Economics 435
The Financial System
(10/4/21)

Instructor: Prof. Menzie Chinn
UW Madison
Fall 2021

Future Value and Present Value

- If the present value is $100 and the interest rate is 5%, then the *future value* one year from now is:

$$\$100 + \$100(0.05) = \$105$$

- This also shows that the higher the interest rate, the higher the future value.

- In general:

$$FV = PV + PV(i) = PV(1 + i)$$

- And:

$$PV = \frac{FV}{(1 + i)}$$

Future Value and Compound Interest

- What if you leave your $100 in the bank for two years at 5% yearly interest rate?
- The future value is:

$$\$100 + \$100(0.05) + \$100(0.05) + \$5(0.05) = \$110.25$$

$$\$100(1.05)(1.05) = \$100(1.05)^2$$

- In general

$$FV_n = PV(1 + i)^n$$

$$PV = \frac{FV}{(1 + i)^n}$$

Complications

- What if payments, X_t, occur all the way along until the end?

- What if the interest rate, i_t, is not constant?

$$PV_t = \left[\frac{X_{t+1}}{(1+i_t)} + \frac{X_{t+2}}{(1+i_t) \times (1+i_{t+1})} + ... + \frac{X_{t+n}}{(1+i_t) \times (1+i_{t+1})...(1+i_{t+n-1})} \right]$$

- But at time t, one doesn't know t+n information … so:

$$PV_t = \left[\mathcal{E}_t \frac{X_{t+1}}{(1+i_t)} + \mathcal{E}_t \frac{X_{t+2}}{(1+i_t) \times (1+i_{t+1})} + ... + \mathcal{E}_t \frac{X_{t+n}}{(1+i_t) \times (1+i_{t+1})...(1+i_{t+n-1})} \right]$$

Bond Basics

- The most common type of bond is a coupon bond.
    - Issuer is required to make annual payments, called coupon payments.
    - The annual interest the borrower pays (i_c), is the coupon rate.
    - The date on which the payments stop and the loan is repaid (n), is the maturity date or term to maturity.
    - The final payment is the principal, face value, or par value of the bond.

4-5

Bond Prices

1. Zero-coupon or discount bond
- Promise a single payment on a future date
- Example: Treasury bill

2. Fixed-payment loan
- Sequence of fixed payments
- Example: Mortgage or car loan

3. Coupon bond
- periodic interest payments + principal repayment at maturity
- Example: U.S. Treasury Bonds and most corporate bonds

4. Consol
- periodic interest payments forever, principal never repaid
- Example: U.K. government has some outstanding

Zero-Coupon Bonds

- U.S. Treasury bills (T-bills) are the most straightforward type of bond.
  - Each T-bill represents a promise by the U.S. government to pay $100 on a fixed future date.
  - No coupon payments - zero-coupon bonds
  - Also called pure discount bonds (or discount bonds) since the price is less than face value - they sell at a discount.
- Price of $100 face value zero-coupon bond

$$= \frac{\$100}{(1+i)^n}$$

Zero-Coupon Bonds

Assume $i = 5\%$

Price of a One-Year Treasury Bill

$$= \frac{100}{(1+0.05)} = \$95.24$$

Price of a Six-Month Treasury Bill

$$= \frac{100}{(1+0.05)^{1/2}} = \$97.59$$

Zero-Coupon Bonds

- For a zero-coupon bond, the relationship between the price and the interest rate is the same as we saw on present value calculations.

- When the price moves, the interest rate moves with it, in the opposite direction.

- We can compute the interest rate from the price using the present value formula.

  The price of a one-year T-bill is $95.

  $i = (\$100/\$95) - 1 = 0.0526 = 5.26\%$

Fixed-Payment Loans

- Home mortgages and car loans are fixed-payment loans.
  - They promise a fixed number of equal payments at regular intervals.
  - Amortized loans - the borrower pays off part of the principal along with the interest for the life of the loan.

- Value of a Fixed Payment Loan =

$$\frac{FixedPayment}{(1+i)} + \frac{FixedPayment}{(1+i)^2} + \cdots + \frac{FixedPayment}{(1+i)^n}$$

- The sum of the present value of the payments.

Coupon Bonds

- The issuer of a coupon bond promises to make a series of periodic interest payments (coupon payments), plus a principal payment at maturity.

Price of Coupon Bond =

$$P_{CB} = \left[\frac{CouponPayment}{(1+i)^1} + \frac{CouponPayment}{(1+i)^2} + \ldots\ldots + \frac{CouponPayment}{(1+i)^n} \right]$$

$$+ \frac{FaceValue}{(1+i)^n}$$

Consols

- Consols or perpetuities, are like coupon bonds whose payments last forever.
- The borrower pays only interest, never repaying the principal.
- The U.S. government sold consols once in 1900, but the Treasury has bought them all back.
- The price of a consol is the present value of all future interest payments.

$$P_{Consol} = \frac{\text{Yearly Coupon Payment}}{i}$$

Bond Yields

- We know how to calculate bond prices given an interest rate.

- We also need to be able to go in the other direction.

  - Calculate the return to an investment, implicit in the bond's price.

- We will combine information about the promised payments with the price to obtain the *yield*:

  - A measure of the cost of borrowing and the reward for lending.

  - We will use the terms *yield* and *interest rate* interchangeably.

Yield to Maturity

- The most useful measure of the return on holding a bond is called the yield to maturity:

  – The yield bondholders receive if they hold the bond to its maturity when the final principal payment is made.

$$\text{Price of 1yr 5\% Coupon Bond} = \frac{\$5}{(1+i)} + \frac{\$100}{(1+i)}$$

- The value of i that solves the equation is the yield to maturity.

Current Yield

Example:

1 year, 5% coupon bond selling for $99

$$\text{Current Yield} = \frac{5}{99} = 0.0505 \text{ , or } 5.05\%$$

Yield to maturity for this bond is 6.06 percent found as the solution to:

$$\frac{\$5}{(1+i)} + \frac{\$100}{(1+i)} = \$99$$

Holding Period Returns

- The *one-year holding period return* is the sum of the yearly coupon payment divided by the price paid for the bond and the change in the price divided by the price paid.

$$= \frac{\text{Yearly Coupon Payment}}{\text{Price Paid}} + \frac{\text{Change in Price of the Bond}}{\text{Price of the Bond}}$$

$$= \text{Current Yield} + \text{Capital Gain (as a \%)}$$

Data on "Treasury Notes and Bonds"

http://online.wsj.com/mdc/public/page/2_3020-treasury.html

Treasury Notes
(issued w/maturities
2-10 yrs) and Bonds
(issued w/maturities
of 10-30 yrs) are
"Coupon Bonds" in
our nomenclature

WSJ | MARKETS   Menzie Chinn ▾

Home World U.S. Politics Economy Business Tech **Markets** Opinion Books & Arts Real Estate Life & Work WSJ.Magazine Sports

U.S. Treasury Quotes   Friday, October 01, 2021

Treasury Notes & Bonds | Treasury Bills

Treasury note and bond data are representative over-the-counter quotations as of 3pm Eastern time. For notes and bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par.

MATURITY	COUPON	BID	ASKED	CHG	ASKED YIELD
10/15/2021	2.875	100.0200	100.0240	-0.0100	0.2719
10/31/2021	1.250	100.0200	100.0240	-0.0060	0.1841
10/31/2021	1.500	100.0240	100.0300	-0.0080	0.2207
10/31/2021	2.000	100.0360	100.0420	-0.0080	0.188
11/15/2021	2.000	100.0640	100.0700	-0.0060	0.083
11/15/2021	2.875	100.0940	100.1000	-0.0100	0.135
11/15/2021	8.000	100.2840	100.2900	-0.0220	0.057
11/30/2021	1.500	100.0640	100.0700	-0.0080	0.095
11/30/2021	1.750	100.0760	100.0820	-0.0080	0.094
11/30/2021	1.875	100.0840	100.0900	-0.0080	0.069
12/15/2021	2.625	100.1560	100.1620	-0.0100	0.043

WSJ | MARKETS   Menzie Chinn ▾

Home World U.S. Politics Economy Business Tech **Markets** Opinion Books & Arts Real Estate Life & Work WSJ.Magazine Sports

11/15/2030	0.875	95.0800	95.0900	0.8280	1.429
2/15/2031	1.125	97.0740	97.0840	0.8380	1.438
2/15/2031	5.375	135.0520	135.0620	0.8920	1.359
5/15/2031	1.625	101.2120	101.2220	0.1960	1.436
8/15/2031	1.250	97.3160	98.0060	0.1760	1.466
2/15/2036	4.500	137.1000	137.1100	0.9660	1.583
2/15/2037	4.750	141.2740	141.2840	0.9740	1.652
5/15/2037	5.000	145.1940	145.2040	0.9820	1.668
2/15/2038	4.375	137.2040	137.2140	0.9920	1.723
5/15/2038	4.500	139.2220	139.2420	0.9960	1.735
2/15/2039	3.500	125.1260	125.1460	0.9840	1.788
5/15/2039	4.250	137.0300	137.0500	1.0160	1.783
8/15/2039	4.500	141.0700	141.0900	1.0000	1.790
11/15/2039	4.375	139.1160	139.1360	1.0060	1.812
2/15/2040	4.625	143.2340	143.2540	1.0180	1.811
5/15/2040	1.125	87.0120	87.0320	0.9300	1.955
5/15/2040	4.375	139.2820	139.3020	0.9880	1.832

Data on "Treasury Notes and Bonds"

http://online.wsj.com/mdc/public/page/2_3020-treasury.html

WSJ MARKETS

Menzie Chinn ▾

Home   World   U.S.   Politics   Economy   Business   Tech   **Markets**   Opinion   Books & Arts   Real Estate   Life & Work   WSJ Magazine   Sports   🔍

2/15/2048	3.000	119.2400	119.2600	1.0480	2.026
5/15/2048	3.125	122.1700	122.1900	1.0540	2.023
8/15/2048	3.000	120.0020	120.0220	1.7280	2.027
11/15/2048	3.375	128.0960	128.1160	1.0660	2.012
2/15/2049	3.000	120.1320	120.1520	1.0580	2.022
5/15/2049	2.875	117.2740	117.2940	1.0600	2.025
8/15/2049	2.250	104.1460	104.1660	1.0260	2.036
11/15/2049	2.375	107.0820	107.1020	1.0320	2.032
2/15/2050	2.000	99.0100	99.0300	1.7060	2.042
5/15/2050	1.250	82.1700	82.1900	0.9860	2.058
8/15/2050	1.375	85.0420	85.0620	0.9960	2.058
11/15/2050	1.625	90.1900	90.2100	1.0060	2.053
2/15/2051	1.875	96.0500	96.0700	1.0200	2.047
5/15/2051	2.375	107.1840	107.2040	1.0520	2.030
8/15/2051	2.000	99.0320	99.0520	1.7160	2.038

Source: Tullett Prebon

Accessed
10/2/2021

Data on Treasury Bills

https://www.wsj.com/market-data/bonds/treasuries

Confusingly,
Treasury
Bills (issued
w/maturity 1
yr or less) are
"discount
bonds" in our
nomenclature

Accessed
10/2/2021

WSJ | MARKETS   Menzie Chinn ▾

English Edition ▾ | Print Edition | Video | Podcasts | Latest Headlines

Home   World   U.S.   Politics   Economy   Business   Tech   **Markets**   Opinion   Books & Arts   Real Estate   Life & Work   WSJ. Magazine   Sports   🔍

MarketData > Bonds & Rates > U.S. Treasury Quotes

Quotes & Companies   View All Companies

U.S. Treasury Quotes   Friday, October 01, 2021

Treasury Notes & Bonds | **Treasury Bills**

Treasury bill bid and ask data are representative over-the-counter quotations as of 3pm Eastern time quoted as a discount to face value. Treasury bill yields are to maturity and based on the asked quote.

MATURITY	BID	ASKED	CHG	ASKED YIELD
10/5/2021	0.023	0.013	+0.005	0.013
10/7/2021	0.030	0.020	+0.005	0.020
10/12/2021	0.030	0.020	+0.005	0.020
10/14/2021	0.043	0.033	+0.025	0.033
10/19/2021	0.098	0.088	+0.038	0.089
10/21/2021	0.113	0.103	+0.043	0.104

WSJ | MARKETS   Menzie Chinn ▾

Home   World   U.S.   Politics   Economy   Business   Tech   **Markets**   Opinion   Books & Arts   Real Estate   Life & Work   WSJ. Magazine   Sports   🔍

2/3/2022	0.048	0.038	+0.003	0.038
2/10/2022	0.048	0.038	+0.005	0.038
2/17/2022	0.045	0.035	unch.	0.036
2/24/2022	0.050	0.040	+0.003	0.041
3/3/2022	0.050	0.040	+0.005	0.041
3/10/2022	0.053	0.043	+0.008	0.043
3/17/2022	0.050	0.040	+0.005	0.041
3/24/2022	0.050	0.040	+0.005	0.041
3/31/2022	0.058	0.048	+0.005	0.048
4/21/2022	0.055	0.045	+0.008	0.046
5/19/2022	0.058	0.048	+0.005	0.048
6/16/2022	0.065	0.055	+0.005	0.056
7/14/2022	0.073	0.063	+0.005	0.063
8/11/2022	0.078	0.068	+0.007	0.069
9/8/2022	0.085	0.075	+0.008	0.076

"On the run"

Source: Tullett Prebon

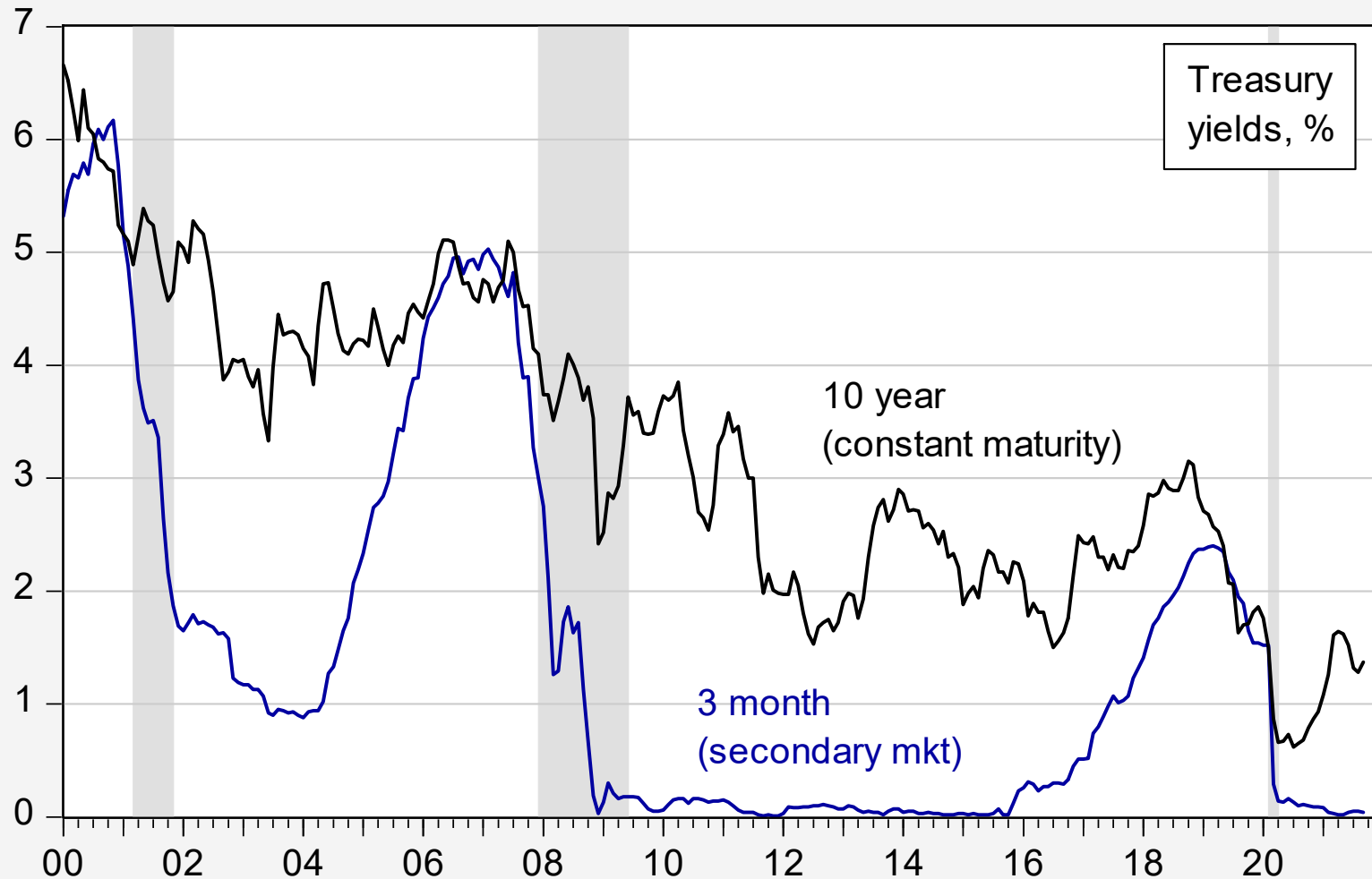# Data on Treasurys

https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield

Secondary Market, Constant Maturity

Treasury yields, %

10 year (constant maturity)

3 month (secondary mkt)

Real and Nominal Interest Rates

- The nominal interest rate you agree on (*i*) must be based on *expected inflation (π^e)* over the term of the loan plus the real interest rate you agree on (*r*).
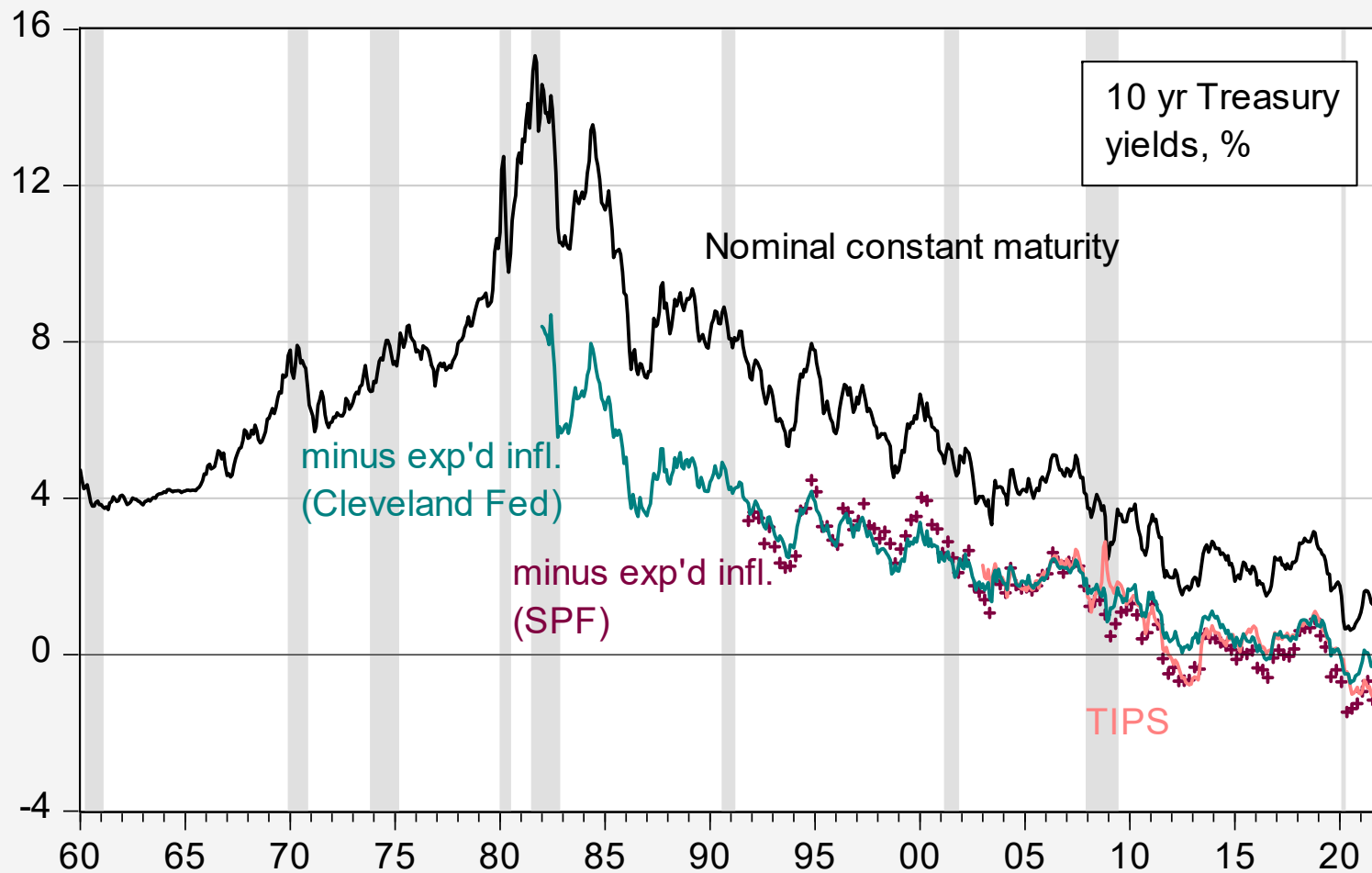
$$i = r + \pi^e$$

- This is called the *Fisher Equation.*
- The higher expected inflation, the higher the nominal interest rate.

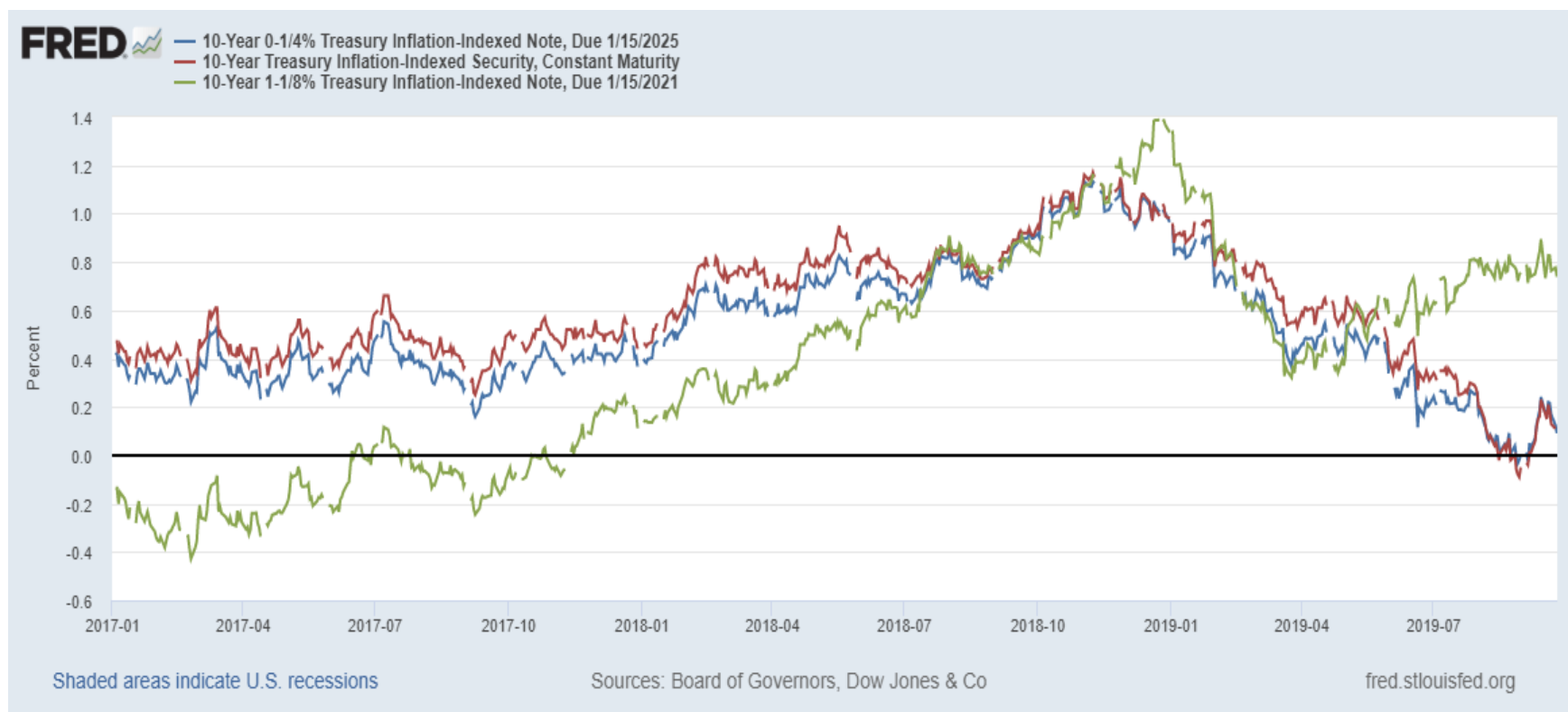# Data on Treasury Inflation Protected Securities (TIPS)

http://online.wsj.com/mdc/public/page/2_3020-tips.html

WSJ | MARKETS — Menzie Chinn

Home World U.S. Politics Economy Business Tech **Markets** Opinion Books & Arts Real Estate Life & Work WSJ. Magazine Sports

Treasury Inflation-Protected Securities Friday, October 01, 2021

Treasury Inflation-Protected Securities, or TIPS, are securities whose principal is tied to the Consumer Price Index (CPI). The principal increases with inflation and decreases with deflation. When the security matures, the U.S. Treasury pays the original or adjusted principal, whichever is greater. TIPS pay interest every six months. The accrued principal assumes an initial investment of $1,000. Figures after periods in bid and ask quotes represent 32nds; 101.26 means 101 26/32, or 101.8125% of 100% face value; 99.01 means 99 1/32, or 99.03125% of face value.

MATURITY	COUPON	BID	ASKED	CHG	YIELD	ACCRUED PRINCIPAL
2022 Jan 15	0.125	100.23	100.25	unch.	-2.665	1206
2022 Apr 15	0.125	101.13	101.15	unch.	-2.594	1122
2022 Jul 15	0.125	102.14	102.16	unch.	-2.999	1187
2023 Jan 15	0.125	103.13	103.15	+1	-2.530	1182
2023 Apr 15	0.625	104.22	104.24	unch.	-2.394	1099
2023 Jul 15	0.375	105.14	105.16	+2	-2.615	1173
2024 Jan 15	0.625	106.25	106.27	+2	-2.279	1170
2024 Apr 15	0.500	106.29	106.31	+3	-2.118	1082
2024 Jul 15	0.125	106.28	106.30	+4	-2.281	1149
2024 Oct 15	0.125	107.04	107.06	+4	-2.153	1064
2025 Jan 15	0.250	107.19	107.21	+4	-1.999	1152
2032 Apr 15	3.375	146.30	147.03	+24	-0.882	1538
2040 Feb 15	2.125	149.22	149.27	+41	-0.471	1263
2041 Feb 15	2.125	151.16	151.20	+43	-0.430	1246
2042 Feb 15	0.750	122.22	122.27	+40	-0.333	1208
2043 Feb 15	0.625	120.01	120.06	+42	-0.291	1187
2044 Feb 15	1.375	138.10	138.16	+47	-0.290	1171
2045 Feb 15	0.750	123.31	124.04	+47	-0.252	1159
2046 Feb 15	1.000	131.09	131.14	+52	-0.250	1152
2047 Feb 15	0.875	129.08	129.13	+52	-0.248	1131
2048 Feb 15	1.000	133.28	134.01	+55	-0.248	1107
2049 Feb 15	1.000	135.06	135.10	+57	-0.247	1084
2050 Feb 15	0.250	113.28	114.00	+55	-0.228	1061
2051 Feb 15	0.125	110.26	110.30	+56	-0.235	1048

Nominal vs. Real

10 yr Treasury yields, %

Nominal constant maturity

minus exp'd infl. (Cleveland Fed)

minus exp'd infl. (SPF)

TIPS

# Constant Maturity vs. On the Run vs. Off the Run

Alternative Approach to Bond Prices

- There are other bonds besides government bonds

- In the IS-LM approach incorporating portfolio demand for government bonds, other nongovernment bond supply/demand factors

- A more general approach lumps all bonds together

Factors That Shift Bond Supply

Figure 6.2 A Shift in the Supply of Bonds
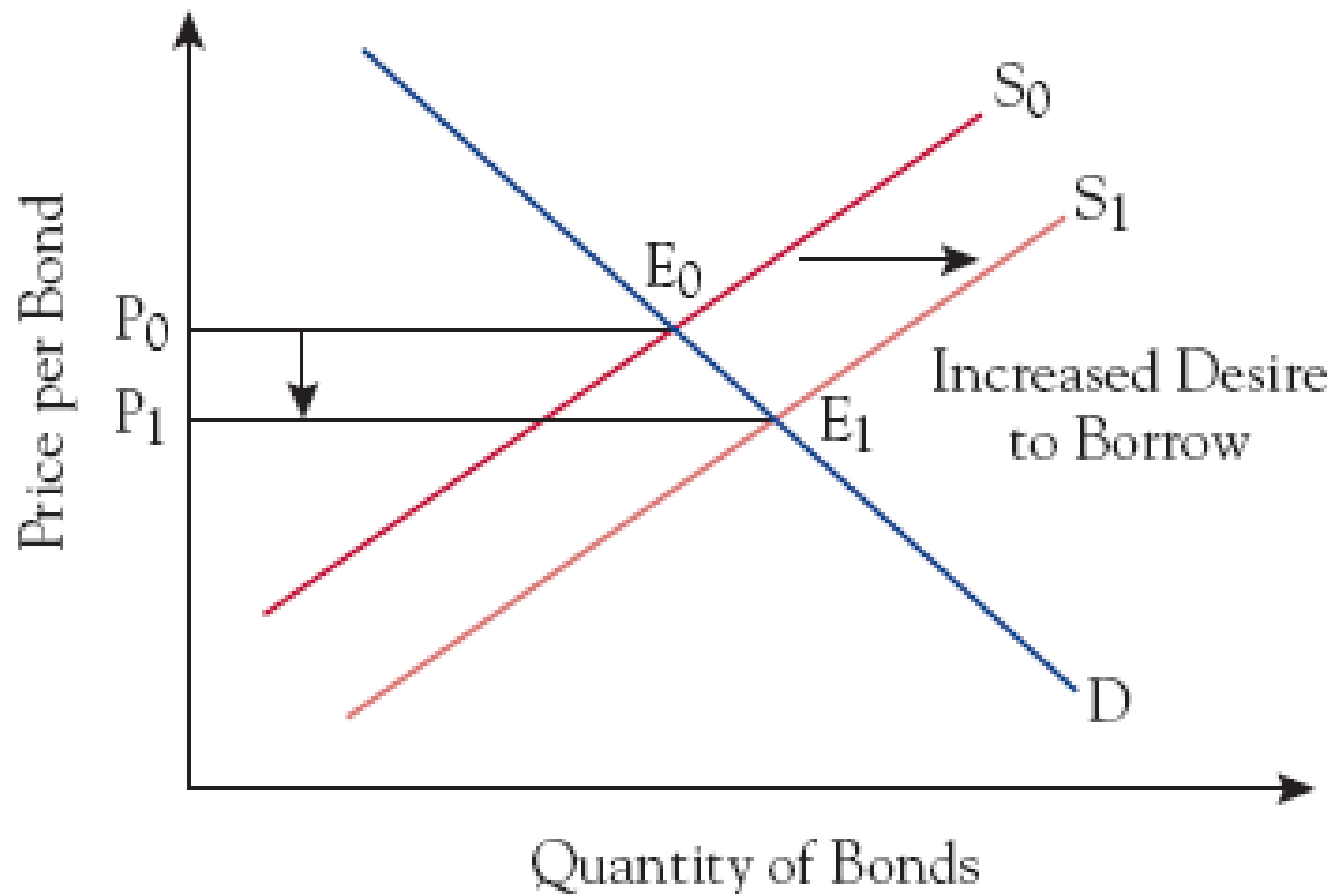
Factors That Shift Bond Demand

Figure 6.3 A Shift in the Demand for Bonds

6-2