Expectations Hypothesis of the Term Structure

**Generalized Expectations Theory**
The expectations hypothesis of the term structure (EHTS) is a particular approach to explaining why the yield curve has a particular shape at any given instant. To illustrate this concept, consider first the simplest case, where one is examining a simple two period example. Define the following:

- \( i_t \) is today’s (time \( t \)) interest rate on a one-period bond,
- \( i_{t+1} \) is the interest rate on a one-period bond expected for next period (\( t+1 \)), and
- \( i_{2t} \) is today’s (time \( t \)) interest rate on the two-period bond.

The expected return over the two periods from investing $1 in the two-period bond and holding it for the two periods can be calculated as:

\[
(1 + i_{2t})(1 + i_{2t}) - 1 = 1 + 2i_{2t} + (i_{2t})^2 - 1 \approx 2i_{2t}
\]

The alternative strategy is to buy a one-period bond, then roll it over into another one-period bond next period; the expected return from that is:

\[
i_t + i^e_{t+1}
\]

Both bonds will be held only if the expected returns are equal,

\[
2i_{2t} = i_t + i^e_{t+1}
\]

Solving for the two-period interest rate in terms of one-period interest rates yields:

\[
i_{2t} = \frac{\left( i_t + i^e_{t+1} \right)}{2}
\]

(1)

This says that when investors equalize expected returns, then the interest rate on the two-period bond is equal to the average of the interest rate on the one-period bond and the expected interest rate on the one-period bond next period. This logic can be generalized to yield the expression for an \( n \) period bond.

\[
i_{nt} = \frac{\left( i_t + i^e_{t+1} + i^e_{t+2} + i^e_{t+(n-1)} \right)}{n}
\]

(2)

The example in the book is for the case where the one-year interest rates expected over the next five years are 5%, 6%, 7%, 8% and 9%, respectively. Then if the EHTS holds, the interest rate on a 5-year bond should be given by:

\[
i_{5t} = \frac{(0.05 + 0.06 + 0.07 + 0.08 + 0.09)}{5} = 0.07
\]

**Liquidity Premium Theory**

The segmented markets hypothesis posits that investors do not arbitrage completely over instruments of different maturities. Unfortunately, this approach gives little guidance on the manner in which interest rates...
deviate from those predicted by the EHTS. However, if one believes that longer term instruments are riskier than short term (remember what we said about how changes in interest rates affect yields to maturity), one can rewrite (2) as (3):

\[
i_{nt} = \frac{\left( i_t + i_{t+1}^e + i_{t+2}^e + i_{t+(n-1)}^e \right)}{n} + \ell_{nt}
\]  

(3)

Where \( \ell_{nt} \) is the liquidity (term) premium for the \( n \)-period bond at time \( t \), which is always positive and rises with the term to maturity of the bond, \( n \). If the term premium is 1% at \( n=5 \), then the interest rate – given the same values as in our previous example – will be:

\[
i_{5t} = \frac{0.05 + 0.06 + 0.07 + 0.08 + 0.09}{5} + 0.01 = 0.08
\]

The implications of the liquidity premium theory can be illustrated by reference to Figure 5:

Here are real-world data (green is current, orange is previous)

Source: Bloomberg, plot for 19 September 2006 data.