_{t+1}=\frac{D_{t+2}}{1+r f+r p}+\frac{E_{t+1} P_{t+2}}{1+r f+r p} \tag{i}
\end{equation*}
$$

Substituting into (2) yields:

$$
\begin{equation*}
P_{t}=\frac{D_{t+1}}{1+r f+r p}+\frac{E_{t}\left(E_{t+1} D_{t+2}\right)}{(1+r f+r p)(1+r f+r p)}+\frac{E_{t}\left(E_{t+1} P_{t+2}\right)}{(1+r f+r p)(1+r f+r p)} \tag{ii}
\end{equation*}
$$

Note that by the "Law of Iterated Expectations", viz.,

$$
E_{t}\left(E_{t+1}\left(Z_{t+2}\right)\right)=E_{t} Z_{t+2}
$$

Equation (2) becomes:

$$
\begin{equation*}
P_{t}=\frac{D_{t+1}}{1+r f+r p}+\frac{E_{t} D_{t+2}}{(1+r f+r p)^{2}}+\frac{E_{t} P_{t+2}}{(1+r f+r p)^{2}} \tag{iii}
\end{equation*}
$$

Repeating, yields:

$$
P_{t}=\frac{D_{t+1}}{1+r f+r p}+\frac{E_{t} D_{t+2}}{(1+r f+r p)^{2}}+\frac{E_{t} D_{t+3}}{(1+r f+r p)^{3}}+\frac{E_{t} P_{t+3}}{(1+r f+r p)^{3}}
$$

4.2 ( 6 minutes) Consider the following graph of the S\&P 500, the 3 month Treasury rate, and the VIX, a measure of perceived risk in the stock market. Can you explain why the S\&P500 behaved the way it did in March 2020 onward? Specifically, why the drop, and then why the acceleration in the growth rate of the stock index? Explain in the context of your answer to problem 4.1.

The answer to problem 4.1 is:

$$
P_{t}=\frac{D_{t+1}}{1+r f+r p}+\frac{E_{t} D_{t+2}}{(1+r f+r p)^{2}}+\frac{E_{t} D_{t+3}}{(1+r f+r p)^{3}}+\frac{E_{t} P_{t+3}}{(1+r f+r p)^{3}}
$$

The risk premium ( $r p$ ) rises with the VIX in March 2020, driving down P. After the VIX falls in May 2020 onward, prices rise. But the risk free interest rate (rf) has fallen from March 2020 onward, so P rises from that reason as well.


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