Present Value

Let \( i = .10 \)
In one year \( $100 \times (1 + 0.10) = $110 \)
In two years \( $110 \times (1 + 0.10) = $121 \)
or \( 100 \times (1 + 0.10)^2 \)
In three years \( $121 \times (1 + 0.10) = $133 \)
or \( 100 \times (1 + 0.10)^3 \)
In \( n \) years
\( $100 \times (1 + i)^n \)

Yield to Maturity

**Simple loan**

\[
PV = \text{amount borrowed} = $100 \\
CF = \text{cash flow in one year} = $110 \\
n = \text{number of years} = 1 \\
\]$100 = \frac{$110}{(1 + i)^1} \\
(1 + i)^1 \times $100 = $110 \\
(1 + i) = \frac{$110}{$100} \\
i = 0.10 = 10\%
\]

For simple loans, the simple interest rate equals the yield to maturity.

**Fixed payment loan**

The same cash flow payment every period throughout the life of the loan

\[
LV = \text{loan value} \\
FP = \text{fixed yearly payment} \\
n = \text{number of years until maturity} \\
LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \ldots + \frac{FP}{(1+i)^n}
\]

Coupon bond

Using the same strategy used for the fixed-payment loan:

\[
P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}
\]

Consol

\[
P_c = \frac{C}{i_c} \\
P_c = \text{price of the consol} \\
C = \text{yearly interest payment} \\
i_c = \text{yield to maturity of the consol} \\
\]

Can rewrite above equation as \( i_c = \frac{C}{P_c} \)

For coupon bonds, this equation gives current yield an easy-to-calculate approximation of yield to maturity.
Discount bond

For any one year discount bond

\[ i = \frac{F - P}{P} \]

F = Face value of the discount bond

P = current price of the discount bond

The yield to maturity equals the increase in price over the year divided by the initial price.

As with a coupon bond, the yield to maturity is negatively related to the current bond price.

Rate of return

The payments to the owner plus the change in value expressed as a fraction of the purchase price

\[ RET = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t} \]

RET = return from holding the bond from time \( t \) to time \( t + 1 \)

\( P_t \) = price of bond at time \( t \)

\( P_{t+1} \) = price of the bond at time \( t + 1 \)

C = coupon payment

\[ \frac{C}{P_t} \] = current yield = \( i \)

\[ \frac{P_{t+1} - P_t}{P_t} \] = rate of capital gain = \( g \)