

Economics 310  
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 University of Wisconsin-Madison

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 Social Sciences 7418

## Problem Set 2 Answers (corrected Z.3.b, 3/1)

3.68 Define the following events:

$A$ : {male worker}  
 $B$ : {female worker}  
 $C$ : {service worker}  
 $D$ : {managerial/professional worker}  
 $E$ : {operator/fabricator/laborer}  
 $F$ : {technical/sales/administrative worker}

- a.  $P(A \cap C) = .05$   
 b.  $P(D) = P(A \cap D) + P(B \cap D) = .16 + .16 = .32$   
 c.  $P[(B \cap D) \cup (B \cap E)] = .16 + .03 = .19$   
 d.  $P(F^c) = 1 - P(F) = 1 - [P(A \cap F) + P(B \cap F)] = 1 - (.10 + .18) = 1 - .28 = .72$

3.36 The 36 possible outcomes obtained when tossing two dice are listed below:

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)  
 (2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)  
 (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)  
 (4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)  
 (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)  
 (6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

$A$ : {(1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (3, 6), (4, 1), (4, 3), (4, 5), (5, 2), (5, 4), (5, 6), (6, 1), (6, 3), (6, 5)}

$B$ : {(3, 6), (4, 5), (5, 4), (5, 6), (6, 3), (6, 5), (6, 6)}

$A \cap B$ : {(3, 6), (4, 5), (5, 4), (5, 6), (6, 3), (6, 5)}

If  $A$  and  $B$  are independent, then  $P(A)P(B) = P(A \cap B)$ .

$$P(A) = \frac{18}{36} = \frac{1}{2} \quad P(B) = \frac{7}{36} \quad P(A \cap B) = \frac{6}{36} = \frac{1}{6}$$

$$P(A)P(B) = \frac{1}{2} \cdot \frac{7}{36} = \frac{7}{72} \neq \frac{1}{6} = P(A \cap B). \text{ Thus, } A \text{ and } B \text{ are not independent.}$$

3.40 a. The frequency table would be:

	Auto	Trad	Ener	Insu	Tele	Elec	Appl	FinS	Ret	Totals
Germany	2			1		1			1	5
Japan	1	6			1	1	2			11
Netherlands			1					1		2
France				1						1
United Kingdom			1							1
Totals	3	6	2	2	1	2	2	1	1	20

b. Define the following events:

$G$ : {Country is Germany}       $J$ : {Country is Japan}       $N$ : {Country is Netherlands}

$F$ : {Country is France}       $U$ : {Country is United Kingdom}

$AU$ : {Business is automobiles}

$T$ : {Business is trading}

$EN$ : {Business is energy}

$I$ : {Business is insurance}

$TE$ : {Business is telecommunications}

$EL$ : {Business is electronics}

$AP$ : {Business is appliances}

$FN$ : {Business is financial services}

$R$ : {Business is retailing}

$$P(J) = 11/20 = .55$$

c.  $P(AU) = 3/20 = .15$

d.  $P(I \cup EL) = (2 + 2)/20 = 4/20 = .20$

e.  $P(J \cap AU) = 1/20 = .05$

f.  $P(G \cup EL) = 6/20 = .30$

g.  $P(AU | J) = \frac{P(AU \cap J)}{P(J)} = \frac{1/20}{11/20} = 1/11 = .091$

h.  $P(J | AU) = \frac{P(AU \cap J)}{P(AU)} = \frac{1/20}{3/20} = 1/3 = .333$

i. If  $J$  and  $AU$  are independent events, then  $P(J | AU) = P(J)$

From part h,  $P(J | AU) = .333$  and from part b,  $P(J) = .55$

Since these probabilities are not equal,  $J$  and  $AU$  are not independent.

3.46 Define the following events:

$A$ : {Patient receives PMI sheet}

$B$ : {Patient was hospitalized}

$$P(A) = .20, P(A \cap B) = .12$$

$$P(B | A) = \frac{P(A \cap B)}{P(A)} = \frac{.12}{.20} = .60$$

3.78 a. We will define the following events:

$A$ : {The first activation device works properly; i.e., activates the sprinkler when it should}

$B$ : {The second activation device works properly}

From the statement of the problem, we know

$$P(A) = .91 \text{ and } P(B) = .87$$

Furthermore, since the activation devices work independently, we conclude that

$$P(A \cap B) = P(A)P(B) = (.91)(.87) = .7917$$

Now, if a fire starts near a sprinkler head, the sprinkler will be activated if either the first activation device or the second activation device, or both, operates properly. Thus,

$$\begin{aligned} P(\text{Sprinkler head will be activated}) &= P(A \cup B) = P(A) + P(B) - P(A \cap B) \\ &= .91 + .87 - .7917 = .9883 \end{aligned}$$

b. The event that the sprinkler head will not be activated is the complement of the event that the sprinkler will be activated. Thus,

$$\begin{aligned} P(\text{Sprinkler head will not be activated}) &= 1 - P(\text{Sprinkler head will be activated}) \\ &= 1 - .9883 = .0117 \end{aligned}$$

c. From part a,  $P(A \cap B) = P(A)P(B) = .7917$

d. In terms of the events we have defined, we wish to determine

$$P(A \cap B^c) = P(A)P(B^c) \text{ (by independence)} = .91(1 - .87) = .91(.13) = .1183$$

### Problem Z.1

Let  $A_1$  denote the event that the policyholder will have an accident within a year of purchase; and let  $A$  denote the event that the policyholder is accident-prone.  $A^c$  is the complement of the event  $A$ .

a) Hence the desired probability,  $P(A_1)$ , is given by

$$P(A_1) = P(A_1|A)P(A) + P(A_1|A^c)P(A^c) = (.4)(.3) + (.2)(.7) = .26$$

b) The desired probability is  $P(A|A_1)$ , which is given by

$$P(A|A_1) = P(A, A_1) / P(A_1) = P(A) * P(A_1|A) / P(A_1) = (.3)(.4) / (.26) = 6/13$$

- 4.18 a. Yes. For all values of  $x$ ,  $0 \leq p(x) \leq 1$  and  $\sum p(x) = .01 + .02 + .03 + .05 + .08 + .09 + .11 + .13 + .12 + .10 + .08 + .06 + .05 + .03 + .02 + .01 + .01 = 1.00$ .
- b.  $P(x = 16) = .06$
- c.  $P(x \leq 10) = p(5) + p(6) + p(7) + p(8) + p(9) + p(10)$   
 $= .01 + .02 + .03 + .05 + .08 + .09 = .28$
- d.  $P(5 \leq x \leq 15) = p(5) + p(6) + p(7) + p(8) + p(9) + p(10) + p(11) + p(12) + p(13)$   
 $+ p(14) + p(15)$   
 $= .01 + .02 + .03 + .05 + .08 + .09 + .11 + .13 + .12 + .10 + .08$   
 $= .82$
- 4.68 a.  $\mu = \sum xp(x) = 10(.2) + 12(.3) + 18(.1) + 20(.4) = 15.4$
- $\sigma^2 = \sum (x - \mu)^2 p(x)$   
 $= (10 - 15.4)^2(.2) + (12 - 15.4)^2(.3) + (18 - 15.4)^2(.1) + (20 - 15.4)^2(.4) = 18.44$
- $\sigma = \sqrt{18.44} \approx 4.294$
- b.  $P(x < 15) = p(10) + p(12) = .2 + .3 = .5$
- c.  $\mu \pm 2\sigma = 15.4 \pm 2(4.294) \Rightarrow (6.812, 23.988)$
- d.  $P(6.812 < x < 23.988) = .2 + .3 + .1 + .4 = 1.0$

- 4.82 a. For company A,

$$\begin{aligned} E(x) &= \sum_{\text{All } x} xp(x) = 2(.05) + 3(.15) + 4(.20) + 5(.35) + 6(.25) \\ &= .10 + .45 + .80 + 1.75 + 1.50 = 4.60 \end{aligned}$$

For company B,

$$\begin{aligned} E(x) &= \sum_{\text{All } x} xp(x) = 2(.15) + 3(.30) + 4(.30) + 5(.20) + 6(.05) \\ &= .30 + .90 + 1.20 + 1.00 + .30 = 3.70 \end{aligned}$$

- b. The expected profit equals the expected value of  $x$  times the profit for each job.

For company A,  $4.6(\$10,000) = \$46,000$

For company B,  $3.7(\$15,000) = \$55,500$

- c. For company A,

$$\begin{aligned} \sigma^2 &= \sum_{\text{All } x} (x - \mu)^2 p(x) = (2 - 4.6)^2 .05 + (3 - 4.6)^2 .15 + (4 - 4.6)^2 .20 \\ &\quad + (5 - 4.6)^2 .35 + (6 - 4.6)^2 .25 \\ &= .338 + .384 + .072 + .056 + .49 = 1.34 \end{aligned}$$

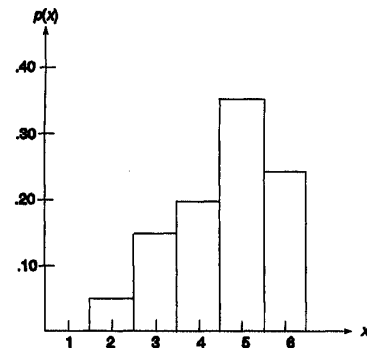
$$\sigma = \sqrt{\sigma^2} = \sqrt{1.34} = 1.16$$

For company B,

$$\begin{aligned} \sigma^2 &= \sum_{\text{All } x} (x - \mu)^2 p(x) = (2 - 3.7)^2 .15 + (3 - 3.7)^2 .30 + (4 - 3.7)^2 .30 \\ &\quad + (5 - 3.7)^2 .20 + (6 - 3.7)^2 .05 \\ &= .4335 + .147 + .027 + .338 + .2645 = 1.21 \end{aligned}$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{1.21} = 1.10$$

- d. For company A, the graph of  $p(x)$  is given here.

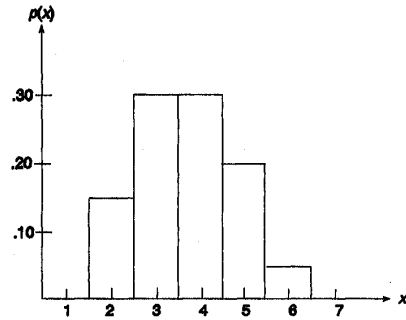


For company A,

$$\mu \pm 2\sigma \Rightarrow 4.6 \pm 2(1.16) \Rightarrow 4.6 \pm 2.32 \Rightarrow (2.28, 6.92)$$

$$P(2.28 < x < 6.92) = p(3) + p(4) + p(5) + p(6) = .15 + .20 + .35 + .25 = .95$$

For company B, the graph of  $p(x)$  is given here.



For company B,

$$\mu \pm 2\sigma \Rightarrow 3.70 \pm 2(1.10) \Rightarrow 3.67 \pm 2.2 \Rightarrow (1.5, 5.9)$$

$$P(1.5 < x < 5.9) = p(2) + p(3) + p(4) + p(5) = .15 + .30 + .30 + .20 = .95$$

### Problem Z.2

The probability distribution for  $n = 5$  and  $p = .5$  is shown in the following tables (see Exercise 4.4), along with the partial sums,  $\sum_{x=0}^a p(x)$ .

$x$	$p(x)$	$a$	$\sum_{x=0}^a p(x)$
0	.03125	0	$p(0) = .03125$
1	.15625	1	$p(0) + p(1) = .18750$
2	.31250	2	$p(0) + p(1) + p(2) = .50000$
3	.31250	3	$p(0) + p(1) + p(2) + p(3) = .8125$
4	.15625	4	$p(0) + p(1) + p(2) + p(3) + p(4) = .96875$
5	.03125	5	$p(0) + p(1) + p(2) + p(3) + p(4) + p(5) = 1$

### Problem Z.3

Use Table 1, Appendix II.

- a  $P[x < 12] = P[x \leq 11] = .748$
- b  $P[x \leq 6] = .610$
- c  $P[x > 4] = 1 - P[x \leq 4] = 1 - .633 = .367$
- d  $P[x \geq 6] = 1 - P[x \leq 5] = 1 - .034 = .966$
- e  $P[3 < x < 7] = P[x \leq 6] - P[x \leq 3] = .828 - .172 = .656$

**Correction to Z.3.b:  $P[x \leq 6] = 0.095$   
[answer given was for  $p=0.40$ , not  $p=0.60$ ]**

- 4.46 Define  $x$  as the number of physically healthy patients that seek medical assistance. The random variable  $x$  is a binomial random variable (the patients are independently chosen with two possible outcomes).

- a. When  $n = 15$  and  $p = .1$ ,

$$\begin{aligned} P(x \geq 5) &= 1 - P(x \leq 4) \\ &= 1 - .987 \text{ (Table II, Appendix B)} \\ &= .013 \end{aligned}$$

- b. When  $n = 15$  and  $p = .4$ ,

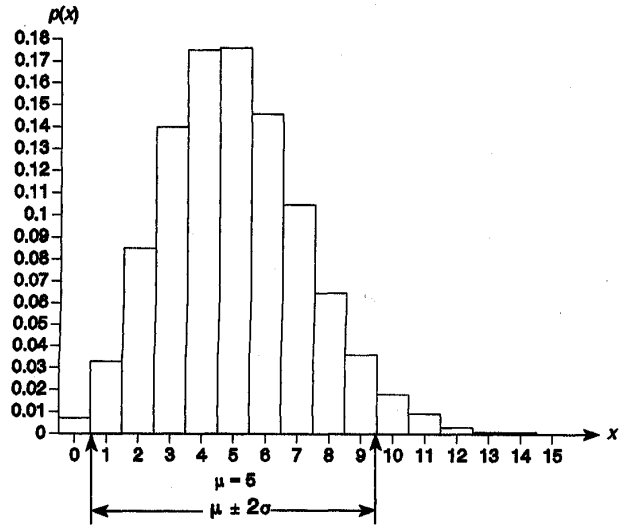
$$\begin{aligned} P(x \geq 5) &= 1 - P(x \leq 4) \\ &= 1 - .217 \text{ (Table II, Appendix B)} \\ &= .783 \end{aligned}$$

- c. We did find 5 of 15 patients seeking medical assistance when they were physically healthy. In part a, we found the probability of finding 5 or more was only .013 when  $p = .10$ . Since this did occur,  $p$  is probably larger than .10.

- 4.56 a. To graph the Poisson probability distribution with  $\lambda = 5$ , we need to calculate  $p(x)$  for  $x = 0$  to 15. Using Table III, Appendix B,

$$\begin{aligned} p(0) &= .007 \\ p(1) &= P(x \leq 1) - P(x \leq 0) = .040 - .007 = .033 \\ p(2) &= P(x \leq 2) - P(x \leq 1) = .125 - .040 = .085 \\ p(3) &= P(x \leq 3) - P(x \leq 2) = .265 - .125 = .140 \\ p(4) &= P(x \leq 4) - P(x \leq 3) = .440 - .265 = .175 \\ p(5) &= P(x \leq 5) - P(x \leq 4) = .616 - .440 = .176 \\ p(6) &= P(x \leq 6) - P(x \leq 5) = .762 - .616 = .146 \\ p(7) &= P(x \leq 7) - P(x \leq 6) = .867 - .762 = .105 \\ p(8) &= P(x \leq 8) - P(x \leq 7) = .932 - .867 = .065 \\ p(9) &= P(x \leq 9) - P(x \leq 8) = .968 - .932 = .036 \\ p(10) &= P(x \leq 10) - P(x \leq 9) = .986 - .968 = .018 \\ p(11) &= P(x \leq 11) - P(x \leq 10) = .995 - .986 = .009 \\ p(12) &= P(x \leq 12) - P(x \leq 11) = .998 - .995 = .003 \\ p(13) &= P(x \leq 13) - P(x \leq 12) = .999 - .998 = .001 \\ p(14) &= P(x \leq 14) - P(x \leq 13) = 1.000 - .999 = .001 \\ p(15) &= P(x \leq 15) - P(x \leq 14) = 1.000 - 1.000 = .000 \end{aligned}$$

The graph is shown at right:



b.  $\mu = \lambda = 5$

$$\sigma = \sqrt{\lambda} = \sqrt{5} = 2.2361$$

$$\mu \pm 2\sigma \Rightarrow 5 \pm 2(2.2361) \Rightarrow 5 \pm 4.4722 \Rightarrow (.5278, 9.4722)$$

c.  $P(.5278 < x < 9.4722) = P(1 \leq x \leq 9) = P(x \leq 9) - P(x = 0)$   
 $= .968 - .007 = .961$

- d. First, we need to find the mean number of customers per hour. If the mean number of customers per 10 minutes is 6.2, then the mean number of customers per hour is  $6.2(6) = 37.2 = \lambda$ .

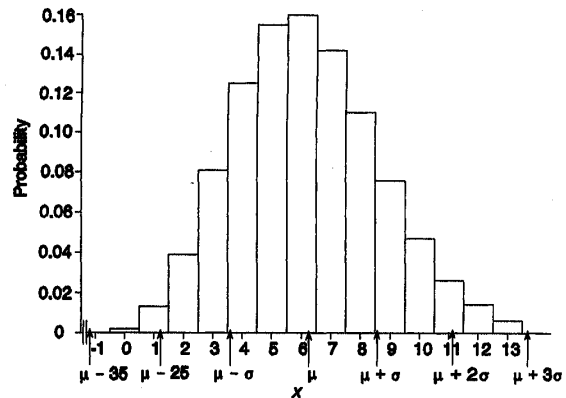
$$\mu = \lambda = 37.2 \text{ and } \sigma = \sqrt{\lambda} = \sqrt{37.2} = 6.099$$

$$\mu \pm 3\sigma \Rightarrow 37.2 \pm 3(6.099) \Rightarrow 37.2 \pm 18.297 \Rightarrow (18,903, 55.498)$$

Using Chebyshev's Rule, we know at least 8/9 or 88.9% of the observations will fall within 3 standard deviations of the mean. The number 75 is way beyond the 3 standard deviation limit. Thus, it would be very unlikely that more than 75 customers entered the store per hour on Saturdays.

- 4.58 a. Using Table III and  $\lambda = 6.2$ ,  $P(x = 2) = P(x \leq 2) - P(x \leq 1) = .054 - .015 = .039$   
 $P(x = 6) = P(x \leq 6) - P(x \leq 5) = .574 - .414 = .160$   
 $P(x = 10) = P(x \leq 10) - P(x \leq 9) = .949 - .902 = .047$

- b. The plot of the distribution is:



- c.  $\mu = \lambda = 6.2$ ,  $\sigma = \sqrt{\lambda} = \sqrt{6.2} = 2.490$   
 $\mu \pm \sigma \Rightarrow 6.2 \pm 2.49 \Rightarrow (3.71, 8.69)$   
 $\mu \pm 2\sigma \Rightarrow 6.2 \pm 2(2.49) \Rightarrow 6.2 \pm 4.98 \Rightarrow (1.22, 11.18)$   
 $\mu \pm 3\sigma \Rightarrow 6.2 \pm 3(2.49) \Rightarrow 6.2 \pm 7.47 \Rightarrow (-1.27, 13.67)$

See the plot in part b.

- d. First, we need to find the mean number of customers per hour. If the mean number of customers per 10 minutes is 6.2, then the mean number of customers per hour is  $6.2(6) = 37.2 = \lambda$ .

$$\mu = \lambda = 37.2 \text{ and } \sigma = \sqrt{\lambda} = \sqrt{37.2} = 6.099$$

$$\mu \pm 3\sigma \Rightarrow 37.2 \pm 3(6.099) \Rightarrow 37.2 \pm 18.297 \Rightarrow (18,903, 55.498)$$

Using Chebyshev's Rule, we know at least 8/9 or 88.9% of the observations will fall within 3 standard deviations of the mean. The number 75 is way beyond the 3 standard deviation limit. Thus, it would be very unlikely that more than 75 customers entered the store per hour on Saturdays.

- 5.10 a. If  $x$  is uniformly distributed over the interval \$10,000 to \$15,000, then  $d = \$15,000$  and  $c = \$10,000$ .

$$E(x) = \frac{c + d}{2} = \frac{\$10,000 + \$15,000}{2} = \$12,500$$

The average monthly reimbursements to the employees is \$12,500.

$$b. f(x) = \begin{cases} \frac{1}{d - c} = \frac{1}{\$15,000 - \$10,000} = \frac{1}{\$5,000} = .0002 & (\$10,000 \leq x \leq \$15,000) \\ 0 & \text{otherwise} \end{cases}$$

$$P(x > \$12,000) = (\$15,000 - \$12,000) \cdot 0.0002 = .6$$

$$c. P(x > a) = .20 \Rightarrow (\$15,000 - a) \cdot 0.0002 = .20$$

$$\Rightarrow 3 - .0002a = .2$$

$$\Rightarrow .0002a = 2.8$$

$$\Rightarrow a = \$14,000$$

- 5.16 a.  $P(z = 1) = 0$ , since a single point does not have an area.

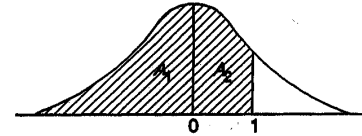
$$b. P(z \leq 1) = P(z \leq 0) + P(0 < z \leq 1)$$

$$= A_1 + A_2$$

$$= .5 + .3413$$

$$= .8413$$

(Table IV, Appendix B)



$$c. P(z < 1) = P(z \leq 1) = .8413 \text{ (Refer to part b.)}$$

$$d. P(z > 1) = 1 - P(z \leq 1) = 1 - .8413 = .1587 \text{ (Refer to part b.)}$$

5.78  $x$  is normal random variable with  $\mu = 40$ ,  $\sigma^2 = 36$ , and  $\sigma = 6$ .

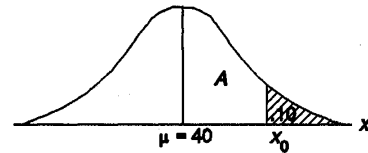
a.  $P(x \geq x_0) = .10$

So,  $A = .5000 - .1000 = .4000$ .

$$z_0 = 1.28 \quad (\text{See part a.})$$

To find  $x_0$ , substitute the values into the z-score formula:

$$z_0 = \frac{x_0 - \mu}{\sigma} \Rightarrow 1.28 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 1.28(6) + 40 = 47.68$$

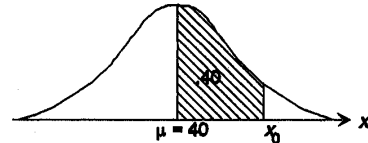


b.  $P(\mu \leq x \leq x_0) = .40$

Look up the area .4000 in the body of Table IV, Appendix B; (take the closest value)  $z_0 = 1.28$ .

To find  $x_0$ , substitute the values into the z-score formula:

$$z_0 = \frac{x_0 - \mu}{\sigma} \Rightarrow 1.28 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 40 + 6(1.28) = 47.68$$



c.  $P(x < x_0) = .05$

So,  $A = .5000 - .0500 = .4500$ .

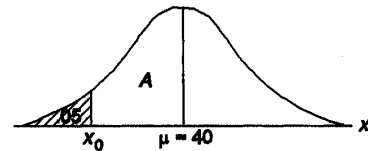
Look up the area .4500 in the body of Table IV, Appendix B;  $z_0 = -1.645$ . (.45 is halfway between .4495 and .4505; therefore, we average the z-scores

$$\frac{1.64 + 1.65}{2} = 1.645$$

$z_0$  is negative since the graph shows  $x_0$  is on the left side of 0.

To find  $x_0$ , substitute the values into the z-score formula:

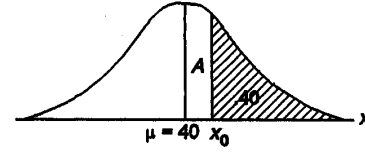
$$z_0 = \frac{x_0 - \mu}{\sigma} \Rightarrow -1.645 = \frac{x_0 - 40}{6} \Rightarrow x_0 = -1.645(6) + 40 = 30.13$$



d.  $P(x > x_0) = .40$

So,  $A = .5000 - .4000 = .1000$ .

Look up the area .1000 in the body of Table IV, Appendix B; (take the closest value)  $z_0 = .25$ .

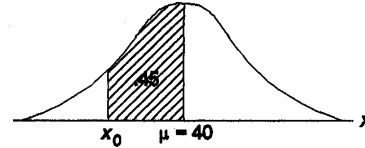


To find  $x_0$ , substitute the values into the z-score formula:

$$z_0 = \frac{x_0 - \mu}{\sigma} \Rightarrow .25 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 40 + 6(.25) = 41.5$$

e.  $P(x_0 \leq x < \mu) = .45$

Look up the area .4500 in the body of Table IV, Appendix B;  $z_0 = -1.645$ . (.45 is halfway between .4495 and .4505; therefore, we average the z-scores



$$\frac{1.64 + 1.65}{2} = 1.645$$

$z_0$  is negative since the graph shows  $x_0$  is on the left side of 0.

To find  $x_0$ , substitute the values into the z-score formula:

$$z_0 = \frac{x_0 - \mu}{\sigma} \Rightarrow -1.645 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 40 - 6(1.645) = 30.13$$

5.32 a. Using Table IV, Appendix B, and  $\mu = 75$  and  $\sigma = 7.5$ ,

$$P(x > 80) = P\left[z > \frac{80 - 75}{7.5}\right] = P(z > .67) = .5 - .2486 = .2514$$

Thus, 25.14% of the scores exceeded 80.

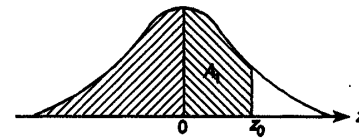
b.  $P(x \leq x_0) = .98$ . Find  $x_0$ .

$$P(x \leq x_0) = P\left[z \leq \frac{x_0 - 75}{7.5}\right] = P(z \leq z_0) = .98$$

$$A_1 = .98 - .5 = .4800$$

Looking up area .4800 in Table IV,  $z_0 = 2.05$ .

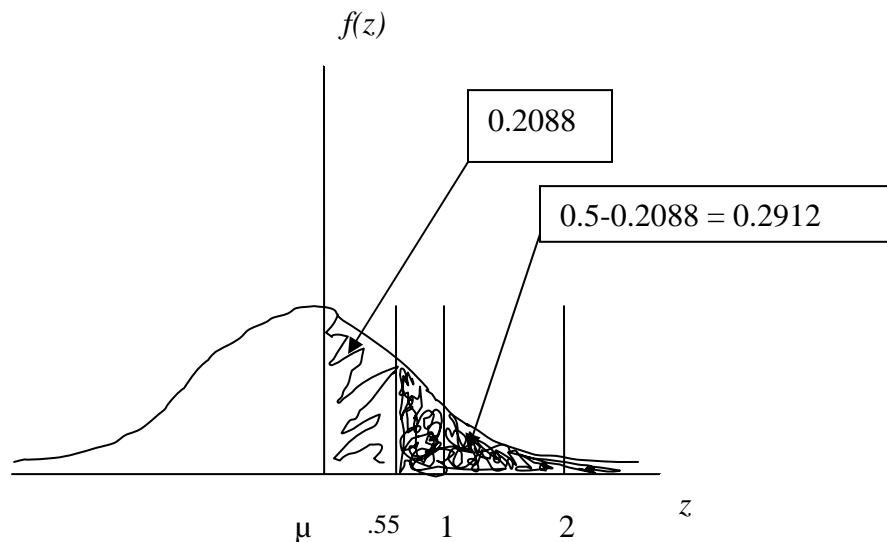
$$z_0 = \frac{x_0 - 75}{7.5} \Rightarrow 2.05 = \frac{x_0 - 75}{7.5} \Rightarrow x_0 = 90.375$$



Z.4

a. Calculate z:

$$z = \frac{0.03 - \mu}{\sigma} = \frac{0.030 - 0.018}{0.022} \approx 0.5454 \approx 0.55$$



So the probability that  $z > 0.55$  is **0.2912**.

b. The probability that payroll employment growth is greater than 3% at annual rate for each of the next four quarters is given by:

$$\begin{aligned} &= P(x > 0.03) \times P(x > 0.03) \times P(x > 0.03) \times P(x > 0.03) \\ &= P(x > 0.03)^4 \\ &= (0.2912)^4 = 0.0072 \end{aligned}$$

or less than one percent.

Note, this answer assumes independence of trials (i.e., in this context, employment growth one quarter is independent of employment growth in another quarter; this is not true in general, so the true probability is likely to be greater than this).