Problem Set 4
Solutions

Economics 310 - Professor Menzie D. Chinn
Fall 2003
University of Wisconsin-Madison

7.8 a. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix B, $z_{.025} = 1.96$. The confidence interval is:

$$\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}} \Rightarrow 33.9 \pm 1.96\frac{3.3}{\sqrt{100}} \Rightarrow 33.9 \pm .647 \Rightarrow (33.253, 34.547)$$

b. $$\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}} \Rightarrow 33.9 \pm 1.96\frac{3.3}{\sqrt{400}} \Rightarrow 33.9 \pm .323 \Rightarrow (33.577, 34.223)$$

c. For part **a**, the width of the interval is 2(.647) = 1.294. For part **b**, the width of the interval is 2(.323) = .646. When the sample size is quadrupled, the width of the confidence interval is halved.

7.20 a. $P(t \geq t_0) = .025$ where df = 11
$t_0 = 2.201$

b. $P(t \geq t_0) = .01$ where df = 9
$t_0 = 2.821$

c. $P(t \leq t_0) = .005$ where df = 6

Because of symmetry, the statement can be rewritten

$P(t \geq -t_0) = .005$ where df = 6
$t_0 = -3.707$

d. $P(t \leq t_0) = .05$ where df = 18
$t_0 = -1.734$

7.28 a. The population is the set of all DOT permanent count stations in the state of Florida.

b. Yes. There are several types of routes included in the sample. There are 3 recreational areas, 7 rural areas, 5 small cities, and 5 urban areas.

c. The 95% confidence interval for the mean traffic count at the 30th highest hour is (1,633.05, 2,778.85). We are 95% confident that the mean traffic count at the 30th highest hour is between 1,633.05 and 2,778.85.

d. We must assume that the distribution of the traffic counts at the 30th highest hour is normal. From the stem-and-leaf display, the data look fairly mound-shaped. Thus, the assumption of normality is probably met.

e. The 95% confidence interval for the mean traffic count at the 100th highest hour is (1,532.39, 2,658.81). We are 95% confident that the mean traffic count at the 100th highest hour is between 1,532.39 and 2,658.81.

We must assume that the distribution of the traffic counts at the 100th highest hour is normal. From the stem-and-leaf display, the data look fairly mound-shaped. Thus, the assumption of normality is probably met.

7.34 a. The sample size is large enough if the interval $\hat{p} \pm 3\sigma_{\hat{p}}$ does not include 0 or 1.

$$\hat{p} \pm 3\sigma_{\hat{p}} \Rightarrow \hat{p} \pm 3\sqrt{\frac{pq}{n}} \Rightarrow \hat{p} \pm 3\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .88 \pm 3\sqrt{\frac{.88(1 - .88)}{121}} \Rightarrow .88 \pm .089 \Rightarrow (.791, .969)$$

Since the interval lies within the interval (0, 1), the normal approximation will be adequate.

b. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .05$. From Table IV, Appendix B, $z_{.05} = 1.645$. The 90% confidence interval is:

$$\hat{p} \pm z_{.05}\sqrt{\frac{pq}{n}} \Rightarrow \hat{p} \pm 1.645\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .88 \pm 1.645\sqrt{\frac{.88(.12)}{121}} \Rightarrow .88 \pm .049 \Rightarrow (.831, .929)$$

c. We must assume that the sample is a random sample from the population of interest.

7.40 a. The point estimate of p is $\hat{p} = x/n = 35/55 = .636$.

b. We must check to see if the sample size is sufficiently large:

$$\hat{p} \pm 3\sigma_{\hat{p}} \approx \hat{p} \pm 3\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .636 \pm 3\sqrt{\frac{.636(.364)}{55}} \Rightarrow .636 \pm .195 \Rightarrow (.441, .831)$$

Since the interval is wholly contained in the interval (0, 1) we may assume that the normal approximation is reasonable.

For confidence coefficient, .99, $\alpha = .01$ and $\alpha/2 = .01/2 = .005$. From Table IV, Appendix B, $z_{.005} = 2.575$. The confidence interval is:

$$\hat{p} \pm z_{.005}\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .636 \pm 2.575\sqrt{\frac{.636(.364)}{55}} \Rightarrow .636 \pm .167 \Rightarrow (.469, .803)$$

c. We are 99% confident that the true proportion of fatal accidents involving children is between .469 and .803.

d. The sample proportion of children killed by air bags who were not wearing seat belts or were improperly restrained is 24/35 = .686. This is rather large proportion. Whether a child is killed by an airbag could be related to whether or not he/she was properly restrained. Thus, the number of children killed by air bags could possibly be reduced if the child were properly restrained.

7.56 For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix B, $z_{.025} = 1.96$. For this study,

$$n = \frac{(z_{\alpha/2})^2\sigma^2}{B^2} \approx \frac{1.96^2(5^2)}{1^2} = 96.04 \approx 97$$

The sample size needed is 97.

8.10 a. To determine if the mean job satisfaction rating for older workers is different from 4.3, we test:

H_0: $\mu = 4.3$
H_a: $\mu \neq 4.3$

b. A Type I error is rejecting the null hypothesis when it is true. In this problem, we would be concluding that the mean job satisfaction rating for older workers is different from 4.3 when, in fact, the mean job satisfaction rating is 4.3.

c. A Type II error is accepting the null hypothesis when it is false. In this problem, we would be concluding that the mean job satisfaction rating for older workers is equal to 4.3, when, in fact, the mean job satisfaction rating is different from 4.3.

8.16 a. The decision rule is to reject H_0 if $\bar{x} > 270$. Recall that

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}}$$

Therefore, reject H_0 if $\bar{x} > 270$

can be written reject H_0 if $z > \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}}$

$$z > \frac{270 - 255}{63/\sqrt{81}}$$
$$z > 2.14$$

The decision rule in terms of z is to reject H_0 if $z > 2.14$.

b.
$$\begin{aligned} P(z > 2.14) &= .5 - P(0 < z < 2.14) \\ &= .5 - .4838 \\ &= .0162 \end{aligned}$$

8.18 a. H_0: $\mu = .36$
H_a: $\mu < .36$

The test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{.323 - .36}{\sqrt{.034}/\sqrt{64}} = -1.61$

The rejection region requires $\alpha = .10$ in the lower tail of the z-distribution. From Table IV, Appendix B, $z_{.10} = 1.28$. The rejection region is $z < -1.28$.

Since the observed value of the test statistic falls in the rejection region ($z = -1.61 < -1.28$), H_0 is rejected. There is sufficient evidence to indicate the mean is less than .36 at $\alpha = .10$.

b. H_0: $\mu = .36$
H_a: $\mu \neq .36$

The test statistic is $z = -1.61$ (see part **a**).

The rejection region requires $\alpha/2 = .10/2 = .05$ in the each tail of the z-distribution. From Table IV, Appendix B, $z_{.05} = 1.645$. The rejection region is $z < -1.645$ or $z > 1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.61 \nless -1.645$), H_0 is not rejected. There is insufficient evidence to indicate the mean is different from .36 at $\alpha = .10$.

8.26 a. To determine if the mean lifetime of the new cartridges exceeds that of the old, we test:

H_0: $\mu = 1{,}502.5$
H_a: $\mu > 1{,}502.5$

b. The test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{1{,}511.4 - 1{,}502.5}{35.7/\sqrt{225}} = 3.74$

The rejection region requires $\alpha = .005$ in the upper tail of the z-distribution. From Table IV, Appendix B, $z_{.005} = 2.58$. The rejection region is $z > 2.58$.

Since the observed value of the test statistic falls in the rejection region ($z = 3.74 > 2.58$), H_0 is rejected. There is sufficient evidence to indicate the mean lifetime of the new cartridges exceeds that of the old at $\alpha = .005$.

c. No. The mean lifetime of the old cartridges was 1,511.4 pages and the mean of the new cartridges is 1,502.5 pages. In practical terms, there is not much difference between these two numbers.

d. Yes. There is much less variability among the new cartridges. Thus, they are more similar to each other and the quality is higher.

8.34 First, find the value of the test statistic:

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{10.7 - 10}{3.1/\sqrt{50}} = 1.60$$

p-value $= P(z \leq -1.60 \text{ or } z \geq 1.60) = 2P(z \geq 1.60) = 2(.5 - .4452) = 2(.0548) = .1096$
(using Table IV, Appendix B)

There is no evidence to reject H_0 for $\alpha \leq .10$.

8.48 a. H_0: $\mu = 6$
H_a: $\mu < 6$

The test statistic is $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{4.8 - 6}{1.3/\sqrt{5}} = -2.064$

The necessary assumption is that the population is normal.

The rejection region requires $\alpha = .05$ in the lower tail of the t-distribution with df $= n - 1 = 5 - 1 = 4$. From Table VI, Appendix B, $t_{.05} = 2.132$. The rejection region is $t < -2.132$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -2.064 \nless -2.132$), H_0 is not rejected. There is insufficient evidence to indicate the mean is less than 6 at $\alpha = .05$.

b. H_0: $\mu = 6$
H_a: $\mu \neq 6$

The test statistic is $t = -2.064$ (from **a**).

The assumption is the same as in **a**.

The rejection region requires $\alpha/2 = .05/2 = .025$ in each tail of the t-distribution with df $= n - 1 = 5 - 1 = 4$. From Table VI, Appendix B, $t_{.025} = 2.776$. The rejection region is $t < -2.776$ or $t > 2.776$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -2.064 \nless -2.776$), H_0 is not rejected. There is insufficient evidence to indicate the mean is different from 6 at $\alpha = .05$.

c. For part **a**, the p-value $= P(t \leq -2.064)$.

From Table VI, with df $= 4$, $.05 < P(t \leq -2.064) < .10$ or $.05 < p\text{-value} < .10$.

For part **b**, the p-value $= P(t \leq -2.064) + P(t \geq 2.064)$.

From Table VI, with df $= 4$, $2(.05) < p\text{-value} < 2(.10)$ or $.10 < p\text{-value} < .20$.

8.52 Some preliminary calculations:

$$\bar{x} = \frac{\sum x}{n} = \frac{25{,}587}{10} = 2{,}558.7$$

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1} = \frac{65{,}474{,}113 - \frac{25{,}587^2}{10}}{10 - 1} = 517.3444$$

$$s = \sqrt{517.3444} = 22.7452$$

To determine whether the true mean pouring temperature differs from the target setting, we test:

H_0: $\mu = 2{,}550$
H_a: $\mu \neq 2{,}550$

The test statistic is $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2{,}558.7 - 2{,}550}{22.7452/\sqrt{10}} = 1.21$

The rejection region requires $\alpha/2 = .01/2 = .005$ in each tail of the t-distribution with df $= n - 1$ $10 - 1 = 9$. From Table VI, Appendix B, $t_{.005} = 3.250$. The rejection region is $t < -3.250$ or $t > 3.250$.

Since the observed value of the test statistic does not fall in the rejection region ($t = 1.21 \not> 3.250$), H_0 is not rejected. There is insufficient evidence to indicate the true mean pouring temperature differs from the target setting at $\alpha = .01$.

8.58 b. First, check to see if n is large enough.

$$p_0 \pm 3\sigma_{\hat{p}} \Rightarrow p_0 \pm 3\sqrt{\frac{p_0 q_0}{n}} \Rightarrow .70 \pm 3\sqrt{\frac{(.70)(.30)}{100}} \Rightarrow .70 \pm .14 \Rightarrow (.56, .84)$$

Since the interval lies within the interval (0, 1), the normal approximation will be adequate.

H_0: $p = .70$
H_a: $p < .70$

The test statistic is $z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.63 - .70}{\sqrt{\frac{.70(.30)}{100}}} = -1.53$

The rejection region requires $\alpha = .05$ in the lower tail of the z-distribution. From Table IV, Appendix B, $z_{.05} = 1.645$. The rejection region is $z < -1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($-1.53 \not< -1.645$), H_0 is not rejected. There is insufficient evidence to indicate that the proportion is less than .70 at $\alpha = .05$.

c. p-value $= P(z \le -1.53) = .5 - .4370 = .0630$

Since p is not less than $\alpha = .05$, H_0 is not rejected.

Z.1

a) Confidence interval is an interval estimator for unknown parameter such as population mean. Here we need to modify the question into "99 % confidence interval for the expected value(or the population mean) of quarterly GDP growth" since quarterly GDP growth itself is a random variable. As the summary statistics shows sample size is 226 and mean is 0.008358 and standard deviation is 0.010075. The (1-α) confidence interval is $[\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}]$

Here α=0.01, α/2=0.005 and $z_{\alpha/2}$=2.575, then 0.99 confidence interval is [0.008358±2.575*0.010075/15] or simply [0.0066284, 0.0100875]

b) For this question, let's define period 2 as the 1995q1-2000q4 period and period 1 as the 1973q1-1994q4. The growth rate for period 2 was 0.008927, with standard deviation of 0.004692.
The growth rate for period 1 was 0.007111. In this question, we're asked to test if the growth rate for period 2 is different from the growth rate for period 1, which implies that we want to claim that the growth rate for two periods are different. Then we formulate its negation as the null hypothesis to be tested H_0: $\mu_1 = \mu_2$. Our claim is then the alternative H_1.: $\mu_1 \neq \mu_2$ where μ_1 is the expected growth rate for period 1 and μ_2 is the expected growth rate for period 2. Then we need to conduct a test for the two unknown parameters μ_1 and μ_2 ,which is beyond this course level. To make it easier, let's **assume** that we know μ_1 and it's equal to average growth rate for period 1(but notice that it is not generally true since average growth rate is a random variable. Here we just assume this to make it solvable without going further.) Now we want to test H_0: μ_2=0.007111 against H_1.: $\mu_2 \neq$0.007111. Then the test statistic with small sample (note, n=24) is

$$t = \frac{\bar{x} - \mu_1}{s/\sqrt{n}} = \frac{0.008927 - 0.007111}{0.004692/\sqrt{24}} = 1.896$$

With α significance level, we reject the null if $t > t_{\alpha/2}$ or $t < -t_{\alpha/2}$, $t_{\alpha/2}$=2.069 with n-1=23 degrees of freedom and 0.05 significance level. Since t < 2.069, we accept the null.
If our alternative is $\mu_2 >$ 0.007111, then we reject the null if $t > t_\alpha$, t_α=1.714 with n-1=23 degrees of freedom and 0.05 significance level. Since t > 1.714, we reject the null with one-tail test.

c) As we see the above test statistic we need a sample mean($\bar{x}$) and a sample standard deviation(s) to construct a test statistic. In this question the key is we are interested in only population growth rate μ_2 and replace population standard deviation σ with sample standard deviation s. However, if we have only one value in the sample, then sample standard deviation s is 0 since we observe just "single" value. Hence we can't construct any test statistic.