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Notes on Exchange Rate Determination

Define the spot exchange rate, E_t, as the value of the home currency; hence, from an American resident's perspective, it is measured as the number of foreign currency units needed to buy one single U.S. dollar, e.g., E{€/$}, E{¥/$} or E{£/$}. Of course, from the perspective of a resident of the Euro area, the exchange rate would be E{$/€}. Thus, when E rises, the domestic currency is gaining value, i.e., appreciating. When E falls, the domestic currency is depreciating.

Perfect capital mobility is then defined as:

$$i_t^D - i_t^F = -\frac{E_{t+1}^e - E_t}{E} \tag{1}$$

where the "D" superscript denotes the U.S. dollar, and "F" superscript denotes the foreign currency unit.

This condition holding is also called the "interest rate parity" condition, because it states that the rate of return in dollars obtained by investing in dollar debt equals the expected rate of return in dollars obtained by investing in foreign currency denominated debt.

Assume that the exchange rate in the short run (E_t) can deviate from its long run value (E_{LR}), but when it does, it tends to gradually move towards its long run value. Hence, if the dollar is stronger than its long run value now, it will tend to weaken over time, holding everything else constant. This can be expressed as equation 2.

$$-\frac{E_{t+1}^e - E_t}{E} = \Theta\left[\frac{E_t}{E_{LR}} - 1\right] \tag{2}$$

Equation 2 doesn't take into account inflation, so we'll rewrite (2) as (2'):

$$-\frac{E_{t+1}^e - E_t}{E} = \Theta\left[\frac{E_t}{E_{LR}} - 1\right] + \pi_{t+1}^e - \pi_{t+1}^{F,e} \tag{2'}$$

where π_{t+1}^e is the expected domestic inflation rate, and $\pi_{t+1}^{F,e}$ is the expected foreign inflation rate. Equation (2') says that when expected inflation at home is higher, the currency tends to weaken more rapidly than otherwise. Combining equation 1 (interest rate parity) with equation 2' yields equation 3:

$$i_t^D - i_t^F = \Theta\left[\frac{E_t}{E_{LR}} - 1\right] + \pi_{t+1}^e - \pi_{t+1}^{F,e} \tag{3}$$

Solving this for Et, the short run exchange rate:

$$E_t = E_{LR}\left[1 + \frac{1}{\Theta}[(i_t^D - \pi_{t+1}^e) - (i_t^F - \pi_{t+1}^{F,e})]\right] \tag{4}$$

Equation (3) indicates that the short run exchange rate equals the long run when real interest rates are equal across countries. When the domestic real interest rate rises relative to that in the foreign country, then E_t rises (the home currency strengthens or "appreciates").

One question might be "What determines the long run exchange rate?" One answer is that given by the "Theory of Purchasing Power Parity":

$$E = \frac{P^F}{P} \tag{5}$$

We know that this theory does not hold in the short term. But there is some evidence that it holds for the long term. Then we can reasonably assume:

$$E_{LR} = \frac{P^F}{P} \tag{5'}$$

Notice that many factors can affect the exchange rate, once one incorporates equation (5') into equation (3): interest rates, inflation rates, and price levels expected in the long run (and anything that affects these variables).

$$E_t = \left(\frac{P^F}{P}\right)\left[1 + \frac{1}{\Theta}[(i_t^D - \pi_{t+1}^e) - (i_t^F - \pi_{t+1}^{F,e})]\right] \tag{6}$$

This model has an interesting feature. When the local interest rate rises, due to a one time decrease in money supply (and the foreign interest rate remains the same), then the domestic currency will immediately strengthen, past its long run value, so that it can depreciate over time according to equation (1) (interest rate parity). This tendency for the exchange rate to initially over-respond to a monetary shock is called "exchange rate overshooting".