Economics 310
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University of Wisconsin-Madison

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Social Sciences 7418

Problem Set 4 Answer Key

This problem set is due in lecture on **Wednesday, December 1st**. No late problem sets will be accepted. **Be sure to show your work** (that is, do not use a spreadsheet or statistical program to generate your answers), and to write your name, ID number, as well as the name of your Teaching Assistant, on your problem set. Numbered problems refer to the textbook problems.

- 8.16
- 8.20
- 8.30
- 8.50
- 8.56
- 8.62
- 8.72
- 8.80
- 8.82
- 9.2
- 9.22
- 9.28
- 9.36
- 9.44
- 9.60
- 9.70

8.16 a. A Type I error would be concluding the proposed user is unauthorized when, in fact, the proposed user is authorized.

A Type II error would be concluding the proposed user is authorized when, in fact, the proposed user is unauthorized.

In this case, a more serious error would be a Type II error. One would not want to conclude that the proposed user is authorized when he/she is not.

b. The Type I error rate is 1%. This means that the probability of concluding the proposed user is unauthorized when, in fact, the proposed user is authorized is .01.

The Type II error rate is .00025%. This means that the probability of concluding the proposed user is authorized when, in fact, the proposed user is unauthorized is .0000025.

c. The Type I error rate is .01%. This means that the probability of concluding the proposed user is unauthorized when, in fact, the proposed user is authorized is .0001.

The Type II error rate is .005%. This means that the probability of concluding the proposed user is authorized when, in fact, the proposed user is unauthorized is .00005.

8.20 a. H_0: $\mu = .36$
H_a: $\mu < .36$

The test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{.323 - .36}{\sqrt{.034}/\sqrt{64}} = -1.61$

The rejection region requires $\alpha = .10$ in the lower tail of the z-distribution. From Table IV, Appendix B, $z_{.10} = 1.28$. The rejection region is $z < -1.28$.

Since the observed value of the test statistic falls in the rejection region ($z = -1.61 < -1.28$), H_0 is rejected. There is sufficient evidence to indicate the mean is less than .36 at $\alpha = .10$.

b. H_0: $\mu = .36$
H_a: $\mu \neq .36$

The test statistic is $z = -1.61$ (see part **a**).

The rejection region requires $\alpha/2 = .10/2 = .05$ in the each tail of the z-distribution. From Table IV, Appendix B, $z_{.05} = 1.645$. The rejection region is $z < -1.645$ or $z > 1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.61 \not< -1.645$), H_0 is not rejected. There is insufficient evidence to indicate the mean is different from .36 at $\alpha = .10$.

8.30 a. To determine whether the true mean rating for this instructor-related factor exceeds 4, we test:

H_0: $\mu = 4$
H_a: $\mu > 4$

The test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{4.7 - 4}{1.62/\sqrt{40}} = 2.73$

The rejection region requires $\alpha = .05$ in the upper tail of the z-distribution. From Table IV, Appendix B, $z_{.05} = 1.645$. The rejection region is $z > 1.645$.

Since the observed value of the test statistic falls in the rejection region ($z = 2.73 > 1.645$), H_0 is rejected. There is sufficient evidence to indicate that the true mean rating for this instructor-related factor exceeds 4 at $\alpha = .05$.

b. If the sample size is large enough, one could almost always reject H_0. Thus, we might be able to detect very small differences if the sample size is large enough. This would be statistical significance. However, even though statistical significance is found, it does not necessarily mean that there is practical significance. A statistical significance can sometimes be found between the hypothesized value of a mean and the estimated value of the mean, but, in practice, this difference would mean nothing. This would be practical significance.

c. Since the sample size is sufficiently large ($n = 40$), the Central Limit Theorem indicates that the sampling distribution of is approximately normal. Also, since the sample size is large, s is a good estimator of σ. Thus, the analysis used is appropriate.

8.50 a. H_0: $\mu = 6$
H_a: $\mu < 6$

The test statistic is $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{4.8 - 6}{1.3/\sqrt{5}} = -2.064$

The necessary assumption is that the population is normal.

The rejection region requires $\alpha = .05$ in the lower tail of the t-distribution with df $= n - 1 = 5 - 1 = 4$. From Table VI, Appendix B, $t_{.05} = 2.132$. The rejection region is $t < -2.132$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -2.064 \not< -2.132$), H_0 is not rejected. There is insufficient evidence to indicate the mean is less than 6 at $\alpha = .05$.

b. H_0: $\mu = 6$
H_a: $\mu \neq 6$

The test statistic is $t = -2.064$ (from **a**).

The assumption is the same as in **a**.

The rejection region requires $\alpha/2 = .05/2 = .025$ in each tail of the t-distribution with $df = n - 1 = 5 - 1 = 4$. From Table VI, Appendix B, $t_{.025} = 2.776$. The rejection region is $t < -2.776$ or $t > 2.776$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -2.064 \not< -2.776$), H_0 is not rejected. There is insufficient evidence to indicate the mean is different from 6 at $\alpha = .05$.

8.56 a. To determine if the mean repellency percentage of the new mosquito repellent is less than 95, we test:

H_0: $\mu = 95$
H_a: $\mu < 95$

The test statistic is $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{83 - 95}{15/\sqrt{5}} = -1.79$

The rejection region requires $\alpha = .10$ in the lower tail of the t distribution. From Table VI, Appendix B, with $df = n - 1 = 5 - 1 = 4$, $t_{.10} = 1.533$. The rejection region is $t < -1.533$.

Since the observed value of the test statistic falls in the rejection region ($t = -1.79 < -1.533$), H_0 is rejected. There is sufficient evidence to indicate that the true mean repellency percentage of the new mosquito repellent is less than 95 at $\alpha = .10$.

b. We must assume that the population of percent repellencies is normally distributed.

8.62 b. First, check to see if n is large enough.

$$p_0 \pm 3\sigma_{\hat{p}} \Rightarrow p_0 \pm 3\sqrt{\frac{p_0 q_0}{n}} \Rightarrow .70 \pm 3\sqrt{\frac{(.70)(.30)}{100}} \Rightarrow .70 \pm .14 \Rightarrow (.56, .84)$$

Since the interval lies within the interval (0, 1), the normal approximation will be adequate.

H_0: $p = .70$
H_a: $p < .70$

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{.63 - .70}{\sqrt{\dfrac{.70(.30)}{100}}} = -1.53$

The rejection region requires $\alpha = .05$ in the lower tail of the z-distribution. From Table IV, Appendix B, $z_{.05} = 1.645$. The rejection region is $z < -1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($-1.53 \not< -1.645$), H_0 is not rejected. There is insufficient evidence to indicate that the proportion is less than .70 at $\alpha = .05$.

c. p-value $= P(z \le -1.53) = .5 - .4370 = .0630$

Since p is not less than $\alpha = .05$, H_0 is not rejected.

8.72 a. Let p = proportion of the subjects who felt that the masculinization of face shape decreased attractiveness of the male face. If there is no preference for the unaltered or morphed male face, then $p = .5$.

For this problem, $\hat{p} = x/n = 58/67 = .866$.

First we check to see if the normal approximation is adequate:

$$p_0 \pm 3\sigma_{\hat{p}} \Rightarrow p_0 \pm 3\sqrt{\frac{p_0 q_0}{n}} \Rightarrow .5 \pm 3\sqrt{\frac{.5(.5)}{67}} \Rightarrow (.317, .683)$$

Since the interval falls completely in the interval (0, 1), the normal distribution will be adequate.

To determine if the subjects showed a preference for either the unaltered or morphed face, we test:

H_0: $p = .5$
H_a: $p \neq .5$

b. The test statistic is $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{.866 - .50}{\sqrt{\dfrac{.50(.50)}{67}}} = 5.99$

c. By definition, the p-value is the probability of observing our test statistic or anything more unusual, given H_0 is true. For this problem, $p = P(z \leq -5.99) + P(z \geq 5.99)$ $= .5 - P(-5.99 < z < 0) + .5 - P(0 < z < 5.99) \approx .5 - .5 + .5 - .5 = 0$. This corresponds to what is listed in the Exercise.

d. The rejection region requires $\alpha/2 = .01/2 = .005$ in each tail of the z distribution. From Table IV, Appendix B, $z_{.005} = 2.575$. The rejection region is $z < -2.575$ or $z > 2.575$.

Since the observed value of the test statistic falls in the rejection region ($z = 5.99 > 2.575$), H_0 is rejected. There is sufficient evidence to indicate that the subjects showed a preference for either the unaltered or morphed face at $\alpha = .01$.

8.80 a. From Exercise 8.79, we want to test H_0: $\mu = 75$ against H_a: $\mu < 75$ using $\alpha = .10$, $\sigma = 15$, $n = 49$, and $\bar{x} = 72.257$.

If $\mu = 74$,

$$\beta = P(\bar{x}_0 > 72.257 \text{ when } \mu = 74) = P\left(z > \frac{72.257 - 74}{15/\sqrt{49}}\right) = P(z > -.81)$$
$$= .5 + .2910 = .7910$$

If $\mu = 72$,

$$\mu = P(\bar{x}_0 > 72.257 \text{ when } \mu = 72) = P\left(z > \frac{72.257 - 72}{15/\sqrt{49}}\right) = P(z > .12)$$
$$= .5 - .0478 = .4522$$

If $\mu = 70$,

$\beta = P(\bar{x}_0 > 72.257 \text{ when } \mu = 70) = .1469$
(Refer to Exercise 8.69, part **c**.)

If $\mu = 68$,

$$\beta = P(\bar{x}_0 > 72.257 \text{ when } \mu = 68) = P\left(z > \frac{72.257 - 68}{15/\sqrt{49}}\right) = P(z > 1.99)$$
$$= .5 - .4767 = .0233$$

If $\mu = 66$,

$$\beta = P(\bar{x}_0 > 72.257 \text{ when } \mu = 66) = P\left(z > \frac{72.257 - 66}{15/\sqrt{49}}\right) = P(z > 2.92)$$
$$= .5 - .4982 = .0018$$

In summary,

μ	74	72	70	68	66
β	.7910	.4522	.1469	.0233	.0018

8.82 a. To determine if the mean size of California homes built in late 1999 exceeds the national average, we test:

H_0: $\mu = 2230$
H_a: $\mu > 2230$

The test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{2347 - 2230}{257/\sqrt{100}} = 4.55$

The rejection region requires $\alpha = .01$ in the upper tail of the z-distribution. From Table IV, Appendix B, $z_{.05} = 2.33$. The rejection region is $z > 2.33$.

Since the observed value of the test statistic falls in the rejection region ($z = 4.55 > 2.33$), H_0 is rejected. There is sufficient evidence to indicate the mean size of California homes built in late 1999 exceeds the national average at $\alpha = .01$.

b. To compute the power, we must first set up the rejection regions in terms of .

$$\bar{x}_0 = \mu_0 + z_\alpha \sigma_{\bar{x}} \approx \mu_0 + 2.33\left(\frac{s}{\sqrt{n}}\right) = 2{,}230 + 2.33\left(\frac{257}{\sqrt{100}}\right) = 2{,}289.88$$

We would reject H_0 if $\bar{x} > 2{,}289.88$

The power of the test when $\mu = 2{,}330$ would be:

$$\text{Power} = P(\bar{x} > 2289.88 \mid \mu = 2{,}330) = P\left(z > \frac{\bar{x}_0 - \mu_a}{\sigma_{\bar{x}}}\right) = P\left(z > \frac{2{,}289.88 - 2{,}330}{257/\sqrt{100}}\right)$$
$$= P(z > -1.56) = .5 + .4406 = .9406$$

c. The power of the test when $\mu = 2{,}280$ would be:

$$\text{Power} = P(> 2289.88 \mid \mu = 2{,}280) = P\left(z > \frac{\bar{x}_0 - \mu_a}{\sigma_{\bar{x}}}\right) = P\left(z > \frac{2{,}289.88 - 2{,}280}{257/\sqrt{100}}\right)$$
$$= P(z > 0.38) = .5 - .1480 = .3520$$

9.2 a. $\mu_{\bar{x}_1} = \mu_1 = 12$ $\qquad \sigma_{\bar{x}_1} = \dfrac{\sigma_1}{\sqrt{n_1}} = \dfrac{4}{\sqrt{64}} = .5$

b. $\mu_{\bar{x}_2} = \mu_2 = 10$ $\qquad \sigma_{\bar{x}_2} = \dfrac{\sigma_2}{\sqrt{n_2}} = \dfrac{3}{\sqrt{64}} = .375$

c. $\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2 = 12 - 10 = 2$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{4^2}{64} + \frac{3^2}{64}} = \sqrt{\frac{25}{64}} = .625$$

d. Since $n_1 \geq 30$ and $n_2 \geq 30$, the sampling distribution of $\bar{x}_1 - \bar{x}_2$ is approximately normal by the Central Limit Theorem.

9.22 a. The bacteria counts are probably normally distributed because each count is the median of five measurements from the same specimen.

b. Let μ_1 = mean of the bacteria count for the discharge and μ_2 = mean of the bacteria count upstream. Since we want to test if the mean of the bacteria count for the discharge exceeds the mean of the count upstream, we test:

H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 > 0$

c. Using MINITAB, the descriptive statistics are:

```
Descriptive Statistics: Plant, Upstream

Variable            N       Mean     Median     TrMean      StDev    SE Mean
Plant               6      32.10      31.75      32.10       3.19       1.30
Upstream            6     29.617     30.000     29.617      2.355      0.961

Variable       Minimum    Maximum         Q1         Q3
Plant            28.20      36.20      29.40      35.23
Upstream        26.400     32.300     27.075     31.850
```

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{(6-1)3.19^2 + (6-1)2.355^2}{6+6-2} = 7.861$$

The test statistic is $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{s_p^2\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}} = \dfrac{(32.10 - 29.617) - 0}{\sqrt{7.861\left(\dfrac{1}{6}+\dfrac{1}{6}\right)}} = 1.53$

No α level was given, so we will use $\alpha = .05$. The rejection region requires $\alpha = .05$ in the upper tail of the t-distribution with df $= n_1 + n_2 - 2 = 6 + 6 - 2 = 10$. From Table VI, Appendix B, $t_{.05} = 1.812$. The rejection region is $t > 1.812$.

Since the observed value of the test statistic does not fall in the rejection region ($t = 1.53 \not> 1.812$), H_0 is not rejected. There is insufficient evidence to indicate the mean bacteria count for the discharge exceeds the mean of the count upstream at $\alpha = .05$.

d. We must assume:

1. The mean counts per specimen for each location is normally distributed.
2. The variances of the 2 distributions are equal.
3. Independent and random samples were selected from each population.

9.28 a. H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 < 0$

The rejection region requires $\alpha = .10$ in the lower tail of the z-distribution. From Table IV, Appendix B, $z_{.10} = 1.28$. The rejection region is $z < -1.28$.

b. H_0: $\mu_1 - \mu_2 = 0$
H_a: $\mu_1 - \mu_2 < 0$

The test statistic is $z = \dfrac{\bar{d} - 0}{\dfrac{s_d}{\sqrt{n_d}}} = \dfrac{-3.5 - 0}{\dfrac{\sqrt{21}}{\sqrt{38}}} = -4.71$.

The rejection region is $z < -1.28$ (Refer to part a.)

Since the observed value of the test statistic falls in the rejection region ($z = -4.71 < -1.28$), H_0 is rejected. There is sufficient evidence to indicate $\mu_1 - \mu_2 < 0$ at $\alpha = .10$.

c. Since the sample size of the number of pairs is greater than 30, we do not need to assume that the population of differences is normal. The sampling distribution of $\bar{d}$ is approximately normal by the Central Limit Theorem. We must assume that the differences are randomly selected.

d. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix B, $z_{.05} = 1.645$. The 90% confidence interval is:

$$\bar{d} \pm z_{.05}\frac{s_d}{\sqrt{n_d}} \Rightarrow -3.5 \pm 1.645\frac{\sqrt{21}}{\sqrt{38}} \Rightarrow -3.5 \pm 1.223 \Rightarrow (-4.723,\ -2.277)$$

e. The confidence interval provides more information since it gives an interval of possible values for the difference between the population means.

9.36 a. To determine whether male students' attitudes toward their fathers differ from their attitudes toward their mothers, on average, we test:

H_0: $\mu_d = 0$
H_a: $\mu_d \neq 0$

b. Using MINITAB, the descriptive statistics are:

```
Descriptive Statistics: Father, Mother, Diff

Variable             N       Mean     Median     TrMean      StDev    SE Mean
Father              13      3.846      4.000      3.909      1.068      0.296
Mother              13      4.154      4.000      4.182      0.801      0.222
Diff                13     -0.308     -1.000     -0.273      1.032      0.286

Variable       Minimum    Maximum         Q1         Q3
Father           2.000      5.000      3.000      5.000
Mother           3.000      5.000      3.500      5.000
Diff            -2.000      1.000     -1.000      1.000
```

The test statistic is $t = \dfrac{\bar{d} - D_o}{\dfrac{s_d}{\sqrt{n_d}}} = \dfrac{-.308 - 0}{\dfrac{1.032}{\sqrt{13}}} = -1.08$

The rejection region requires $\alpha/2 = .05/2 = .025$ in each tail of the t-distribution with df $= n_d - 1 = 13 - 1 = 12$. From Table VI, Appendix B, $t_{.025} = 2.179$. The rejection region is $t < -2.179$ or $t > 2.179$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -1.08 \not< -2.179$), H_0 is not rejected. There is insufficient evidence to indicate that male students' attitudes toward their fathers differ from their attitudes toward their mothers at $\alpha = .05$.

c. In order for the above test to be valid, we must assume that

1. The population of differences is normal
2. The differences are randomly selected

d. A stem-and-leaf display of the differences is:

```
Character Stem-and-Leaf Display

Stem-and-leaf of Diff      N  = 13
Leaf Unit = 0.10


  1   -2 0
(6)   -1 000000
 6     0 00
 4     1 0000
```

We know that the data are not normal, because there are only 4 different values for the differences. Although the stem-and-leaf display does not look particularly normal, it is rather mound-shaped. The t-test procedure works pretty well even if the data are not exactly normal.

9.44 For confidence coefficient .95, $\alpha = 1 - .95 = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix B, $z_{.025} = 1.96$. The 95% confidence interval for $p_1 - p_2$ is approximately:

a. $(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1} + \dfrac{\hat{p}_2\hat{q}_2}{n_2}} \Rightarrow (.65 - .58) \pm 1.96\sqrt{\dfrac{.65(1-.65)}{400} + \dfrac{.58(1-.58)}{400}}$

$\Rightarrow .07 - .067 \Rightarrow (.003, .137)$

b. $(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} \Rightarrow (.31 - .25) - 1.96\sqrt{\frac{.31(1-.31)}{180} + \frac{.25(1-.25)}{250}}$

$\Rightarrow .06 \pm .086 \Rightarrow (-.026, .146)$

c. $(\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} \Rightarrow (.46 - .61) \pm 1.96\sqrt{\frac{.46(1-.46)}{100} + \frac{.61(1-.61)}{120}}$

$\Rightarrow -.15 \pm .131 \Rightarrow (-.281, -.019)$

9.60 First, find the sample sizes needed for width 5, or margin of error 2.5.

For confidence coefficient .9, $\alpha = 1 - .9 = .1$ and $\alpha/2 = .1/2 = .05$. From Table IV, Appendix B, $z_{.05} = 1.645$.

$$n_1 = n_2 = \frac{(z_{\alpha/2})^2(\sigma_1^2 + \sigma_2^2)}{ME^2} = \frac{(1.645)^2(10^2 + 10^2)}{2.5^2} = 86.59 \approx 87$$

Thus, the necessary sample size from each population is 87. Therefore, sufficient funds have not been allocated to meet the specifications since $n_1 = n_2 = 100$ are large enough samples.

9.70 a. With $\nu_1 = 2$ and $\nu_2 = 30$,
$P(F \geq 5.39) = .01$ (Table XI, Appendix B)

b. With $\nu_1 = 24$ and $\nu_2 = 10$,
$P(F \geq 2.74) = .05$ (Table IX, Appendix B)

Thus, $P(F < 2.74) = 1 - P(F \geq 2.74) = 1 - .05 = .95$.

c. With $\nu_1 = 7$ and $\nu_2 = 1$,
$P(F \geq 236.8) = .05$ (Table VIII, Appendix B)

Thus, $P(F < 236.8) = 1 - P(F \geq 236.8) = 1 - .05 = .95$.

d. With $\nu_1 = 40$ and $\nu_2 = 40$,
$P(F > 2.11) = .01$ (Table XI, Appendix B)

- Problem X.1

During the late 1990s, there was a belief that Europe lagged behind the United States in productivity growth, especially over the 1995q1-2000q4 period (shaded area in the graph below).

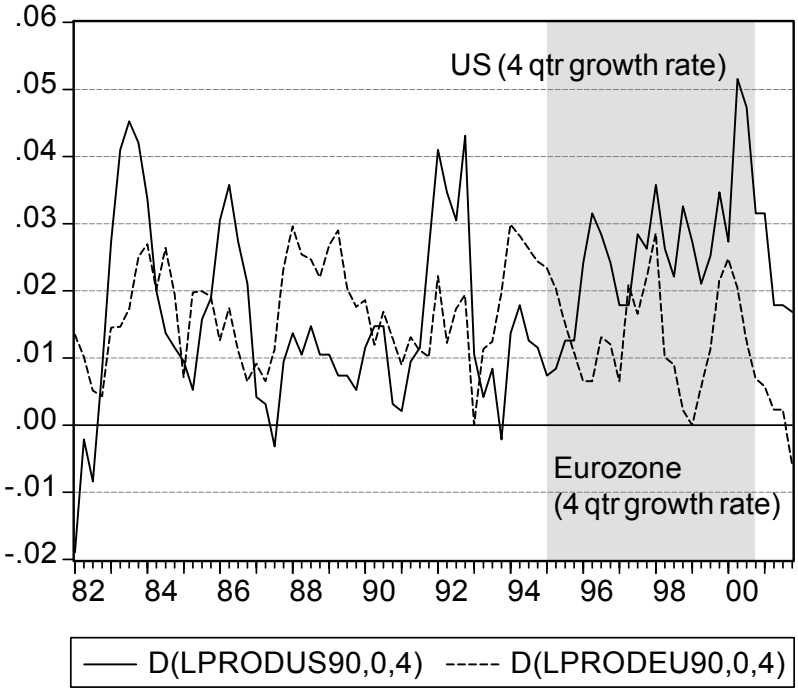


Figure 1: US and Eurozone 4 quarter growth rates.

Here are some statistics below on quarter-on-quarter annualized growth rates and differences in growth rates between the two economies.

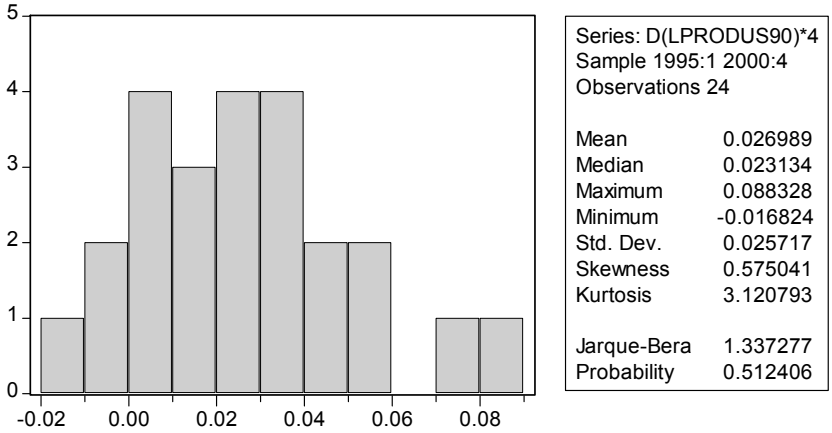


Figure 2: Histogram for United States qoq annualized GDP growth

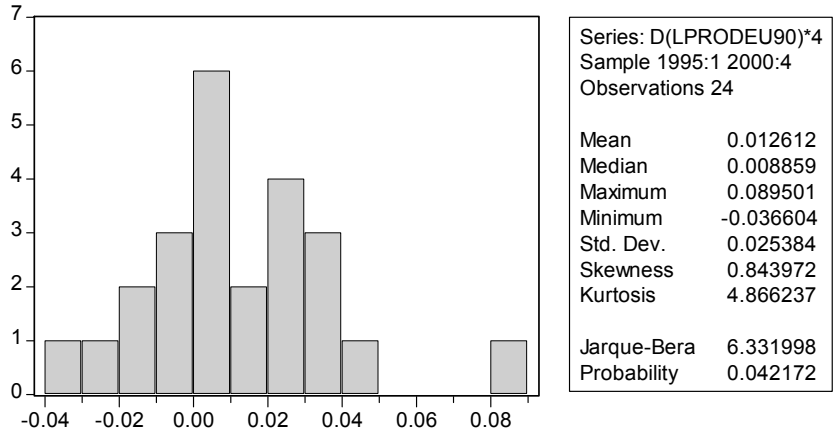


Figure 3: Histogram for Eurozone qoq annualized GDP growth

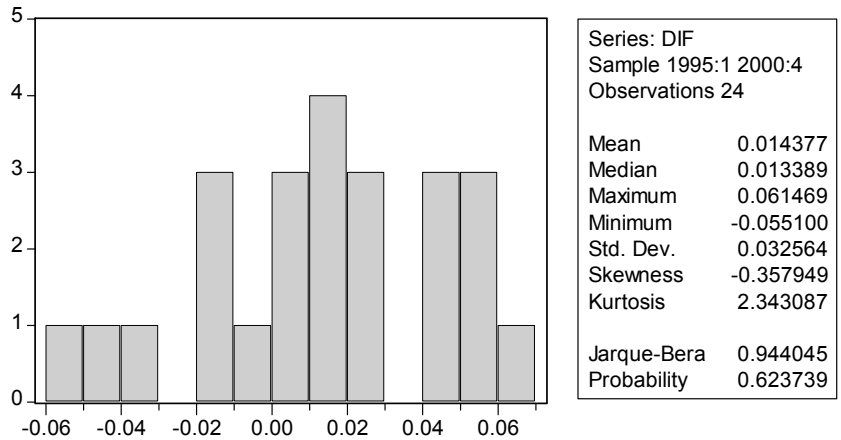


Figure 4: Histogram for DIF = (d(lprodus90)-d(lprodeu90))*4

X.1.a. Conduct a test for difference in means between the US and Eurozone, assuming independent sampling, and setting the probability of Type I error equal to 2.5%.
The hypothesis test that corresponds to this text is:

$$H_0 : dy_{US} = dy_{Eu}$$
$$H_A : dy_{US} \neq dy_{Eu}$$

The general formula for this test, assuming independent random sampling, is given by:

$$t = \frac{(\overline{X}_1 - \overline{X}_s) - D_0}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where} \quad s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

or in this case where 1 refers to the US and 2 refers to Eu, and D_0 = 0,

$$t = \frac{(\overline{X}_{US} - \overline{X}_{Eu}) - 0}{\sqrt{s_p^2\left(\frac{1}{n_{US}} + \frac{1}{n_{Eu}}\right)}} = \frac{(.0270 - .0126)}{\sqrt{(0.001306)\left(\frac{1}{24} + \frac{1}{24}\right)}} = \frac{0.0144}{0.01043} = 1.38$$

The critical value for $t_{\alpha/2} = t_{.0125}$ for 46 (=24+24-2) degrees of freedom cannot be read off the t-table directly, for two reasons. First, there are no 46 df entries, just 40 and 60 df entries.

Use 40 df; then there is no entry for $t_{.0125}$ (remember this is a two-tailed test). One way to proceed is to realize that even using a lower probability of Type I, error, say 0.025, means the critical t value is 2.021, and 1.38 < 2.021, one fails to reject the null hypothesis in favor of the alternative.

The other approach is to be more careful, and to linearly interpolate between the two values; .0025 (=0.0125-0.01) is 1/6 of the distance between .00250 and 0.010, so one could say that 2.021 + (5/6)× (2.423-2.021) = 2.356 is the critical t value.

X.1.b. Conduct a test for difference in means between the US and Eurozone assuming paired sampling, and once again setting the probability of Type I error equal to 2.5%.

The hypothesis test is now:

$H_0: d\hat{y} = 0$ where $d\hat{y} = dy_{US} - dy_{Eu}$
$H_A: d\hat{y} \neq 0$

$$t = \frac{\bar{x}_D - 0}{s_D / \sqrt{n}} = \frac{0.0144 - 0}{0.0326 / \sqrt{23}} \approx 2.12$$

Now it becomes obvious that being careful is important. Using the $t_{.025}$ critical value for 23 df (=24-1), one would reject the null. However, linearly interpolating, one finds that the $t_{.0125}$ critical value is approximately 2.069+(5/6)×(2.500-2.069) = 2.428.

Notice, if one performed the **one** tail test corresponding to the hypothesis test:
$H_0: d\hat{y} = 0$
$H_A: d\hat{y} > 0$
then one could use the entry in the t-tables, where the critical value is 2.069 < 2.428, and one would reject the null hypothesis in favor of the alternative.

X.1.c. Which testing procedure is more appropriate? Why?

The paired differences test is more appropriate because the data are sampled in a systematically related manner.

- Problem X.2
  Consider the following data pertaining to annualized quarter-on-quarter growth rates of real US GDP (in 1996 chained dollars) over the last 25 years.

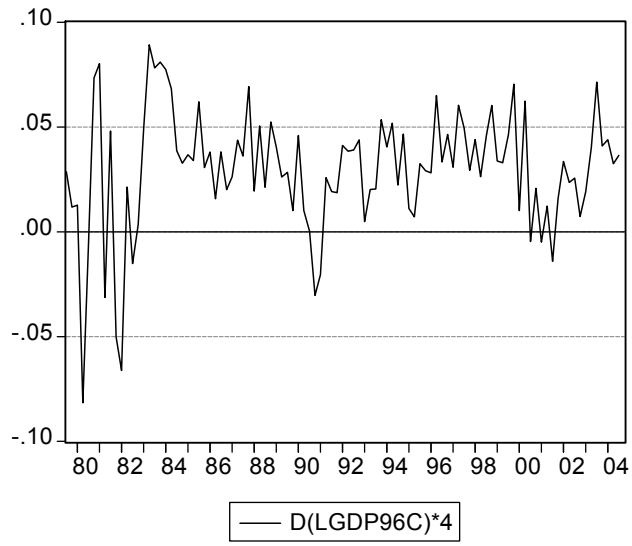


Figure 5: US qoq annualized GDP growth rate, 1979q1-2004q3

X.2.a. Suppose one wanted to test the hypothesis that the variability of US GDP growth was greater than 2.5%. Write out the hypothesis test, given the following information.

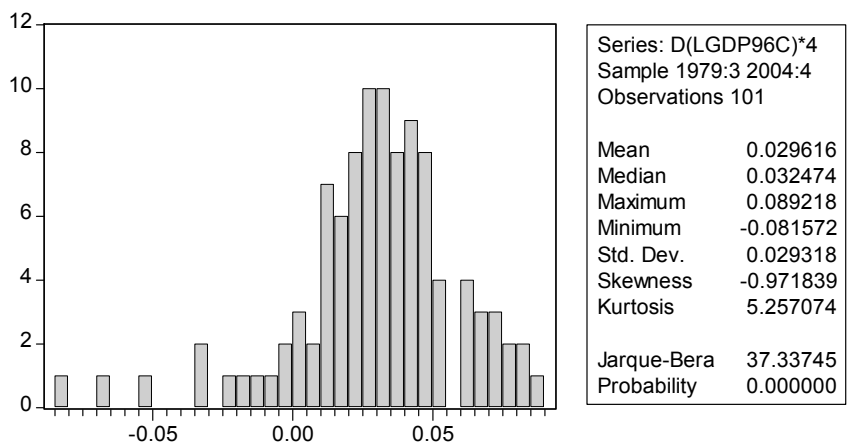


Figure 6: Histogram for United States qoq annualized GDP growth, 1979q3-2004q3.

$$H_0 : \sigma^2 = (0.025)^2$$
$$H_0 : \sigma^2 > (0.025)^2$$

X.2.b. Conduct the hypothesis test, using the data provided. What is your conclusion, setting α = 0.05?

We calculate the χ^2 statistic:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = 137.36$$

The critical value for 100 df (=101-1) is 124.32; hence, we reject the null hypothesis that the standard deviation of the annualized growth rate is 2.5% in favor of the alternative that it is greater than 2.5%.

X.2.c. What assumptions do you need to impose in order to arrive at your conclusion?

It is necessary to assume that the growth rate is normally distributed.

- X.3. There is a widespread view that the US macroeconomy has been more "stable" since 1984.

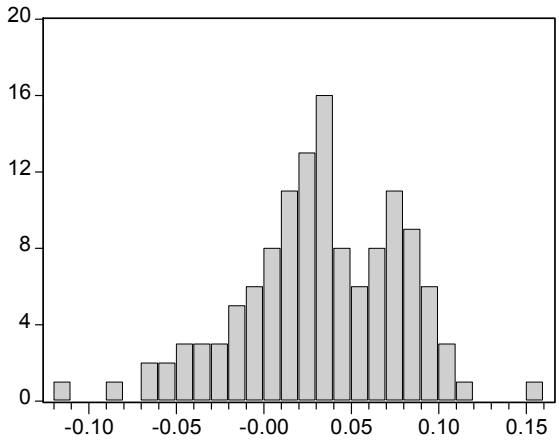
X.3.a. Write out a hypothesis test that the economy has been more stable in the post-1984 era.

Let sample 1 correspond to the pre-1984 era, and sample 2 correspond to the post-1984 era. Then

$H_0 : \sigma_1^2 = \sigma_s^2$

$H_0 : \sigma_1^2 > \sigma_2^2$

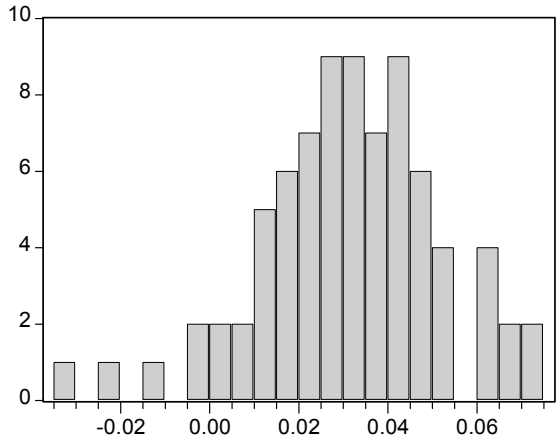
X.3.b. Conduct the appropriate test, using the statistics provided below, and setting Type I error equal to 1%.



Series: D(LGDP96C)*4
Sample 1953:1 1984:4
Observations 127

Mean	0.032815
Median	0.031627
Maximum	0.154627
Minimum	-0.110207
Std. Dev.	0.045075
Skewness	-0.371950
Kurtosis	3.224857
Jarque-Bera	3.195890
Probability	0.202312

Figure 7: Histogram for United States qoq annualized GDP growth, 1953q1-1984q4



Series: D(LGDP96C)*4
Sample 1985:1 2004:4
Observations 79

Mean	0.030981
Median	0.032474
Maximum	0.071446
Minimum	-0.030350
Std. Dev.	0.020221
Skewness	-0.426968
Kurtosis	3.492274
Jarque-Bera	3.197991
Probability	0.202099

Figure 8: Histogram for United States qoq annualized GDP growth, 1985q1-2004q3

Calculate the F-statistic:

$$F = \frac{s_1^2}{s_2^2} = \frac{(0.0451)^2}{(0.0202)^2} \approx 4.985$$

The critical F-statistic for α=0.01 and 126 (=127-1) numerator degrees of freedom and 78 (=79-1) denominator degrees of freedom is between approximately 1.53 (120 denominator df) and 1.73 (60 denominator df). Using linear interpolation, one finds that the critical value is approximately 1.67. Since the F-statistic is far in excess of this value, we can reject the null hypothesis that the variances are the same in the two samples in favor of the alternative that the variance in the early subsample is greater than that in the later.

27.11.2004