Problem Set 2
Solutions

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3.4 a. $\binom{9}{4} = \frac{9!}{4!(9-4)!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 126$

b. $\binom{7}{2} = \frac{7!}{2!(7-2)!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 21$

c. $\binom{4}{4} = \frac{4!}{4!(4-4)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 1} = 1$

d. $\binom{5}{0} = \frac{5!}{0!(5-0)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 1$

e. $\binom{6}{5} = \frac{6!}{5!(6-5)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1} = 6$

3.10 a. There are a total of $9 \times 2 = 18$ sample points for this experiment. There are 9 sources of CO poisoning, and each source of poisoning has 2 possible outcomes, fatal or nonfatal. Suppose we introduce some notation to make it easier to write down the sample points. Let *FI* = Fire, *AU* = Auto exhaust, *FU* = Furnace, *K* = Kerosene or spaceheater, *AP* = Appliance, *OG* = Other gas-powered motors, *FP* = Fireplace, *O* = Other, and *U* = Unknown. Also, let *F* = Fatal and *N* = Nonfatal. The 18 sample points are:

FI, F	*AU, F*	*FU, F*	*K, F*	*AP, F*	*OG, F*	*FP, F*	*O, F*	*U, F*
FI, N	*AU, N*	*FU, N*	*K, N*	*AP, N*	*OG, N*	*FP, N*	*O, N*	*U, N*

b. The set of all sample points is called the sample space.

c. The event *A* is made up of the following sample points: *FI, F* and *FI, N*

Then, $P(A) = P(FI, F) + P(FI, N) = 63/981 + 53/981 = 116/981 = .118$

d. The event *B* is made up of the following sample points:
(*FI, F*); (*AU, F*); (*FU, F*); (*K, F*); (*AP, F*); (*OG, F*); (*FP, F*); (*O, F*); (*U, F*)

Then, $P(B) = P(FI, F) + P(AU, F) + P(FU, F) + P(K, F) + P(AP, F)$
$+ P(OG, F) + P(FP, F) + P(O, F) + P(U, F)$
$= 63/981 + 60/981 + 18/891 + 9/981 + 9/981 + 3/981 + 0/981 + 3/981$
$+ 9/981$
$= 174/981 = .177$

e. The event C is made up of the following sample points: (AU, F) and (AU, N)

Then, $P(C) = P(AU, F) + P(AU, N) = 60/981 + 178/981 = 238/981 = .243$

f. The event D is made up of the following sample point: AU, F

Then, $P(D) = P(AU, F) = 60/981 = .061$

g. The event E is made up of the following sample point: FI, N

Then, $P(E) = P(FI, N) = 53/981 = .054$

3.14 We will denote the five successful utility companies as S_1, S_2, S_3, S_4, and S_5 and the two failing companies as F_1 and F_2. There are

$$\binom{7}{3} = \frac{7!}{3!4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 35$$

possible ways to choose three companies from the seven, as shown below:

(S_1, S_2, S_3)	(S_1, S_3, S_4)	(S_1, S_4, S_5)	(S_1, S_5, F_1)
(S_1, S_2, S_4)	(S_1, S_3, S_5)	(S_1, S_4, F_1)	(S_1, S_5, F_2)
(S_1, S_2, S_5)	(S_1, S_3, F_1)	(S_1, S_4, F_2)	
(S_1, S_2, F_1)	(S_1, S_3, F_2)		(S_1, F_1, F_2)
(S_1, S_2, F_2)			
(S_2, S_3, S_4)	(S_2, S_4, S_5)	(S_2, S_5, F_1)	(S_2, F_1, F_2)
(S_2, S_3, S_5)	(S_2, S_4, F_1)	(S_2, S_5, F_2)	
(S_2, S_3, F_1)	(S_2, S_4, F_2)		
(S_2, S_3, F_2)			
(S_3, S_4, S_5)	(S_3, S_5, F_1)	(S_3, F_1, F_2)	
(S_3, S_4, F_1)	(S_3, S_5, F_2)		
(S_3, S_4, F_2)			
(S_4, S_5, F_1)	(S_5, F_1, F_2)		
(S_4, S_5, F_2)			
(S_4, F_1, F_2)			

a. Each outcome is equally likely, so each sample point has probability 1/35. From the 35 events listed, 10 do not contain F_1 or F_2. Therefore, P(selecting none) = 10/35.

b. From the 35 events listed, 20 contain either F_1 or F_2, but not both. Therefore, P(selecting one) = 20/35.

c. From the 35 events listed, 5 contain both F_1 and F_2. Therefore, P(selecting both) = 5/35.

3.22 a. The outcome "On" and "High" is $A \cap D$.

b. The outcome "Low" or "Medium" is D^c.

3.30 a. $P \cap S \cap A$

Products 6 and 7 are contained in this intersection.

b. $P(\text{possess all the desired characteristics}) = P(P \cap S \cap A)$

$$= P(6) + P(7) = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$$

c. $A \cup S$

$$P(A \cup S) = P(2) + P(3) + P(5) + P(6) + P(7) + P(8) + P(9) + P(10)$$

$$= \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{8}{10} = \frac{4}{5}$$

d. $P \cap S$

$$P(P \cap S) = P(2) + P(6) + P(7) = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10}$$

3.34 If we flip 3 coins, the possible outcomes are:

(H,H,H), (H,H,T), (H,T,H), (T,H,H), (H,T,T), (T,H,T), (T,T,H), (T,T,T),

A: {Observe at least one head}
B: {Observe exactly two heads}
C: {Observe exactly two tails}
D: {Observe at most one head}

a. Then,

$$P(A) = 1/8 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8 = 7/8$$
$$P(B) = 1/8 + 1/8 + 1/8 = 3/8$$
$$P(C) = 1/8 + 1/8 + 1/8 = 3/8$$
$$P(D) = 1/8 + 1/8 + 1/8 + 1/8 = 4/8 = 1/2$$
$$P(A \cap B) = P(B) = 3/8$$
$$P(A \cap D) = 1/8 + 1/8 + 1/8 = 3/8$$
$$P(B \cap C) = 0$$
$$P(B \cap D) = 0$$

b.

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{3/8}{7/8} = 3/7$$
$$P(A \mid D) = \frac{P(A \cap D)}{P(D)} = \frac{3/8}{4/8} = 3/4$$
$$P(C \mid B) = \frac{P(B \cap C)}{P(B)} = \frac{0}{3/8} = 0$$

c. For pair A and B: A and B are not independent because $P(B \mid A) \neq P(B)$ or $3/7 \neq 3/8$.

For pair A and C:

$$P(A \cap C) = P(C) = 3/8$$
$$P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{3/8}{3/8} = 1$$

A and C are not independent because $P(A \mid C) \neq P(A)$ or $1 \neq 7/8$.

For pair A and D: A and D are not independent because $P(A \mid D) \neq P(A)$ or $3/4 \neq 7/8$.

For pair B and C: B and C are not independent because $P(C \mid B) \neq P(C)$ or $0 \neq 3/8$.

For pair B and D:

$$P(B \cap D) = 0$$
$$P(B \mid D) = \frac{P(B \cap D)}{P(D)} = \frac{0}{4/8} = 0$$

B and D are not independent because $P(B \mid D) \neq P(B)$ or $0 \neq 3/8$.

For pair C and D:

$$P(C \cap D) = P(C) = 3/8$$
$$P(C \mid D) = \frac{P(C \cap D)}{P(D)} = \frac{3/8}{4/8} = 3/4$$

C and D are not independent because $P(C \mid D) \neq P(C)$ or $3/4 \neq 3/8$.

3.44 a. $P(B) = \dfrac{5{,}021}{833{,}303} = .0060$

b. $P(A \cap B) = \dfrac{1{,}808}{833{,}303} = .0022$

c. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{341{,}180}{833{,}303} + \frac{5{,}021}{833{,}303} - \frac{1{,}808}{833{,}303} = \frac{344{,}393}{833{,}303} = .4133$$

d. $P(A \mid B) = \dfrac{(A \cap B)}{P(B)} = \dfrac{1{,}808/833{,}303}{5{,}021/833{,}303} = \dfrac{1{,}808}{5{,}021} = .3601$

e. No. If A and B are independent, then $P(A \mid B) = P(A)$. Here, $P(A \mid B) \neq P(A)$ or .3601 $\neq$ .4094. Thus, A and B are not independent.

3.48 Define the following events:

A: {tax filer is audited}
B: {income under \$25,000}
C: {income from \$25,000–\$49,999}
D: {income from \$50,000–\$99,999}
E: {income is \$100,000 or more}

We know the following information:

$P(B) = 13.2/62 = .2129$
$P(C) = 27.3/62 = .4403$
$P(D) = 17.0/62 = .2742$
$P(E) = 4.5/62 = .0726$

$P(A \mid B) = .0117$
$P(A \mid C) = .0095$
$P(A \mid D) = .0116$
$P(A \mid E) = .0285$

a.
$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap C) + P(A \cap D) + P(A \cap E) \\ &= P(B)P(A \mid B) + P(C)P(A \mid C) + P(D)P(A \mid D) + P(E)P(A \mid E) \\ &= .2129(.0117) + .4403(.0095) + .2742(.0116) + .0726(.0285) \\ &= .00249093 + .00418285 + .00318072 + .0020691 = .0119 \end{aligned}$$

b. $P(C \cap A) = P(C)P(A \mid C) = .4403(.0095) = .0042$

$$\begin{aligned} P((D \cup E) \cup A^c) &= P(D \cup E) + P(A^c) - P((D \cup E) \cap A^c) \\ &= P(D) + P(E) - P(D \cap E) + (1 - P(A)) - P(A^c \mid D)P(D) \\ &\quad - P(A^c \mid E)P(E) \\ &= .2742 + .0726 - 0 + (1 - .0119) - (1 - .0116)(.2742) \\ &\quad - (1 - .0285)(.0726) = .9933 \end{aligned}$$

3.82 Define the following events:

C: {Committee judges joint acceptable}
I: {Inspector judges joint acceptable}

The sample points of this experiment are:

$C \cap I$
$C \cap I^c$
$C^c \cap I$
$C^c \cap I^c$

a. The probability the inspector judges the joint to be acceptable is:

$$P(I) = P(C \cap I) + P(C^c \cap I) = \frac{101}{153} + \frac{23}{153} = \frac{124}{153} \approx .810$$

The probability the committee judges the joint to be acceptable is:

$$P(C) = P(C \cap I) + P(C \cap I^c) = \frac{101}{153} + \frac{10}{153} = \frac{111}{153} \approx .725$$

b. The probability that both the committee and the inspector judge the joint to be acceptable is:

$$P(C \cap I) = \frac{101}{153} \approx .660$$

The probability that neither judge the joint to be acceptable is:

$$P(C^c \cap I^c) = \frac{19}{153} \approx .124$$

c. The probability the inspector and committee disagree is:

$$P(C \cap I^c) + P(C^c \cap I) = \frac{10}{153} + \frac{23}{153} = \frac{33}{153} \approx .216$$

The probability the inspector and committee agree is:

$$P(C \cap I) + P(C^c \cap I^c) = \frac{101}{153} + \frac{19}{153} = \frac{120}{153} \approx .784$$

4.14 a. $P(x \leq 3) = p(1) + p(3) = .1 + .2 = .3$

b. $P(x < 3) = p(1) = .1$

c. $P(x = 7) = .2$

d. $P(x \geq 5) = p(5) + p(7) + p(9) = .4 + .2 + .1 = .7$

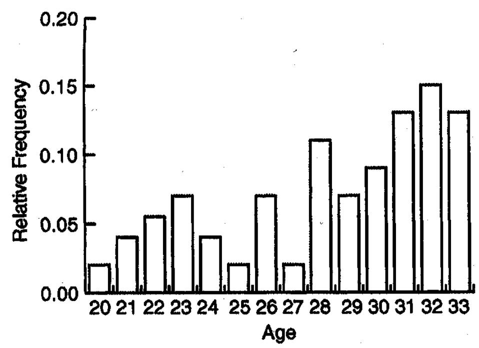
e. $P(x > 2) = p(3) + p(5) + p(7) + p(9) = .2 + .4 + .2 + .1 = .9$

f. $P(3 \leq x \leq 9) = p(3) + p(5) + p(7) + p(9) = .2 + .4 + .2 + .1 = .9$

4.16 a. Yes. Relative frequencies are observed values from a sample. Relative frequencies are commonly used to estimate unknown probabilities. In addition, relative frequencies have the same properties as the probabilities in a probability distribution, namely

1. all relative frequencies are greater than or equal to zero
2. the sum of all the relative frequencies is 1

b. The graph of the probability distribution is:



c. Let x = age of employee. $P(x > 30) = .1273 + .1455 + .1273 = .4000$.

$P(x > 40) = 0$.

$P(x < 30) = .0182 + .0364 + .0545 + .0727 + .0364 + .0182 + .0727 + .0182 + .1090 + .0727 = .5090$

d. $P(x = 25 \text{ or } 26) = P(x = 25) + P(x = 26) = .0182 + .0727 = .0909$

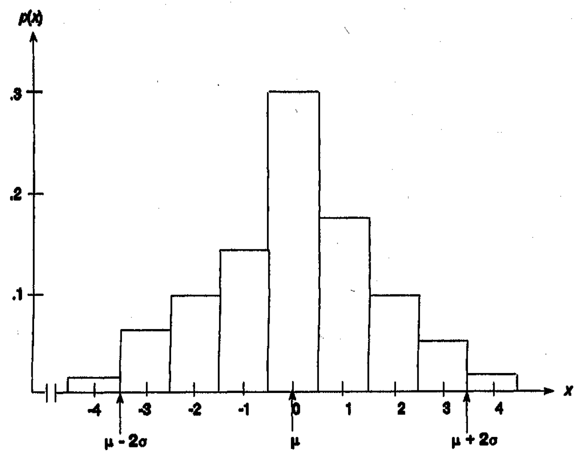
4.24 a.

$$\begin{aligned}\mu = E(x) = \sum xp(x) &= -4(.02) + (-3)(.07) + (-2)(.10) + (-1)(.15) + 0(.3) \\ &\quad + 1(.18) + 2(.10) + 3(.06) + 4(.02) \\ &= -.08 - .21 - .2 - .15 + 0 + .18 + .2 + .18 + .08 = 0\end{aligned}$$

$$\begin{aligned}\sigma^2 = E[(x - \mu)^2] = \sum (x - \mu)^2 p(x) &= (-4 - 0)^2(.02) + (-3 - 0)^2(.07) + (-2 - 0)^2(.10) \\ &\quad + (-1 - 0)^2(.15) + (0 - 0)^2(.30) + (1 - 0)^2(.18) \\ &\quad + (2 - 0)^2(.10) + (3 - 0)^2(.06) + (4 - 0)^2(.02) \\ &= .32 + .63 + .4 + .15 + 0 + .18 + .4 + .54 + .32 = 2.94\end{aligned}$$

$$\sigma = \sqrt{2.94} = 1.715$$

b.



$\mu \pm 2\sigma \Rightarrow 0 \pm 2(1.715) \Rightarrow 0 \pm 3.430 \Rightarrow (-3.430, 3.430)$

c. $P(-3.430 < x < 3.430) = p(-3) + p(-2) + p(-1) + p(0) + p(1) + p(2) + p(3)$
$= .07 + .10 + .15 + .30 + .18 + .10 + .06 = .96$

4.26 a. $E(x) = \sum_{\text{All } x} xp(x)$

Firm A: $E(x) = 0(.01) + 500(.01) + 1000(.01) + 1500(.02) + 2000(.35) + 2500(.30)$
$+ 3000(.25) + 3500(.02) + 4000(.01) + 4500(.01) + 5000(.01)$
$= 0 + 5 + 10 + 30 + 700 + 750 + 750 + 70 + 40 + 45 + 50$
$= 2450$

Firm B: $E(x) = 0(.00) + 200(.01) + 700(.02) + 1200(.02) + 1700(.15) + 2200(.30)$
$+ 2700(.30) + 3200(.15) + 3700(.02) + 4200(.02) + 4700(.01)$
$= 0 + 2 + 14 + 24 + 255 + 660 + 810 + 480 + 74 + 84 + 47$
$= 2450$

b. $\sigma = \sqrt{\sigma^2}$ $\quad \sigma^2 = \sum_{\text{All } x} (x - \mu)^2 p(x)$

Firm A: $\sigma^2 = (0 - 2450)^2(.01) + (500 - 2450)^2(.01) + \cdots + (5000 - 2450)^2(.01)$
$= 60{,}025 + 38{,}025 + 21{,}025 + 18{,}050 + 70{,}875 + 750 + 75{,}625$
$+ 22{,}050 + 24{,}025 + 42{,}025 + 65{,}025$
$= 437{,}500$
$\sigma = 661.44$

Firm B: $\sigma^2 = (0 - 2450)^2(.00) + (200 - 2450)^2(.01) + \cdots + (4700 - 2450)^2(.01)$
$= 0 + 50{,}625 + 61{,}250 + 31{,}250 + 84{,}375 + 18{,}750 + 84{,}375$
$+ 31{,}250 + 61{,}250 + 50{,}625$
$= 492{,}500$
$\sigma = 701.78$

Firm B faces greater risk of physical damage because it has a higher variance and standard deviation.

4.38 a. $p(0) = \binom{3}{0}(.3)^0(.7)^{3-0} = \frac{3!}{0!3!}(.3)^0(.7)^3 = \frac{3 \cdot 2 \cdot 1}{1 \cdot 3 \cdot 2 \cdot 1}(1)(.7)^3 = .343$

$p(1) = \binom{3}{1}(.3)^1(.7)^{3-1} = \frac{3!}{1!2!}(.3)^1(.7)^2 = .441$

$p(2) = \binom{3}{2}(.3)^2(.7)^{3-2} = \frac{5!}{2!1!}(.3)^2(.7)^1 = .189$

$p(3) = \binom{3}{3}(.3)^3(.7)^{3-3} = \frac{5!}{3!0!}(.3)^3(.7)^0 = .027$

b.

x	$p(x)$
0	.343
1	.441
2	.189
3	.027

4.40 a. $P(x = 2) = P(x \leq 2) - P(x \leq 1) = .167 - .046 = .121$ (from Table II, Appendix B)

b. $P(x \leq 5) = .034$

c. $P(x > 1) = 1 - P(x \leq 1) = 1 - .919 = .081$

d. $P(x < 10) = P(x \leq 9) = 0$

e. $P(x \geq 10) = 1 - P(x \leq 9) = 1 - .002 = .998$

f. $P(x = 2) = P(x \leq 2) - P(x \leq 1) = .206 - .069 = .137$

4.44 a. In order for x to be a binomial random variable, the n trials must be identical. We can assume that the process of selecting of a worker is identical from trial to trial. There are two possible outcomes - a worker missed work due to a back injury or not. The probability of success must be the same from trial to trial. We can assume that the probability of missing work due to a back injury is constant. The trials must be independent of each other. We can assume that the outcome of one trials will not affect the outcome of any other. Thus, x is a binomial random variable.

b. From the information given in the problem, the estimate of p is .40.

c. The mean is $\mu = E(x) = np = 10(.40) = 4$.

The standard deviation is $\sigma = \sqrt{np(1 - p)} = \sqrt{10(.40)(.60)} = \sqrt{2.4} = 1.549$

d. Using Table II, Appendix B, with $n = 10$ and $p = .40$,

$P(x = 1) = P(x \leq 1) - P(x \leq 0) = .046 - .006 = .040$

$P(x > 1) = 1 - P(x \leq 1) = 1 - .046 = .954$

4.54 a. For $\lambda = 1$, $P(x \leq 2) = .920$ (from Table III, Appendix B)

b. For $\lambda = 2$, $P(x \leq 2) = .677$

c. For $\lambda = 3$, $P(x \leq 2) = .423$

d. The probability decreases as λ increases. This is reasonable because λ is equal to the mean. As the mean increases, the probability that x is less than a particular value will decrease.

4.60 a. $P(x = 0) = \frac{\lambda^0 e^{-\lambda}}{0!} = \frac{2.63^0 e^{-2.63}}{0!} = .0721$

b. $E(x) = \lambda = 2.63$

$\sigma = \sqrt{\lambda} = \sqrt{2.63} = 1.622$

c. The z-score corresponding to $x = 10$ is $z = \frac{10 - 2.63}{1.622} = 4.544$

Since this z-score is fairly large and the distribution of x is fairly mound-shaped, it would be very unlikely to observe as many as 10 fatalities in any given month. Using the Empirical Rule, almost none of the observations are more than 3 standard deviations above the mean.

d. The experiment consists of counting the number of fatalities per month. We must assume that the probability of a fatality in a month is the same for any month. We must also assume that the number of fatalities in any month is independent of the number of fatalities in any other month.

4.64 a. For $\lambda = 1$, $P(x \geq 3) = 1 - P(x \leq 2) = 1 - .920 = .080$ from Table III, Appendix B.

b. Yes. The probability that the number of arrivals will exceed 2 is .080. Although this is not exceedingly small, only 8% of the minutes will have more than two arrivals.