Economics 310
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University of Wisconsin-Madison

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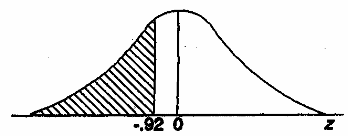
Problem Set 3 Answers

5.50 x is a binomial random variable with $n = 100$ and $p = .4$.

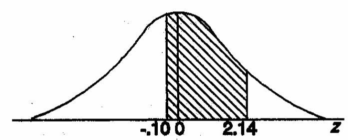
$$\mu \pm 3\sigma \Rightarrow np \pm 3\sqrt{npq} \Rightarrow 100(.4) \pm 3\sqrt{100(.4)(1 - .4)}$$
$$\Rightarrow 40 \pm 3(4.8990) \Rightarrow (25.303, 54.697)$$

Since the interval lies in the range 0 to 100, we can use the normal approximation to approximate the probabilities.

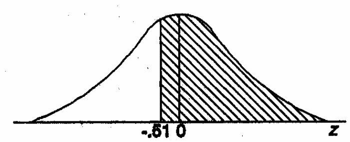
a. $P(x \leq 35) \approx P\left[z \leq \frac{(35 + .5) - 40}{4.899}\right]$
$= P(z \leq -.92)$
$= .5000 - .3212 = .1788$
(Using Table IV in Appendix B.)



b. $P(40 \leq x \leq 50)$
$\approx P\left[\frac{(40 - .5) - 40}{4.899} \leq z \leq \frac{(50 + .5) - 40}{4.899}\right]$
$= P(-.10 \leq z \leq 2.14)$
$= P(-.10 \leq z \leq 0) + P(0 \leq z \leq 2.14)$
$= .0398 + .4838 = .5236$
(Using Table IV in Appendix B.)



c. $P(x \geq 38) \approx P\left[z \geq \frac{(38 - .5) - 40}{4.899}\right]$
$= P(z \geq -.51)$
$= .5000 + .1950 = .6950$
(Using Table IV in Appendix B.)



5.70 Let x = trip time for the taxi service in minutes. The random variable x has an exponential distribution with $\lambda = .05$.

a. $\mu = \frac{1}{\lambda} = \frac{1}{.05} = 20$ minutes

b. $P(x > 30) = e^{-.05(30)} = e^{-1.5} = .22313$ (Table V, Appendix B)

c. P(Both taxis will be gone for more than 30 minutes)
$= P$(First taxi gone more than 30 minute $\cap$ Second taxi gone more than 30 minutes)
$= P(x > 30)P(x > 30)$ (by independence)
$= .22313^2$
$= .049787$

P(At least one taxi will return within 30 minutes)
$= 1 - P$(Both taxis will be gone for more than 30 minutes
$= 1 - .049787$ (Refer to previous part.)
$= .950213$

6.34 a. First we must compute μ and σ. The probability distribution for x is:

x	$p(x)$
1	.3
2	.2
3	.2
4	.3

$$\mu = E(x) = \sum xp(x) = 1(.3) + 2(.2) + 3(.2) + 4(.3) = 2.5$$

$$\begin{aligned}\sigma^2 = E\sum(x-\mu)^2 &= \sum(x-\mu)^2 p(x) \\ &= (1-2.5)^2(.3) + (2-2.5)^2(.2) + (3-2.5)^2(.2) + (4-2.5^2)(.3) \\ &= 1.45\end{aligned}$$

$$\mu_{\bar{x}} = \mu = 2.5,\ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{1.45}}{\sqrt{40}} = .1904$$

b. By the Central Limit Theorem, the distribution of $\bar{x}$ is approximately normal. The sample size, $n = 40$, is sufficiently large. Our answer does depend on n. If n is not sufficiently large, the Central Limit Theorem would not apply.

6.10 a. $\mu = \sum xp(x) = 0\left(\frac{1}{3}\right) + 1\left(\frac{1}{3}\right) + 2\left(\frac{1}{3}\right) = 1$

b.

Sample	$\bar{x}$	Probability	Sample	$\bar{x}$	Probability
0, 0, 0	0	1/27	1, 1, 2	4/3	1/27
0, 0, 1	1/3	1/27	1, 2, 0	1	1/27
0, 0, 2	2/3	1/27	1, 2, 1	4/3	1/27
0, 1, 0	1/3	1/27	1, 2, 2	5/3	1/27
0, 1, 1	2/3	1/27	2, 0, 0	2/3	1/27
0, 1, 2	1	1/27	2, 0, 1	1	1/27
0, 2, 0	2/3	1/27	2, 0, 2	4/3	1/27
0, 2, 1	1	1/27	2, 1, 0	1	1/27
0, 2, 2	4/3	1/27	2, 1, 1	4/3	1/27
1, 0, 0	1/3	1/27	2, 1, 2	5/3	1/27
1, 0, 1	2/3	1/27	2, 2, 0	4/3	1/27
1, 0, 2	1	1/27	2, 2, 1	5/3	1/27
1, 1, 0	2/3	1/27	2, 2, 2	2	1/27
1, 1, 1	1	1/27			

From the above table, the sampling distribution of the sample mean would be:

$\bar{x}$	Probability
0	1/27
1/3	3/27
2/3	6/27
1	7/27
4/3	6/27
5/3	3/27
2	1/27

c.

Sample	m	Probability	Sample	m	Probability
0, 0, 0	0	1/27	1, 1, 2	1	1/27
0, 0, 1	0	1/27	1, 2, 0	1	1/27
0, 0, 2	0	1/27	1, 2, 1	1	1/27
0, 1, 0	0	1/27	1, 2, 2	2	1/27
0, 1, 1	1	1/27	2, 0, 0	0	1/27
0, 1, 2	1	1/27	2, 0, 1	1	1/27
0, 2, 0	0	1/27	2, 0, 2	2	1/27
0, 2, 1	1	1/27	2, 1, 0	1	1/27
0, 2, 2	2	1/27	2, 1, 1	1	1/27
1, 0, 0	0	1/27	2, 1, 2	2	1/27
1, 0, 1	1	1/27	2, 2, 0	2	1/27
1, 0, 2	1	1/27	2, 2, 1	2	1/27
1, 1, 0	1	1/27	2, 2, 2	2	1/27
1, 1, 1	1	1/27			

From the above table, the sampling distribution of the sample median would be:

m	Probability
0	7/27
1	13/27
2	7/27

d. $$E(\bar{x}) = \sum \bar{x}p(\bar{x}) = 0\left[\frac{1}{27}\right] + \frac{1}{3}\left[\frac{3}{27}\right] + \frac{2}{3}\left[\frac{6}{27}\right] + 1\left[\frac{7}{27}\right] + \frac{4}{3}\left[\frac{6}{27}\right] + \frac{5}{3}\left[\frac{3}{27}\right] + 2\left[\frac{1}{27}\right]$$
$$= 1$$

Since $E(\bar{x}) = \mu$, $\bar{x}$ is an unbiased estimator for μ.

$$E(m) = \sum mp(m) = 0\left[\frac{7}{27}\right] + 1\left[\frac{13}{27}\right] + 2\left[\frac{7}{27}\right] = 1$$

Since $E(m) = \mu$, m is an unbiased estimator for μ.

e. $\sigma_{\bar{x}}^2 = \sum(\bar{x} - \mu)^2 p(\bar{x})$

$$= (0-1)^2\left[\frac{1}{27}\right] + \left[\frac{1}{3} - 1\right]^2\left[\frac{3}{27}\right] + \left[\frac{2}{3} - 1\right]^2\left[\frac{6}{27}\right] + (1-1)^2\left[\frac{7}{27}\right]$$

$$+ \left[\frac{4}{3} - 1\right]^2\left[\frac{6}{27}\right] + \left[\frac{5}{3} - 1\right]^2\left[\frac{3}{27}\right] + (2-1)^2\left[\frac{1}{27}\right] = \frac{2}{9} = .2222$$

$$\sigma_m^2 = \sum(m-1)^2 p(m) = (0-1)^2\left[\frac{7}{27}\right] + (1-1)^2\left[\frac{13}{27}\right] + (2-1)^2\left[\frac{7}{27}\right] = \frac{14}{27}$$
$$= .5185$$

f. Since both the sample mean and median are unbiased and the variance is smaller for the sample mean, it would be the preferred estimator of μ.

6.22 For this population and sample size,

$$E(\bar{x}) = \mu = 100, \quad \sigma_{\bar{x}} = \sigma/\sqrt{n} = 10/\sqrt{900} = 1/3$$

a. Approximately 95% of the time, $\bar{x}$ will be within two standard deviations of the mean, i.e., $\mu \pm 2\sigma \Rightarrow 100 \pm 2\left[\frac{1}{3}\right] \Rightarrow 100 \pm \frac{2}{3} \Rightarrow (99.33, 100.67)$. Almost all of the time, the sample mean will be within three standard deviations of the mean, i.e., $\mu \pm 3\sigma \Rightarrow 100 \pm 3\left[\frac{1}{3}\right] \Rightarrow$ $100 \pm 1 \Rightarrow (99, 101)$.

b. No more than three standard deviations, i.e., $3\left[\frac{1}{3}\right] = 1$

c. No, the previous answer only depended on the standard deviation of the sampling distribution of the sample mean, not the mean itself.

6.26 a. $\mu_{\bar{x}} = \mu = 3.5 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{.5}{\sqrt{100}} = .05$

b. $P(3.40 < \bar{x} < 3.60) = P\left[\frac{3.40 - 3.5}{.05} < z < \frac{3.60 - 3.5}{.05}\right]$

$= P(-2 < z < 2) = .4772 + .4772 = .9544$
(using Table IV, Appendix B)

c. $P(\bar{x} > 3.62) = P\left[z > \frac{3.62 - 3.5}{.05}\right] = P(z > 2.40) = .5 - .4918 = .0082$
(using Table IV, Appendix B)

d. $\mu_{\bar{x}} = \mu = 3.5 \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{.5}{\sqrt{200}} = .03536$

The mean of the sampling distribution of $\bar{x}$ would stay the same, but the standard deviation would decrease.

$P(3.40 < \bar{x} < 3.60) = P\left[\frac{3.40 - 3.5}{.03536} < z < \frac{3.60 - 3.5}{.03536}\right]$

$= P(-2.83 < z < 2.83) = .4977 + .4977 = .9954$
(using Table IV, Appendix B)

This probability is larger than when the sample size was 100.

$P(\bar{x} > 3.62) = P\left[z > \frac{3.62 - 3.5}{.03536}\right] = P(z > 3.39) \approx .5 - .5 = 0$
(using Table IV, Appendix B)

This probability is smaller than when the sample size was 100.

7.4 a. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix B, $z_{.025} = 1.96$. The confidence interval is:

$$\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}} \Rightarrow 25.9 \pm 1.96\frac{2.7}{\sqrt{90}} \Rightarrow 25.9 \pm .56 \Rightarrow (25.34, 26.46)$$

b. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix B, $z_{.05} = 1.645$. The confidence interval is:

$$\bar{x} \pm z_{.05}\frac{s}{\sqrt{n}} \Rightarrow 25.9 \pm 1.645\frac{2.7}{\sqrt{90}} \Rightarrow 25.9 \pm .47 \Rightarrow (25.43, 26.37)$$

c. For confidence coefficient .99, $\alpha = .01$ and $\alpha/2 = .01/2 = .005$. From Table IV, Appendix B, $z_{.005} = 2.58$. The confidence interval is:

$$\bar{x} \pm z_{.005}\frac{s}{\sqrt{n}} \Rightarrow 25.9 \pm 2.58\frac{2.7}{\sqrt{90}} \Rightarrow 25.9 \pm .73 \Rightarrow (25.17, 26.63)$$

7.26 a. From the printout, the 95% confidence interval is (−4238%, 18,834%). We are 95% confident that the true mean 5-year revenue growth rate for the 1999 Technology Fast 500 is between −4,238% and 18,834%.

b. The population being sampled from must be normally distributed.

c. From the stem-and-leaf display in the printout, the data appear to be skewed to the right. The data do not appear to have come from a normal distribution. Therefore, the 95% confidence interval in part a may not be valid.

7.88 a. First we must compute $\hat{p}$: $\hat{p} = \dfrac{x}{n} = \dfrac{11,239}{43,732} = .257$

To see if the sample size is sufficiently large:

$$\hat{p} \pm 3\sigma_{\hat{p}} \approx \hat{p} \pm 3\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .257 \pm 3\sqrt{\frac{.257(.743)}{43,732}} \Rightarrow .257 \pm .006 \Rightarrow (.251, .263)$$

Since this interval is wholly contained in the interval (0, 1), we may conclude that the normal approximation is reasonable.

For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix B, $z_{.05} = 1.645$. The confidence interval is:

$$\hat{p} \pm z_{.05}\sqrt{\frac{pq}{n}} \approx \hat{p} \pm 1.645\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .257 \pm 1.645\sqrt{\frac{.257(.743)}{43{,}732}} \Rightarrow .257 \pm .003$$
$$\Rightarrow (.254, .260)$$

We are 90% confident that the true percentage of U.S. adults who currently smoke is between 25.4% and 26.0%.

b. First we must compute $\hat{p}$: $\hat{p} = \frac{x}{n} = \frac{10{,}539}{43{,}732} = .241$

To see if the sample size is sufficiently large:

$$\hat{p} \pm 3\sigma_{\hat{p}} \approx \hat{p} \pm 3\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .241 \pm 3\sqrt{\frac{.241(.759)}{43{,}732}} \Rightarrow .241 \pm .006 \Rightarrow (.235, .247)$$

Since this interval is wholly contained in the interval (0, 1), we may conclude that the normal approximation is reasonable.

For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix B, $z_{.05} = 1.645$. The confidence interval is:

$$\hat{p} \pm z_{.05}\sqrt{\frac{pq}{n}} \approx \hat{p} \pm 1.645\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .241 \pm 1.645\sqrt{\frac{.241(.759)}{43{,}732}} \Rightarrow .241 \pm .003$$
$$\Rightarrow (.238, .244)$$

We are 90% confident that the true percentage of U.S. adults who are former smokers is between 23.8% and 24.4%.

7.58 For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .05$. From Table IV, Appendix B, $z_{.05} = 1.645$.

For a width of .06, $B = .06/2 = .03$

The sample size is $n = \frac{(z_{\alpha/2})^2 pq}{B^2} = \frac{(1.645)^2(.17)(.83)}{.03^2} = 424.2 \approx 425$

You would need to take $n = 425$ samples.

Y.1 The value $P[t > t_a] = a$ is the tabled entry for a particular number of degrees of freedom.

a $t_{.10} = 1.356$ with 12 d.f. **b** $t_{.01} = 2.485$ with 25 d.f. **c** $t_{.05} = 1.746$ with 16 d.f.

Y.2 Calculate $\hat{p} = \frac{x}{n} = \frac{140}{500} = .280$. Then an approximate 95% confidence interval for p is

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .280 \pm 1.96\sqrt{\frac{.28(.72)}{500}} \Rightarrow .280 \pm .039$$

or $.241 < p < .319$.

8.8 Let μ = average Libor rate for 3-month loans. Since many Western banks think that the reported average Libor rate (.39) is too high, they want to show that the average is less than .39. The appropriate hypotheses would be:

H_0: $\mu = .39$
H_a: $\mu < .39$

8.18 a. H_0: $\mu = .36$
H_a: $\mu < .36$

The test statistic is $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{.323 - .36}{\sqrt{.034}/\sqrt{64}} = -1.61$

The rejection region requires $\alpha = .10$ in the lower tail of the z-distribution. From Table IV, Appendix B, $z_{.10} = 1.28$. The rejection region is $z < -1.28$.

Since the observed value of the test statistic falls in the rejection region ($z = -1.61 < -1.28$), H_0 is rejected. There is sufficient evidence to indicate the mean is less than .36 at $\alpha = .10$.

b. H_0: $\mu = .36$
H_a: $\mu \neq .36$

The test statistic is $z = -1.61$ (see part a).

The rejection region requires $\alpha/2 = .10/2 = .05$ in the each tail of the z-distribution. From Table IV, Appendix B, $z_{.05} = 1.645$. The rejection region is $z < -1.645$ or $z > 1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.61 \nless -1.645$), H_0 is not rejected. There is insufficient evidence to indicate the mean is different from .36 at $\alpha = .10$.

Y.3 **a)** The 99% confidence interval is

$$\begin{aligned}\overline{x} \pm z_{\alpha/2}\tfrac{\sigma}{\sqrt{n}} &= .005254 \pm 2.575 \times \tfrac{.007765}{\sqrt{136}} \\ &= .005254 \pm .0017145 \\ &= (.006969, .003540)\end{aligned}$$

b) The t-statistic is

$t = \frac{\overline{x}-\mu}{s/\sqrt{n}} = \frac{.007244-.004358}{(.006453/\sqrt{36})} = \frac{.002706}{.0010755} = 2.516$

For 35 d.f. we have

$t_{.05/2} = t_{.025} = \frac{2.042+2.021}{2} = 2.0315$

$2.516 > 2.0315 \Rightarrow$we reject the hypothesis.

Assumptions are:

i) Underlying variable is normally distributed.

ii) 0.4538 is thought of fixed, not random variable.

c) No, since everything would be multiplied by 4.

The new t-statistic t' is

$t' = \frac{.028976-.018152}{.025812/\sqrt{36}} = 2.516$

which is equal to the t-statistic t found in part b)