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Debt and Deficits

Mathematics (Blanchard & Johnston)

$$-BuS_t \equiv BuD_t = rB_{t-1} + G_t - T_t$$

Where for simplicity *P*=1, *r* is the real interest rate.

$$B_t - B_{t-1} = BuD_t$$

$$\underbrace{B_t - B_{t-1}}_{\text{Change in the debt}} = \underbrace{rB_{t-1}}_{\text{Interest payments}} + \underbrace{G_t - T_t}_{\text{Primary deficit}}$$

$$B_t = (1+r)B_{t-1} + [G_t - T_t]$$

Divide by Y:

$$\frac{B_t}{Y_t} = (1+r)\frac{B_{t-1}}{Y_t} + \frac{G_t - T_t}{Y_t}$$

$$\frac{B_t}{Y_t} = (1+r)\left(\frac{Y_{t-1}}{Y_t}\right)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

$$\frac{B_t}{Y_t} = (1+r)\left(\frac{1}{1+g}\right)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

$$\frac{B_t}{Y_t} - \frac{B_{t-1}}{Y_{t-1}} = (r-g)\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

Sometimes this is written as:

$$\dot{b}_t \equiv b_t - b_{t-1} = (r-g)b_{t-1} + d_t$$

Where the primary surplus is defined as:

$$d_t \equiv \frac{G_t - T_t}{Y_t}$$

The foregoing is in real terms (as P=1). Allowing for inflation, and time-variation in interest rates, inflation rates, and real growth (IMF, *WEO* 2012):

$$b_t = (1+r-g)b_{t-1} + d_t \approx b_t = \frac{(1+i_t)}{(1+\pi_t)(1+r_t)}b_{t-1} + d_t$$

Sustainability (Greenlaw, Hamilton, Hooper, and Mishkin)

Now B, Y, S in nominal terms:

$$B_t + (1 + R_t)(B_t - S_t)$$

$$\frac{B_{t+1}}{Y_{t+1}} = \frac{Y_t}{Y_{t+1}}\frac{1}{Y_t}(1 + R_t)(B_t - S_t)$$
$$b_{t+1} = \left\{\frac{Y_t}{Y_{t+1}}(1 + R_t)\right\}(b_t - s_t)$$
$$b_{t+1} = \{(1 + r'_t)\}(b_t - s_t)$$
$$Y_{t+1} = (1 + g'_t)Y_t$$

Suppose r* constant net borrowing cost, and b* constant debt-to-GDP ratio, satisfied by s*.

$$b^* = (1 + r'^*)(b^* - s^*)$$

$$s^* = \frac{r'^* b^*}{1 + r'^*}$$

Note if $R_t > g'_t$ then debt dynamics are explosive. If $R_t < g'_t$ then dynamics are stable.
What if the interest rate depends on debt-to-GDP ratios? Then increased debt-to-GDP ratios can make an economy more susceptible to tipping into unsustainable debt dynamics.

$$R_{it} = \hat{\alpha}_i + \hat{\gamma}_t + \underset{(3.95)}{0.0313}\, b_{i,t-1} + \underset{(2.30)}{0.0142}\, b^n_{i,t-1} - \underset{(5.16)}{0.184}\, c_{i,t-1} + e_{it} \qquad (17)$$

$$R^2 = 0.69 \qquad \text{log likelihood} = -288.32.$$

$$R_{it} = \hat{\alpha}_i + \hat{\gamma}_t + \underset{(0.30)}{0.0029}\, b_{i,t-1} + \underset{(4.29)}{0.245}\, c_{i,t-1} + \underset{(4.81)}{0.000203}\, b^2_{i,t-1} \qquad (18)$$

$$+ \underset{(2.98)}{0.00793}\, c^2_{i,t-1} - \underset{(10.18)}{0.00636}\, c_{i,t-1} b_{i,t-1} + e_{it}$$

$$R^2 = 0.82 \qquad \text{log likelihood} = -224.28.$$

$$R_{it} = \hat{\alpha}_i + \hat{\gamma}_t + \underset{(7.14)}{0.0370}\, b^n_{i,t-1} - \underset{(3.65)}{0.157}\, c_{i,t-1} + \underset{(0.89)}{0.0000365}(b^n_{i,t-1})^2 \qquad (19)$$

$$+ \underset{(2.35)}{0.0101}\, c^2_{i,t-1} - \underset{(2.10)}{0.00124}\, c_{i,t-1} b^n_{i,t-1} + e_{it}$$

$$R^2 = 0.76 \qquad \text{log likelihood} = -259.74.$$