Present Value, Future Value, and Bonds

For one period:

\[ \text{PV}_1 = \text{FV}_1 + \text{PV}_1 \times i = \text{PV}_1 (1 + i) \]

\[ \text{PV}_1 = \frac{\text{FV}_1}{1 + i} \]

In general:

\[ \text{FV}_{n,t} = \text{PV}_t (1 + i)^n \]

\[ \text{PV}_t = \frac{\text{FV}_{n,t}}{(1 + i)^n} \]

When the interest rate and payments are not constant:

\[ \text{PV}_t = \left[ \frac{X_{t+1}}{(1 + i_t)} + \frac{X_{t+2}}{(1 + i_t)(1 + i_{t+1})} + ... + \frac{X_{t+n}}{(1 + i_t)(1 + i_{t+1})...(1 + i_{t+n-1})} \right] \]

And when the future is not known:

\[ \text{PV}_t = \left[ \frac{\text{E}X_{t+1}}{(1 + i_t)} + \frac{\text{E}X_{t+2}}{(1 + i_t)(1 + \text{E}i_{t+1})} + ... + \frac{\text{E}X_{t+n}}{(1 + i_t)(1 + \text{E}i_{t+1})...(1 + \text{E}i_{t+n-1})} \right] \]

Types of bonds:

- Zero-coupon or discount bonds
- Fixed payment loans
- Coupon bonds
- Consols

For one year discount bonds,

\[ 1 + i_t = \frac{\text{FaceValue}_t}{P_{DB,t}} \quad \text{or} \quad i_t = \frac{\text{FaceValue}_t}{P_{DB,t}} - 1 \]

For fixed payment loan, where fixed payments and interest rates are constant:

\[ P_{FPL,t} = \frac{\text{FixedPaym't}_t}{(1 + i)} + \frac{\text{FixedPaym't}_t}{(1 + i)^2} + ... + \frac{\text{FixedPaym't}_t}{(1 + i)^n} \]

For coupon bonds where the coupons and interest rates are constant,

\[ P_{CB,t} = \frac{\text{CouponPaym't}_t}{(1 + i)} + \frac{\text{CouponPaym't}_t}{(1 + i)^2} + ... + \frac{\text{CouponPaym't}_t}{(1 + i)^n} + \frac{\text{FaceValue}}{(1 + i)^n} \]

For consols,

\[ P_{Consol,t} = \frac{\text{CouponPaym't}_t}{(1 + i)} + \frac{\text{CouponPaym't}_t}{(1 + i)^2} + ... + \frac{\text{CouponPaym't}_t}{(1 + i)^n} = \frac{\text{CouponPaym't}_t}{(i)^n} \]
The value of a bond varies inversely with the interest rate used to calculate the present value of the promised payment.

Types of interest rates:
- Yield to maturity
- Current yield
- Holding period yield

Yield to maturity for coupon bond is the interest rate that solves:
\[
P_{CB,t} = \frac{CouponPaym't}{(1 + i)} + \frac{CouponPaym't}{(1 + i)^2} + \cdots + \frac{CouponPaym't}{(1 + i)^n} + \frac{FaceValue}{(1 + i)^n}
\]

Current yield on a coupon bond is:
\[
= \frac{CouponPaym't}{P_{CB,t}}
\]

Holding period yield on a coupon bond held for one period is:
\[
= \frac{CouponPaym't}{P_{CB,t}} + \frac{P_{CB,t+1} - P_{CB,t}}{P_{CB,t}}
\]

Where the first term is the current yield, and the second is the capital gains.

The expected (or ex ante) real interest rate is given by the Fisher equation:
\[
i_t = r_t + \pi_{t+1}^e
\]
\[
r_t = i_t - \pi_{t+1}^e
\]

The ex post real interest rate is given by:
\[
r^\text{exp}_t = i_t - \pi_{t+1}^e
\]

Determination of interest rates (by way of determination of bond prices)