Problem Set 3 Answers

Due in lecture on Wednesday, December 4. Be sure to put your name on your problem set. Put “boxes” around your answers to the algebraic questions.

1. Consider an economy where firms hold liabilities in nominal debt, and assets in the form of physical capital. Suppose further that firms are collateral constrained, i.e., they can only borrow up to the value of the physical capital minus debt.

1.1 Consider the implications of an unanticipated decrease in the price level on the firm’s ability to borrow. Explain.

Answer: In the Bernanke-Gertler-Gilchrist model,

\[ x_t \leq a_0 f(x_0) + \left( \frac{q_t}{r_t} \right) K - r_0 b_0 \]

In the model \( b_0 \) is the real debt from period 0. If the price level were to jump unexpectedly, this would mean the firm’s net wealth would be higher. As a consequence, the firm can hire more of the variable input in period 1, i.e., \( x_t \).

1.2 What are the implications of a recession on the ability of the firm to borrow, assuming that the price of capital rises when economic activity is high?

Answer: In this model, the higher price of capital is manifested in a higher \( q_t \). Then the firm’s net worth is once again higher, so the firm can hire more of the variable input.

2. Consider a Taylor rule of the following form:

\[ i_t^{FedFunds} = \pi_t + \beta(y_t - y_t^*) + \delta (\pi_t - \pi_t^*) + r_t^* \]

2.1 Calculate the implied target Fed Funds rate. Assume inflation is measured using year on year inflation measured using personal consumption expenditure deflator. Further assume potential GDP in 2013Q3 was 16647.6. Target inflation is 2.0%, and the equilibrium real rate is 2%.

Answer: Here is data from BEA via FRED:

<table>
<thead>
<tr>
<th></th>
<th>D(LOG(PCONS),0,4)</th>
<th>LOG(GDP09)</th>
<th>LOG(GDP09_POT_CBOAUG13ADJ)</th>
<th>GDP09_POT_CBOAUG13ADJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>2012Q3</td>
<td>0.015794</td>
<td>9.650280</td>
<td>9.703984</td>
<td>16382.74</td>
</tr>
<tr>
<td>2012Q4</td>
<td>0.016473</td>
<td>9.651407</td>
<td>9.708052</td>
<td>16449.52</td>
</tr>
<tr>
<td>2013Q1</td>
<td>0.013542</td>
<td>9.655040</td>
<td>9.711966</td>
<td>16514.04</td>
</tr>
<tr>
<td>2013Q2</td>
<td>0.010610</td>
<td>9.661002</td>
<td>9.715934</td>
<td>16579.69</td>
</tr>
<tr>
<td>2013Q3</td>
<td>0.011218</td>
<td>9.668015</td>
<td>9.720022</td>
<td>16647.60</td>
</tr>
</tbody>
</table>
So:

\[ i_t^{FedFunds} = \pi_t + \beta(y_t - y_t^*) + \delta (\pi_t - \pi_t^*) + r_t^* \]

Becomes:

\[ i_t^{FedFunds} = 0.011 + 0.5 \times (9.668 - 9.720) + 0.5 \times (0.011 - 0.020) + 0.02 \]
\[ i_t^{FedFunds} = 0.011 + 0.5 \times (-0.052) + 0.5 \times (-0.009) + 0.02 \]
\[ i_t^{FedFunds} = 0.011 - 0.026 - 0.0045 + 0.02 \]
\[ i_t^{FedFunds} = 0.0005 \]

2.2 What happens to the target Fed Funds rate if the target inflation rate is raised to 3%.

\[ i_t^{FedFunds} = 0.011 + 0.5 \times (9.668 - 9.720) + 0.5 \times (0.011 - 0.030) + 0.02 \]
\[ i_t^{FedFunds} = 0.011 + 0.5 \times (-0.052) + 0.5 \times (-0.019) + 0.02 \]
\[ i_t^{FedFunds} = 0.011 - 0.026 - 0.0095 + 0.02 \]
\[ i_t^{FedFunds} = -0.0045 \]

2.3 Calculate the target Fed Funds rate if \( \beta = 0 \).

**Answer:** There are two possible answers. We start with the original assumption of target inflation of 2%; further assume \( \delta = 0.5 \).

\[ i_t^{FedFunds} = 0.011 + 0.5 \times (0.011 - 0.020) + 0.02 \]
\[ i_t^{FedFunds} = 0.011 + 0.5 \times (-0.009) + 0.02 \]
\[ i_t^{FedFunds} = 0.011 - 0.0045 + 0.02 \]
\[ i_t^{FedFunds} = 0.0265 \]

With the assumption of target inflation of 3%; further assume \( \delta = 0.5 \).

\[ i_t^{FedFunds} = 0.011 + 0.5 \times (0.011 - 0.030) + 0.02 \]
\[ i_t^{FedFunds} = 0.011 + 0.5 \times (-0.019) + 0.02 \]
\[ i_t^{FedFunds} = 0.011 - 0.0095 + 0.02 \]
\[ i_t^{FedFunds} = 0.0215 \]
3. Suppose the demand for reserves is stable. Use a graph of the market for Bank Reserves to show how the Open Market Trading Desk would implement a decision by the FOMC to raise the target federal funds rate. You should assume that the discount and deposit rates are adjusted so that the spreads between them and the target federal funds rate are maintained.

**Answer:** To achieve a higher target for the federal funds rate, the Open Market Trading Desk would carry out open market sales, shifting the supply of reserves to the left until demand and supply of reserves intersect at the new target federal funds rate. (Move from point A to point B on the graph.) The corresponding rise in the discount and deposit rates to maintain the spreads means that the supply and demand curves for reserves become perfectly elastic at higher rates.

4. Consider the following data from Bloomberg for 11/25, where the "last" column denotes the yield to maturity:
Suppose the expectations hypothesis of the term structure holds.

4.1 Calculate the expected one year interest rate, one year from 11/25.

According to the expectations hypothesis of the term structure, for a two year bond,
\[ i_{t+1} = 2i_{2t} - i_t \]
Substituting in 0.26 and 0.17 for the two and one year bond yields results in:
\[ i_{t+1} = 2 \times 0.0028 - 0.0012 = 0.0044 \]
Or
\[ 0.44\% \]

4.2 Calculate the average of expected one year interest rates for periods 5, 6, 7, 8 and 9 years from 11/25/2013.

In general, for the pure expectations hypothesis:
\[ i_{nt} = \frac{(i_{t_2} + i_{t_3} + \ldots + i_{t_{n-1}})}{n} \]
So for the tenen year yield:
\[ i_{10t} = \frac{(i_{t_2} + i_{t_3} + \ldots + i_{t_{9}})}{10} \]
Rearranging:
\[ 10 \times i_{10t} = ([i_{t_5} + i_{t_6} + i_{t_7} + i_{t_8} + i_{t_9}]) \]
Notice that the term in the square bracket is 5 times the five year yield, viz.:
\[ 5 \times i_{5t} = [i_{t_5} + i_{t_6} + i_{t_7} + i_{t_8} + i_{t_9}] \]
Substituting in, one finds:
\[ 10 \times i_{10t} = ([5 \times i_{5t}]) \]
Hence:
\[ 10 \times i_{10t} - 5 \times i_{5t} = \frac{(i_{t_5} + i_{t_6} + i_{t_7} + i_{t_8} + i_{t_9})}{5} \]
5. Suppose the Fed announces, a year after stopping QE3, that it was going to resume buying long term bonds in QE4.

5.1 Further suppose the long term interest rate increased, rather than decreased. Does this mean quantitative/credit easing has necessarily failed? Explain, using the equation for the term structure of interest rates.

**Answer:** the equation for the term structure is:

\[ i_{nt} = \frac{(i_{1t} + i_{1t=1}^\varepsilon + ... + i_{1t+a-1}^\varepsilon)}{n} + rp_{nt} \]

Purchases of the \( n \) year bonds should decrease \( rp_{nt} \). The fact that \( i_{nt} \) increases suggests that the purchases might positively affect the expected one year interest rates (i.e., raise them) by way of increasing expected inflation. This reasoning uses:

\[ i_t = r_t + \pi_{t+1}^e \]

To the extent that QE/CE aims at reducing the real interest rate, then an increase in the long term nominal rate is not necessarily a failure.

5.2 If the pure expectations hypothesis of the term structure holds, and long term interest rates fall in response to QE4, what channel must be in effect?

**Answer:** The pure EHTS is given by:

\[ i_{nt} = \frac{(i_{1t} + i_{1t=1}^\varepsilon + ... + i_{1t+a-1}^\varepsilon)}{n} \]

Then there is no role for affecting the term premium. The only reason the long term yield falls is because QE/CE signals duration commitment via extended guidance, i.e., that future one term bond yields are lower than otherwise.

\[ 0.0412 = \frac{10 \times 0.0273 - 5 \times 0.0134}{5} \]