Economics 435
Fall 2012
University of Wisconsin-Madison

Menzie D. Chinn
Social Sciences 7418

Present Value, Future Value, and Bonds

For one period:

$$FV_t = PV_t + PV_t \times i = PV_t(1+i)$$

$$PV_t = \frac{FV_t}{(1+i)}$$

In general:

$$FV_{n,t} = PV_t(1+i)^n$$

$$PV_t = \frac{FV_{n,t}}{(1+i)^n}$$

When the interest rate and payments are not constant:

$$PV_t = \left[\frac{X_{t+1}}{(1+i_t)} + \frac{X_{t+2}}{(1+i_t)\times(1+i_{t+1})} + \ldots\ldots + \frac{X_{t+n}}{(1+i_t)\times(1+i_{t+1})\ldots(1+i_{t+n-1})} \right]$$

And when the future is not known:

$$PV_t = \left[\frac{\mathcal{E}_t X_{t+1}}{(1+i_t)} + \frac{\mathcal{E}_t X_{t+2}}{(1+i_t)\times(1+\mathcal{E}_t i_{t+1})} + \ldots\ldots + \frac{\mathcal{E}_t X_{t+n}}{(1+i_t)\times(1+\mathcal{E}_t i_{t+1})\ldots(1+\mathcal{E}_t i_{t+n-1})} \right]$$

Types of bonds:

- Zero-coupon or discount bonds
- Fixed payment loans
- Coupon bonds
- Consols

For one year discount bonds,

$$1+i_t = \frac{FaceValue_t}{P_{DB,t}} \quad \text{or} \quad i_t = \frac{FaceValue_t}{P_{DB,t}} - 1$$

For fixed payment loan, where fixed payments and interest rates are constant:

$$P_{FPL,t} = \frac{FixedPaym't}{(1+i)} + \frac{FixedPaym't}{(1+i)^2} + \ldots + \frac{FixedPaym't}{(1+i)^n}$$

For coupon bonds where the coupons and interest rates are constant,

$$P_{CB,t} = \frac{CouponPaym't}{(1+i)} + \frac{CouponPaym't}{(1+i)^2} + \ldots + \frac{CouponPaym't}{(1+i)^n} + \frac{FaceValue}{(1+i)^n}$$

For consols,

$$P_{Consol,t} = \frac{CouponPaym't}{(1+i)} + \frac{CouponPaym't}{(1+i)^2} + \ldots + \frac{CouponPaym't}{(1+i)^n} = \frac{CouponPaym't}{(i)^n}$$

The value of a bond varies inversely with the interest rate used to calculate the present value of the promised payment.

Types of interest rates:

- Yield to maturity
- Current yield
- Holding period yield

Yield to maturity for coupon bond is the interest rate that solves:

$$P_{CB,t} = \frac{CouponPaym't}{(1+i)} + \frac{CouponPaym't}{(1+i)^2} + \ldots + \frac{CouponPaym't}{(1+i)^n} + \frac{FaceValue}{(1+i)^n}$$

Current yield on a coupon bond is:

$$= \frac{CouponPaym't}{P_{CB,t}}$$

Holding period yield on a coupon bond held for one period is:

$$= \frac{CouponPaym't}{P_{CB,t}} + \frac{P_{CB,t+1} - P_{CB,t}}{P_{CB,t}}$$

Where the first term is the current yield, and the second is the capital gains.

The expected (or ex ante) real interest rate is given by the Fisher equation:

$$i_t = r_t + \pi^e_{t+1}$$

$$r_t = i_t - \pi^e_{t+1}$$

The ex post real interest rate is given by:

$$r_t^{\exp ost} = i_t - \pi_{t+1}$$

Determination of interest rates (by way of determination of bond prices)

Figure 6.2 A Shift in the Supply of Bonds

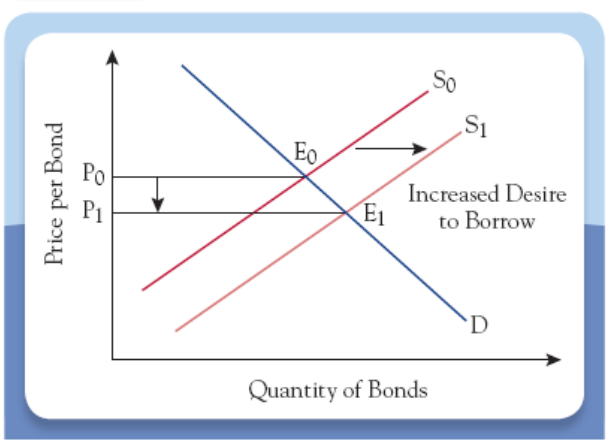

Figure 6.3 A Shift in the Demand for Bonds