

## Problem Set 4 - Solutions

Econ-310, Spring 2004

**8.22 a.** If we wish to test the research hypothesis that the mean GHQ score for all unemployed men exceeds 10, we test:

$$H_0 : \mu = 10$$

$$H_a : \mu > 10$$

This is a one-tailed test. We are only interested in rejecting  $H_0$  if the mean GHQ score for an unemployed men is greater than 10.

**b.** The rejection region requires  $\alpha = 0.05$  in the upper tail of the  $z$ -distribution. From Table IV, Appendix B,  $z_{.05} = 1.645$ . The rejection region is  $z > 1.645$ .

**c.** The test statistic is  $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{10.94 - 10.0}{5.10/\sqrt{49}} = 1.29$

Since the observed value of the test statistic does not fall in the rejection region ( $z = 1.29 \not> 1.645$ ),  $H_0$  is not rejected. There is insufficient evidence to indicate the mean GHQ score for all unemployed men is greater than 10 at  $\alpha = .05$ .

**8.32**  $p$ -value =  $P(z \geq 2.17) + P(z \leq -2.17) = (.5 - .4850)2 = .0300$  (Using Table IV, Appendix B)

**Z.1 a.**  $H_a : \mu > 10$

**b.**  $H_0 : \mu = 10$

**c.** The rejection region requires  $\alpha = 0.10$  in the upper tail of the  $t$ -distribution with  $df = n - 1 = 3$ . From Table VI, Appendix B,  $t_{.10} = 1.638$ . The rejection region is  $t > 1.638$ .

**d.** Some preliminary calculations are:

$$\bar{x} = \frac{\sum x}{n} = \frac{43.9}{4} = 10.975$$

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n - 1} = \frac{486.25 - \frac{43.9^2}{4}}{4 - 1} = 1.4825$$

$$s = \sqrt{1.4825} = 1.217$$

The test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{10.975 - 10}{1.217/\sqrt{4}} = 1.602$$

Since the test statistics does not fall in the rejection region ( $t = .1.602 \not> 1.638$ ),  $H_0$  is not rejected. There is insufficient evidence to conclude  $\mu > 10$  at  $\alpha = .10$ .

**8.54 a.** To determine whether the mean profit change for restaurants with frequency programs is greater than \$1047.34, we test:

$$H_0 : \mu = 1047.34$$

$$H_a : \mu > 1047.34$$

**b.** Some preliminary calculations are:

$$\bar{x} = \frac{\sum x}{n} = \frac{30,113.17}{12} = 2,509.43$$

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} = \frac{126,379,568.8 - \frac{30,113.17^2}{12}}{12-1} = 4,619,331.955$$

$$s = \sqrt{4,619,331.955} = 2149.2631$$

The test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{2509.43 - 1047.34}{2149.2631/\sqrt{12}} = 2.36$$

The rejection region requires  $\alpha = .05$  in the upper tail of the  $t$ -distribution with  $df = n - 1 = 12 - 1 = 11$ . From Table VI, Appendix B,  $t_{.05} = 1.796$ . The rejection region is  $t > 1.796$ . Since the observed value of the test statistic falls in the rejection region ( $t = 2.36 > 1.796$ ),  $H_0$  is rejected. There is sufficient evidence to indicate the mean profit change for restaurants with frequency programs is greater than \$1047.34 for  $\alpha = .05$ .

It appears that the frequency program would be profitable for the company if adopted nationwide.

**Z.2 a.** If we wish to test the research hypothesis that  $p$  is greater than 0.6, we test:

$$H_0 : p = 0.6$$

$$H_a : p > 0.6$$

**b.** This is a one-tailed test. We are only interested in rejecting  $H_0$  if  $p$  is greater than 0.6.

**c.** Some preliminary calculations are:

$$\hat{p} = \frac{x}{n} = \frac{1238}{2000} = 0.619$$

The test statistic is

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.619 - 0.6}{\sqrt{\frac{0.6(1-0.6)}{2000}}} = 1.7344$$

The rejection region requires  $\alpha = .05$  in the upper tail of the  $z$ -distribution. From Table IV, Appendix B,  $z_{.05} = 1.645$ . The rejection region is  $z > 1.645$ . Since the observed value of the test statistic falls in the rejection region ( $t = 1.7344 > 1.645$ ),  $H_0$  is rejected. There is sufficient evidence to indicate that  $p$  is greater than 0.6 for  $\alpha = .05$ .

**8.72** From Exercise 8.71 we want to test  $H_0 : \mu = 500$  against  $H_a : \mu > 500$  using  $\alpha = .05$ ,  $\sigma = 100$ ,  $n = 25$ , and  $\bar{x} = 532.9$ .

a.  $\beta = P(\bar{x}_0 < 532.9 \text{ when } \mu = 575) = P(z < \frac{532.9-575}{100/\sqrt{25}}) = P(z < -2.11) = .5 - .4826 = .0174$

b. Power =  $1 - \beta = 1 - .0174 = .9826$

c. In Exercise 8.71,  $\beta = .1949$  and the power is .8051. The value of  $\beta$  has decreased in this exercise since  $\mu = 575$  is further from the hypothesized value than  $\mu = 550$ . As a result, the power of the test in this exercise has increased (when  $\beta$  decreases, the power of the test increases).

**8.80** Using Table VII, Appendix B:

a. For  $n = 12$ ,  $df = n - 1 = 11$

$$P(\chi^2 = \chi_0^2) = .10 \Rightarrow \chi_0^2 = 17.2750$$

b. For  $n = 9$ ,  $df = n - 1 = 8$

$$P(\chi^2 = \chi_0^2) = .05 \Rightarrow \chi_0^2 = 15.5073$$

c. For  $n = 5$ ,  $df = n - 1 = 4$

$$P(\chi^2 = \chi_0^2) = .025 \Rightarrow \chi_0^2 = 11.1433$$

**Z.3 a.**

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_a : \mu_1 - \mu_2 > 0$$

b. This is a one-tailed test. We are only interested in rejecting  $H_0$  if  $\mu_1$  is larger than  $\mu_2$ .

c. The test statistic is

$$t = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{11.6 - 9.7}{\sqrt{\frac{27.9}{80} + \frac{38.4}{80}}} = \frac{1.9}{0.9104} = 2.087$$

The rejection region requires  $\alpha = .10$  in the upper tail of the  $z$ -distribution. From Table IV, Appendix B,  $z_{.10} = 1.28$ . The rejection region is  $z > 1.28$ .

d. The difference between sample means is 1.9. For sample size of 80 and given sample variance this is a big difference.

e. Since the observed value of the test statistic falls in the rejection region ( $z = 2.087 > 1.28$ ),  $H_0$  is rejected. There is sufficient evidence to indicate that  $\mu_1 > \mu_2$  for  $\alpha = .10$ .

#### 9.4 Assumptions about the two populations:

1. Both sampled population have relative frequency distribution that are approximately normal.

2. The population variances are equal.

Assumptions about the two samples:

The samples are randomly and independently selected from the population.

#### 9.10 Some preliminary calculations:

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{654}{15} = 43.6$$

$$s_1^2 = \frac{\sum x_1^2 - \frac{(\sum x_1)^2}{n_1}}{n_1 - 1} = \frac{28934 - \frac{654^2}{15}}{15 - 1} = \frac{419.6}{14} = 29.9714$$

$$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{858}{16} = 53.625$$

$$s_2^2 = \frac{\sum x_2^2 - \frac{(\sum x_2)^2}{n_2}}{n_2 - 1} = \frac{46450 - \frac{858^2}{16}}{16 - 1} = \frac{439.75}{15} = 29.3167$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{14 * 29.9714 + 15 * 29.3167}{15 + 16 - 2} = \frac{859.3501}{29} = 29.6328$$

a.

$$H_0 : \mu_2 - \mu_1 = 10$$

$$H_a : \mu_2 - \mu_1 > 10$$

The test statistic is

$$t = \frac{\bar{x}_2 - \bar{x}_1 - D_0}{\sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{(53.625 - 43.6) - 10}{\sqrt{29.6328(\frac{1}{15} + \frac{1}{16})}} = \frac{.025}{1.9564} = .013$$

The rejection region requires  $\alpha = .01$  in the upper tail of the  $t$ -distribution with  $df = n_1 + n_2 - 2 = 29$ . From Table VI, Appendix B,  $t_{.01} = 2.462$ . The rejection region is  $t > 2.462$ . Since the test statistics does not fall in the rejection region ( $t = .013 \not> 2.462$ ),  $H_0$  is not rejected. There is insufficient evidence to conclude  $\mu_2 - \mu_1 > 10$  at  $\alpha = .01$ .

**b.** For confidence coefficient .98,  $\alpha = .02$  and  $\alpha/2 = .01$ . From Table VI, Appendix B, with  $df = n_1 + n_2 - 2 = 15 + 16 - 2 = 29$ ,  $t_{.01} = 2.462$ . The 98% confidence interval for  $\mu_2 - \mu_1$  is:

$$\bar{x}_2 - \bar{x}_1 \pm t_{\alpha/2} \sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})} = 10.025 \pm 4.817 = (5.208, 14.842)$$

We are 98% confident that the difference between the mean of population 2 and the mean of population 1 is between 5.208 and 14.842.

**9.24 a.** Let  $\mu_1$  = mean distance to work for men in the central city and  $\mu_2$  = mean distance to work for women in the central city.

For confidence coefficient .99,  $\alpha = .01$  and  $\alpha/2 = .005$ . From Table IV, Appendix B,  $z_{.005} = 2.58$ . The confidence interval is:

$$\begin{aligned} \bar{x}_1 - \bar{x}_2 \pm z_{.005} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} &\Rightarrow (7.4 - 4.5) \pm 2.58 \sqrt{\frac{6.3^2}{159} + \frac{4.2^2}{119}} \\ &\Rightarrow 2.9 \pm 1.63 \Rightarrow (1.27, 4.53) \end{aligned}$$

We are 99% confident that the difference in the mean distance to work for men and women who reside in the central city is between 1.27 and 4.53 miles.

**b.** Let  $\mu_1$  = mean distance to work for men in suburban residences and  $\mu_2$  = mean distance to work for women in suburban residences.

The confidence interval is:

$$\begin{aligned} \bar{x}_1 - \bar{x}_2 \pm z_{.005} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} &\Rightarrow (9.3 - 6.6) \pm 2.58 \sqrt{\frac{7.1^2}{138} + \frac{5.6^2}{93}} \\ &\Rightarrow 2.7 \pm 2.16 \Rightarrow (0.54, 4.86) \end{aligned}$$

We are 99% confident that the difference in the mean distance to work for men and women who reside in suburbs is between 0.54 and 4.86 miles.

**c.** Since neither of the confidence intervals contain 0, there is evidence that on the average women work closer to home than men.

d. We must assume that independent random samples were drawn from each population.

9.26 a.

Pair	Difference
1	3
2	2
3	2
4	4
5	0
6	1

$$\bar{x}_D = \frac{\sum x_D}{n_D} = \frac{12}{6} = 2$$

$$s_D^2 = \frac{\sum x_D^2 - \frac{(\sum x_D)^2}{n_D}}{n_D - 1} = \frac{34 - \frac{12^2}{6}}{6 - 1} = 2$$

b.  $\mu_D = \mu_1 - \mu_2$

c. c. For confidence coefficient .95,  $\alpha = .05$  and  $\alpha/2 = .025$ . From Table VI, Appendix B, with  $df = n_D - 1 = 6 - 1 = 5$ ,  $t_{.025} = 2.571$ . The confidence interval is:

$$\bar{x}_D \pm t_{\alpha/2} \frac{s_D}{\sqrt{n_D}} \Rightarrow 2 \pm 2.571 \frac{\sqrt{2}}{\sqrt{6}} \Rightarrow 2 \pm 1.484 \Rightarrow (.516, 3.484)$$

d.

$$H_0 : \mu_D = 0$$

$$H_a : \mu_D \neq 0$$

The test statistic is  $t = \frac{\bar{x}_D}{s_D/\sqrt{n_D}} = \frac{2}{\sqrt{2}/\sqrt{6}} = 3.46$

The rejection region requires  $\alpha/2 = .05/2 = .025$  in each tail of the  $t$ -distribution with  $df = n_D - 1 = 6 - 1 = 5$ . From Table VI, Appendix B,  $t_{.025} = 2.571$ . The rejection region is  $t < -2.571$  or  $t > 2.571$ .

Since the observed value of the test statistic falls in the rejection region ( $3.46 > 2.571$ ),  $H_0$  is rejected. There is sufficient evidence to indicate that the mean difference is different from 0 at  $\alpha = .05$ .

9.56 a. For confidence coefficient .95,  $\alpha = 1 - .95 = .05$  and  $\alpha/2 = .025$ . From Table IV, Appendix B,  $z_{.025} = 1.96$ .

$$n_1 = n_2 = \frac{(z_{\alpha/2})^2(\sigma_1^2 + \sigma_2^2)}{B^2} = \frac{(1.96)^2(15^2 + 17^2)}{3.2^2} = 192.83 \approx 193$$

b. If the range of each population is 40, we would estimate  $\sigma$  by:

$$\sigma \approx 60/4 = 15$$

For confidence coefficient .99,  $\alpha = 1 - .99 = .01$  and  $\alpha/2 = .005$ . From Table IV, Appendix B,  $z_{.005} = 2.58$ .

$$n_1 = n_2 = \frac{(z_{\alpha/2})^2(\sigma_1^2 + \sigma_2^2)}{B^2} = \frac{(2.58)^2(15^2 + 15^2)}{8^2} = 46.80 \approx 47$$

c. For confidence coefficient .9,  $\alpha = 1 - .9 = .1$  and  $\alpha/2 = .05$ . From Table IV, Appendix B,  $z_{.05} = 1.645$ . For a width of 1, the bound is .5.

$$n_1 = n_2 = \frac{(z_{\alpha/2})^2(\sigma_1^2 + \sigma_2^2)}{B^2} = \frac{(1.645)^2(5.8^2 + 7.5^2)}{.5^2} = 143.96 \approx 144$$

**9.88** Some preliminary calculations are :

$$\bar{x}_1 = \frac{\sum x_1}{n_1} = \frac{64.3}{6} = 10.7167$$

$$s_1^2 = \frac{\sum x_1^2 - \frac{(\sum x_1)^2}{n_1}}{n_1 - 1} = \frac{704.43 - \frac{64.3^2}{6}}{6 - 1} = 3.0697$$

$$\bar{x}_2 = \frac{\sum x_2}{n_2} = \frac{61.8}{6} = 10.3$$

$$s_2^2 = \frac{\sum x_2^2 - \frac{(\sum x_2)^2}{n_2}}{n_2 - 1} = \frac{653.06 - \frac{61.8^2}{6}}{6 - 1} = 3.304$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{5 * 3.0697 + 5 * 3.304}{6 + 6 - 2} = 3.18685$$

a. To determine if there is a difference in the mean strength of the two types of shocks, we test:

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_a : \mu_1 - \mu_2 \neq 0$$

The test statistic is

$$t = \frac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{(10.7167 - 10.3) - 0}{\sqrt{3.18685(\frac{1}{6} + \frac{1}{6})}} = .40$$

The rejection region requires  $\alpha/2 = .05/2 = .025$  in each tail of the  $t$ -distribution with  $df = n_1 + n_2 - 2 = 10$ . From Table VI, Appendix B,  $t_{.025} = 2.228$ . The rejection region is  $t < -2.228$  or  $t > 2.228$ .

Since the test statistics does not fall in the rejection region ( $t = .40 \not> 2.228$ ),  $H_0$  is not rejected. There is insufficient evidence to indicate a difference between the mean strength for the two types of shocks at  $\alpha = .05$ .

**b.** For confidence coefficient .95,  $\alpha = 1 - .95 = .05$  and  $\alpha/2 = .025$ . From Table VI, Appendix B, with  $df = n_1 + n_2 - 2 = 10$ ,  $t_{.025} = 2.228$ . The confidence interval is:

$$\begin{aligned} \bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2} \sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)} &\Rightarrow \pm(10.7167 - 10.3) \pm 2.228 \sqrt{3.18685 \left( \frac{1}{6} + \frac{1}{6} \right)} \\ &\Rightarrow .4167 \pm 2.2963 \Rightarrow (-1.8796, 2.7130) \end{aligned}$$

We are 95% confident the mean strength of the manufacturer's shock exceeds the mean strength of the competitor's shock by anywhere from -1.8796 to 2.7130.

**c.** The confidence interval obtained in part **b** is wider than that found in Exercise 9.34. This interval is wider because the standard deviation is larger for the independent samples than for the matched-pair design.

**d.** The results of an unpaired analysis are not valid when the data are collected from a paired experiment. The assumption of independent samples is not valid.

**9.70 a.** The rejection region requires  $\alpha = .05$  in the upper tail of the  $F$ -distribution with  $\nu_1 = n_1 - 1 = 25 - 1 = 24$  and  $\nu_2 = n_2 - 1 = 20 - 1 = 19$ . From Table IX, Appendix B,  $F_{.05} = 2.11$ . The rejection region is  $F > 2.11$  (if  $s_1^2 > s_2^2$ ).

**b.** The rejection region requires  $\alpha = .05$  in the upper tail of the  $F$ -distribution with  $\nu_1 = n_2 - 1 = 15 - 1 = 14$  and  $\nu_2 = n_1 - 1 = 10 - 1 = 9$ . From Table IX, Appendix B,  $F_{.05} \approx 3.01$ . The rejection region is  $F > 3.01$  (if  $s_2^2 > s_1^2$ ).

**c.** The rejection region requires  $\alpha = .10/2 = .05$  in the upper tail of the  $F$ -distribution. If  $s_1^2 > s_2^2$ ,  $\nu_1 = n_1 - 1 = 21 - 1 = 20$  and  $\nu_2 = n_2 - 1 = 31 - 1 = 30$ . From Table IX, Appendix B,  $F_{.05} = 1.93$ . The rejection region is  $F > 1.93$ . If  $s_1^2 < s_2^2$ ,  $\nu_1 = n_2 - 1 = 31 - 1 = 30$  and  $\nu_2 = n_1 - 1 = 21 - 1 = 20$ . From Table IX, Appendix B,  $F_{.05} = 2.04$ . The rejection region is  $F > 2.04$ .

**d.** The rejection region requires  $\alpha = .01$  in the upper tail of the  $F$ -distribution with  $\nu_1 = n_2 - 1 = 41 - 1 = 40$  and  $\nu_2 = n_1 - 1 = 31 - 1 = 30$ . From Table XI, Appendix B,  $F_{.01} = 2.30$ . The rejection region is  $F > 2.30$  (if  $s_2^2 > s_1^2$ ).



e. The rejection region requires  $\alpha = .05/2 = .025$  in the upper tail of the  $F$ -distribution. If  $s_1^2 > s_2^2$ ,  $\nu_1 = n_1 - 1 = 7 - 1 = 6$  and  $\nu_2 = n_2 - 1 = 16 - 1 = 15$ . From Table X, Appendix B,  $F_{.025} = 3.14$ . The rejection region is  $F > 3.14$ . If  $s_1^2 < s_2^2$ ,  $\nu_1 = n_2 - 1 = 16 - 1 = 15$  and  $\nu_2 = n_1 - 1 = 7 - 1 = 6$ . From Table X, Appendix B,  $F_{.025} = 5.27$ . The rejection region is  $F > 5.27$ .

**Z.4** Let  $\mu_1$  be the productivity growth rate during 1991:1-1994:4 (the earlier period) and  $\mu_2$  be the productivity growth rate during 1997:1 2000:4 (the latter period). If we wish to test that the productivity growth rate was higher in the later period than the earlier period, we test:

$$H_0 : \mu_2 - \mu_1 = 0$$

$$H_a : \mu_2 - \mu_1 > 0$$

Since  $n_1 < 30$  and  $n_2 < 30$  we will use t-test. For t-test to be applicable, we have to assume that the productivity growth rate is normal and the productivity growth rate in the earlier period is independent of the productivity growth rate in the later period. The sample variance for the pooled data is:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{15 * 0.028037^2 + 15 * 0.024957^2}{16 + 16 - 2} = \frac{0.02113}{30} = 0.0007$$

The test statistic is

$$t = \frac{\bar{x}_2 - \bar{x}_1 - D_0}{\sqrt{s_p^2(\frac{1}{n_1} + \frac{1}{n_2})}} = \frac{0.026998 - 0.019428}{\sqrt{0.0007(\frac{1}{16} + \frac{1}{16})}} = 0.809$$

The rejection region requires  $\alpha = .10$  in the upper tail of the  $t$ -distribution with  $df = n_1 + n_2 - 2 = 30$ . From Table VI, Appendix B,  $t_{.10} = 1.310$ . The rejection region is  $t > 1.310$ . Since the test statistics does not fall in the rejection region ( $t = .809 \not> 1.310$ ),  $H_0$  is not rejected. There is insufficient evidence to conclude that the productivity growth rate was higher in the latter period than the earlier period at 10% significance level.