The IS-LM Model

This set of notes outlines the IS-LM model of national income and interest rate determination. This involves extending the real side of the economy (described in the previous handout) and introducing a financial side (that involves money and bond markets). The real side of the economy is extended by introducing and interest sensitive component of aggregate demand, namely investment. First the real side and the financial side are described. These are then put together to determine overall economic equilibrium. The impact of fiscal and monetary policy is then discussed, and policy multipliers derived.

1. The Real Side of the Economy

Eq.No.	Equation	Description
(1)	$Y = Z$	Output equals aggregate demand, an equilibrium condition
(2)	$Z = C + I + G$	Definition of aggregate demand
(3)	$C = c_o + c_1 Y_D$	Consumption function, c_1 is the mpc
(4)	$Y_D \equiv Y - T$	Definition of disposable income
(5)	$T = t_0 + t_1 Y$	Tax function; t_0 is lump sum taxes, t_1 is marginal tax rate.
(6)	$I = b_0 + b_1 Y - b_2 i$	Investment function *(revised)*
(7)	$G = GO_0$	Government spending on goods and services, exogenous

The new parameters are $b_1 = \dfrac{\partial I}{\partial Y}$, and $-b_2 = \dfrac{\partial I}{\partial i}$ "the interest sensitivity of investment". Substituting in all the equations (2)-(7) into (1) yields:

(10) $Y = Z = (c_0 + c_1(Y - T)) + (b_0 + b_1 Y - b_2 i) + (GO_o)$

(11) $Y = Z = (c_0 + c_1(Y - (t_0 + t_1 Y))) + (b_0 + b_1 Y - b_2 i) + (GO_o)$

Equation (10) is a linear version of equation (5.2). Since it's linear, we can solve out for Y. Rearrange, solving for Y as a function of i, one obtains:

(12) $Y = \left(\dfrac{1}{1 - c_1(1 - t_1) - b_1} \right) [\Lambda_0 - b_2 i]$ <IS curve>

where $\Lambda_0 \equiv c_0 - c_1(t_0) + b_0 + GO_o$.

This equation can be re-written as:

(13) $\boxed{i = -\left(\dfrac{1 - c_1(1 - t_1) - b_1}{b_2} \right) Y + \left(\dfrac{1}{b_2} \right) \Lambda_0}$ <IS curve>

All points along the line defined by this equation are points where income and interest rates are such that aggregate demand equals income.

2. The Financial Side of the Economy

Eq.No. Equation   Description

(14) $\dfrac{M^d}{P} = \dfrac{M^s}{P}$ Equilibrium condition

(15) $\dfrac{M^s}{P} = \dfrac{M_0}{P}$ Money supply

For money demand:

(16) $\dfrac{M^d}{P} = \mu_0 + Y - hi$ Money demand

Equation (16) implies that real money demand rises dollar for dollar with real income, $\dfrac{\partial (M^d / P)}{\partial Y} = 1$.

Further note that $\dfrac{\partial (M^d / P)}{\partial i} = -h$. Substitute (15) and (16) into (14), and rearrange to obtain:

(17) $i = \left(\dfrac{\mu_0}{h}\right) - \left(\dfrac{1}{h}\right)\left(\dfrac{M_0}{P}\right) + \left(\dfrac{1}{h}\right)Y$ <LM curve>

All points on this line represent the combinations of income and interest rates that equilibrate money supply and money demand.

3. Equilibrium in IS-LM: Algebraic and Graphical

The IS and LM equations constitute a two equation system with two unknowns. The unknowns can be solved for by substituting one equation into another. The two equations are:

(12) $Y = \left(\dfrac{1}{1 - c_1(1 - t_1) - b_1}\right)[\Lambda_0 - b_2 i]$ <IS curve>

(17) $i = \left(\dfrac{\mu_0}{h}\right) - \left(\dfrac{1}{h}\right)\left(\dfrac{M_0}{P}\right) + \left(\dfrac{1}{h}\right)Y$ <LM curve>

One way to solve this system is to substitute is to (17) in for i in (12).

(18) $\quad Y = \left(\dfrac{1}{1-c_1(1-t_1)-b_1}\right) \times \left[\Lambda_0 - b_2\left\langle \dfrac{\mu_0}{h} - \dfrac{1}{h}\dfrac{M_0}{P} + \dfrac{1}{h}Y\right\rangle\right]$

Notice that this can be solved for Y, by bringing the term in the (.) to the left hand side.

(19) $\quad Y(1-c_1(1-t_1)-b_1) = \Lambda_0 - b_2\left\langle \dfrac{\mu_0}{h} - \dfrac{1}{h}\dfrac{M_0}{P}\right\rangle - b_2\dfrac{1}{h}Y$

Collect up the last term on the right hand side involving "Y" to the left hand side:

(20) $\quad Y\left(1-c_1(1-t_1)-b_1+\dfrac{b_2}{h}\right) = \Lambda_0 + b_2\left\langle \dfrac{1}{h}\dfrac{M_0}{P} - \dfrac{\mu_0}{h}\right\rangle$

Dividing both sides by the term in (.) to obtain:

(21) $\quad \boxed{Y_0 = \hat{\gamma}\left[\Lambda_0 + \dfrac{b_2}{h}\left(\dfrac{M_0}{P}\right) - \dfrac{b_2\mu_0}{h}\right]}$ $\qquad$ <equilibrium income>

Where

$$\hat{\gamma} \equiv \dfrac{1}{1-c_1(1-t_1)-b_1+\dfrac{b_2}{h}}$$

Note further that

$$\dfrac{1}{1-c_1(1-t_1)-b_1+\dfrac{b_2}{h}} \equiv \hat{\gamma} \leq \bar{\gamma} \equiv \dfrac{1}{1-c_1(1-t_1)} \quad \text{if } b_1 = 0$$

Equation (21) indicates that equilibrium income (which arises from the interaction of both the real and financial sides of the economy) is a function of real factors (how much "autonomous" spending occurs) and monetary factors (how much money the central bank has printed up).
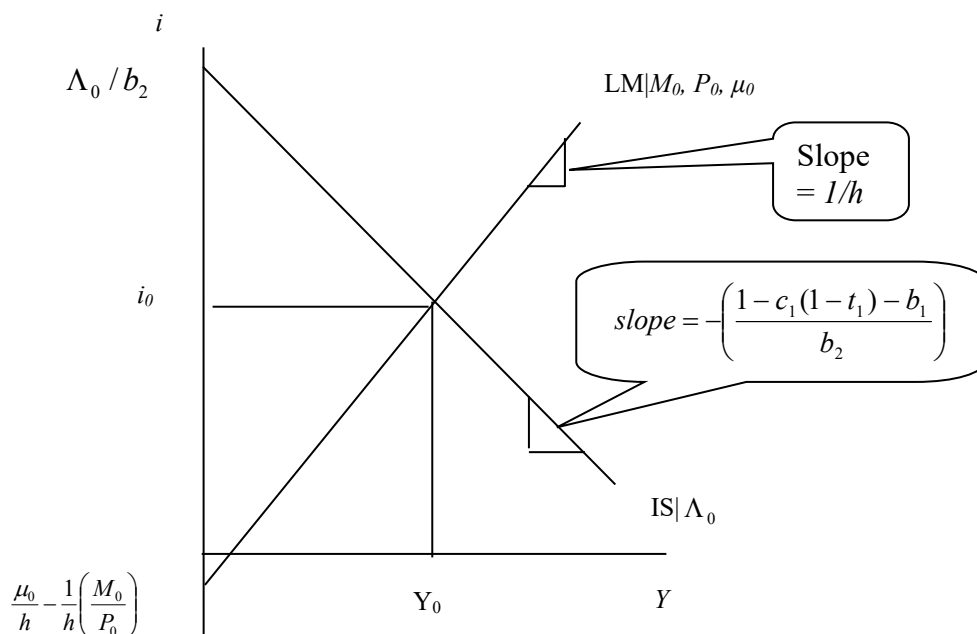
Graphically, equilibrium is depicted in Figure 1:

Figure 1: Equilibrium in IS-LM

4. Policy in IS-LM Model

The easiest way to see the impact of policy is to take the total differential of (21):

$$(21) \quad Y_0 = \hat{\gamma} \left[\Lambda_0 + \frac{b_2}{h} \left(\frac{M_0}{P} \right) - \frac{b_2 \mu_0}{h} \right] \qquad \text{<equilibrium income>}$$

$$(22) \quad \Delta Y = \hat{\gamma} \left[\Delta \Lambda + \frac{b_2}{h} \Delta \left(\frac{M}{P} \right) - \frac{b_2}{h} \Delta \mu \right]$$

For fiscal policy, one has to determine whether it is government spending that is changed, or lump sum taxes. Recall $\Lambda_0 \equiv c_0 - c_1(t_0) + b_0 + GO_o$. If the fiscal policy involves only government spending, then:

$$\Delta Y = \hat{\gamma} \Delta GO \Rightarrow \frac{\Delta Y}{\Delta GO} = \hat{\gamma}$$

If it is lump sum taxes:

$$\Delta Y = -\hat{\gamma} c_1 \Delta t_0 \Rightarrow \frac{\Delta Y}{\Delta t_0} = -\hat{\gamma} c_1$$

4

If monetary policy is being used, the $\Delta\Lambda = 0$, so:

$$\Delta Y = \hat{\gamma}\frac{b_2}{h}\Delta\left(\frac{M}{P}\right) \Rightarrow \frac{\Delta Y}{\Delta(M/P)} = \hat{\gamma}\frac{b_2}{h}$$

How are the effects of these policies depicted graphically? Below are fiscal (government spending) and monetary policies, respectively.
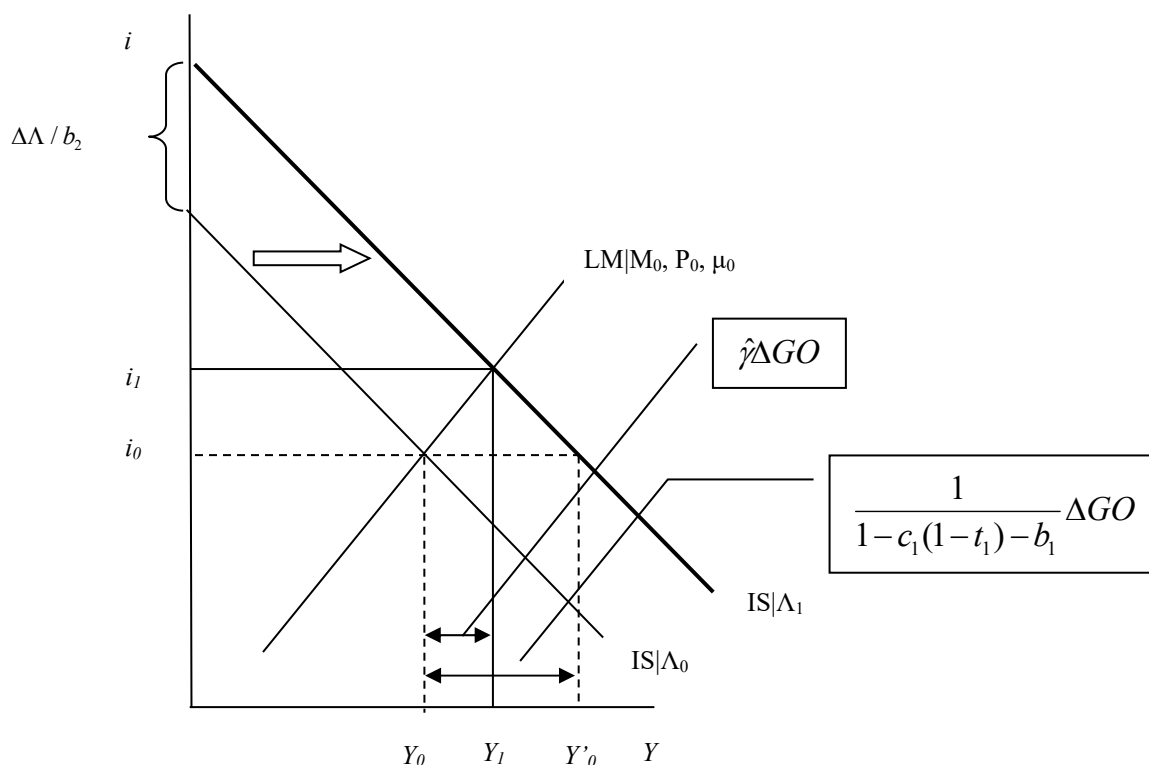

Figure 2: Fiscal (Govt. spending) Policy

Notice that fiscal policy might be less powerful – in the sense of increasing income – than it was in the Keynesian cross model, and will be less powerful if $b_1 = 0$ (that is, investment does not depend on income) and $-b_2 < 0$. It is important that you understand the intuition for why this result occurs: it is because the introduction of a financial sector means that as income rises, money demand rises (while the money supply is fixed); rising interest rates result in decreased investment and net exports and hence a lower income level relative to the counterfactual level of Y'_0.

You will notice, if you experiment with differing-sloped curves, that if the IS curve is steep because the parameters b_2 are small, then fiscal policy will be more effective than when it is flat because these parameters are large. You should think about why that is the case. You will also find that when the LM curve is flat, the fiscal policy is also more effective than when it is steep.
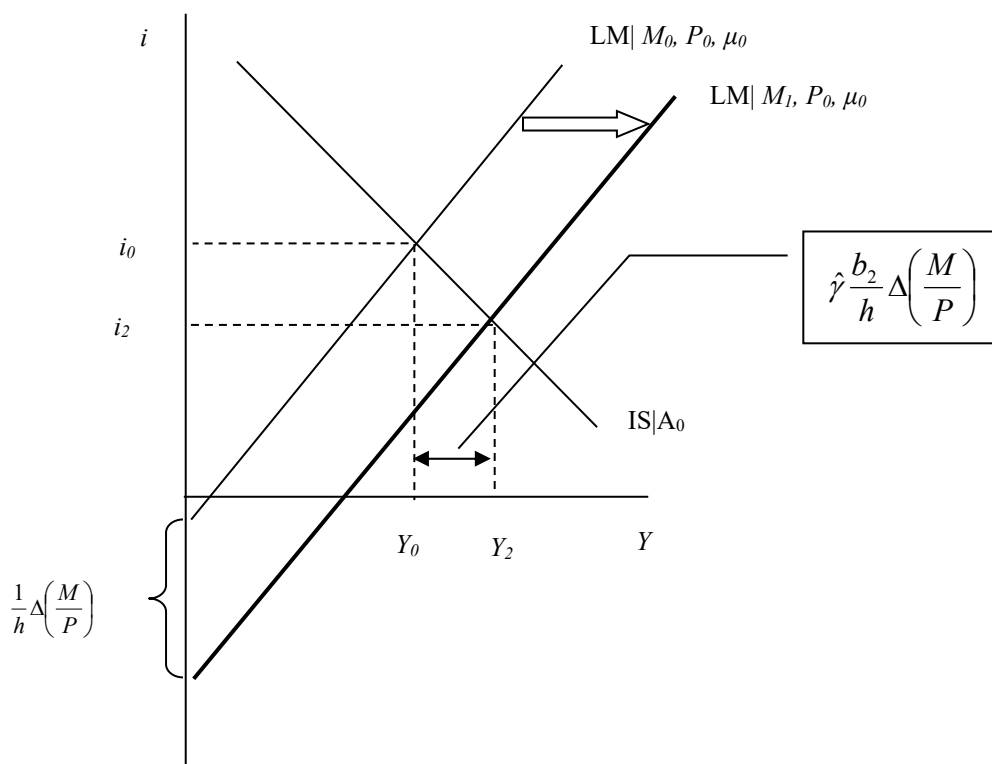
Now consider monetary policy.

Figure 3: Monetary Policy

Monetary policy works by decreasing the interest rate, and thus spurring investment. Notice that when the LM curve is steep because h is small, monetary policy will tend to be powerful than when h is large. *Monetary policy will also be more powerful, the flatter the IS curve is.* You should consider why these outcomes occur.

In news accounts, we don't see monetary policy described as changes in the money stock. Rather, we hear the target interest rate, i^{Target} (in the US, the Fed Funds rate) is being raised, dropped or maintained, after a meeting of the Federal Open Market Committee (FOMC) which takes place every six weeks. How does this fit into the model? One way is to interpret this the Fed as changing the money supply as the IS curve shifts around due to random shocks in order to keep the interest rate at i^{Target}.

In Figure 4 below, when the IS curve is at the position consistent with autonomous spending level Λ_0, the central banks positions the LM curve to intersect the IS curve at i^{Target}, with money supply M_0. When the IS is shifted out due to a higher level of autonomous spending level Λ_1, the central bank increases the money supply to M_1, so that the IS and LM curves still intersect at i^{Target}. When the IS is shifted in, the central bank intervenes again, pulling in the LM curve.

Figure 4. LM and Effective LM Curve

When the central bank undertakes this approach to monetary policy, one can think of a horizontal "effective LM" curve. The central bank in normal times affects output by dropping or raising the target interest rate, and thereby shifting down or up the "effective LM" curve.

Figure 5. Monetary Policy with Effective LM Curve

As the target interest rate is lowered, investment rises, thereby increasing output. Notice that the central bank might increase the target interest rate when output rises due to exogenous shifts in the IS curve. But for now, we'll assume the central bank exogenously sets the target interest rate.

5. The Liquidity Trap

The IS-LM graphs are typically drawn in such a way that the equilibrium interest rate is positive. However, in recent years the target (short term) interest rates have declined to zero, and cannot go further downward (since nominal interest rates for the most part cannot be negative). This is depicted in Figure 6.
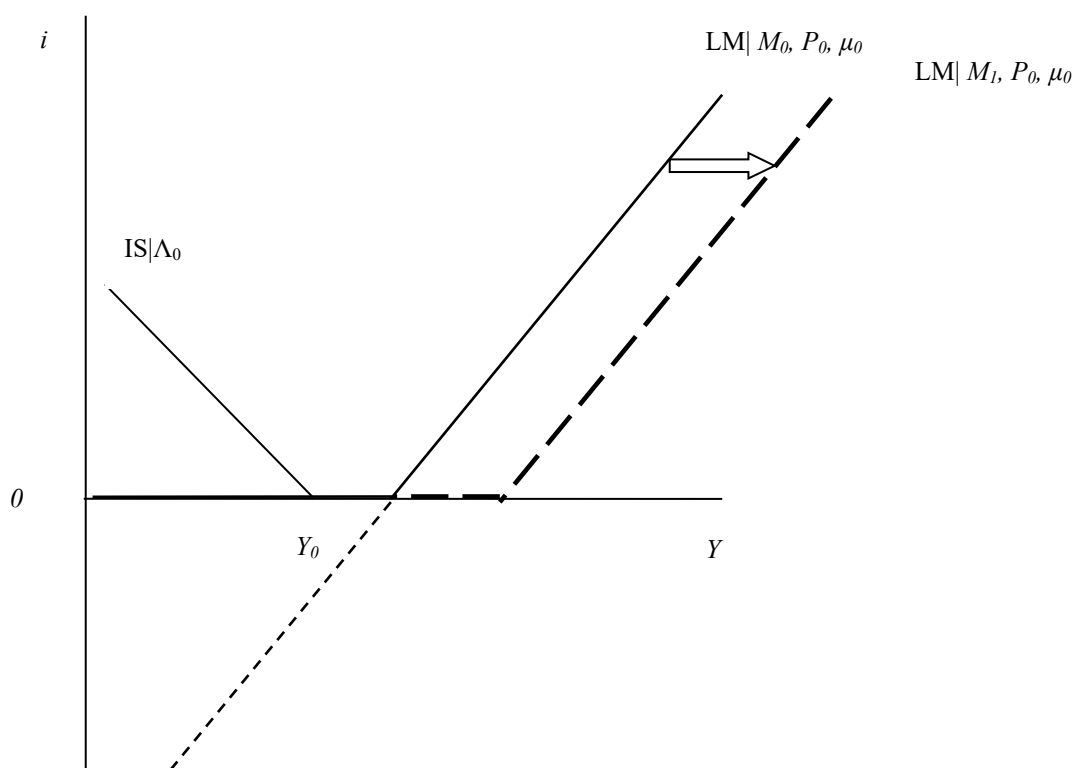

Figure 6. Monetary Policy in the Liquidity Trap

In this situation, equilibrium income is Y_0, and the interest rate is at 0. An increase in the money supply shifts out the LM curve (to the thick dashed line), but cannot further drive down the interest rate. Since interest rates can't decline, then investment cannot be spurred by this channel. Note that fiscal policy *can* increase output (consider a shift outward of the IS curve). Hence, in this simple model, monetary policy is completely ineffective, while fiscal policy is quite effective.

e435_ISLM_f21, 8.9.2021