Economics 310
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University of Wisconsin-Madison

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Problem Set 2 Answers
(corrected Z.3.b, 3/1)

3.68 Define the following events:

A: {male worker}
B: {female worker}
C: {service worker}
D: {managerial/professional worker}
E: {operator/fabricator/laborer}
F: {technical/sales/administrative worker}

a. $P(A \cap C) = .05$

b. $P(D) = P(A \cap D) + P(B \cap D) = .16 + .16 = .32$

c. $P[(B \cap D) \cup (B \cap E)] = .16 + .03 = .19$

d. $P(F^c) = 1 - P(F) = 1 - [P(A \cap F) + P(B \cap F)] = 1 - (.10 + .18) = 1 - .28 = .72$

3.36 The 36 possible outcomes obtained when tossing two dice are listed below:

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

A: {(1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (3, 6), (4, 1), (4, 3), (4, 5), (5, 2), (5, 4), (5, 6), (6, 1), (6, 3), (6, 5)}

B: {(3, 6), (4, 5), (5, 4), (5, 6), (6, 3), (6, 5), (6, 6)}

$A \cap B$: {(3, 6), (4, 5), (5, 4), (5, 6), (6, 3), (6, 5)}

If A and B are independent, then $P(A)P(B) = P(A \cap B)$.

$$P(A) = \frac{18}{36} = \frac{1}{2} \qquad P(B) = \frac{7}{36} \qquad P(A \cap B) = \frac{6}{36} = \frac{1}{6}$$

$P(A)P(B) = \frac{1}{2} \cdot \frac{7}{36} = \frac{7}{72} \neq \frac{1}{6} = P(A \cap B)$. Thus, A and B are not independent.

3.40 a. The frequency table would be:

	Auto	Trad	Ener	Insu	Tele	Elec	Appl	FinS	Ret	Totals
Germany	2			1		1			1	5
Japan	1	6			1	1	2			11
Netherlands			1					1		2
France				1						1
United Kingdom			1							1
Totals	3	6	2	2	1	2	2	1	1	20

b. Define the following events:

G: {Country is Germany} J: {Country is Japan} N: {Country is Netherlands}

F: {Country is France} U: {Country is United Kingdom}

AU: {Business is automobiles} T: {Business is trading}

EN: {Business is energy} I: {Business is insurance}

TE: {Business is telecommunications} EL: {Business is electronics}

AP: {Business is appliances} FN: {Business is financial services}

R: {Business is retailing}

$P(J) = 11/20 = .55$

c. $P(AU) = 3/20 = .15$

d. $P(I \cup EL) = (2 + 2)/20 = 4/20 = .20$

e. $P(J \cap AU) = 1/20 = .05$

f. $P(G \cup EL) = 6/20 = .30$

g. $P(AU \mid J) = \dfrac{P(AU \cap J)}{P(J)} = \dfrac{1/20}{11/20} = 1/11 = .091$

h. $P(J \mid AU) = \dfrac{P(AU \cap J)}{P(AU)} = \dfrac{1/20}{3/20} = 1/3 = .333$

i. If J and AU are independent events, then $P(J \mid AU) = P(J)$

From part h, $P(J \mid AU) = .333$ and from part b, $P(J) = .55$

Since these probabilities are not equal, J and AU are not independent.

3.46 Define the following events:

A: {Patient receives PMI sheet}
B: {Patient was hospitalized}

$P(A) = .20$, $P(A \cap B) = .12$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{.12}{.20} = .60$$

3.78 a. We will define the following events:

A: {The first activation device works properly; i.e., activates the sprinkler when it should}
B: {The second activation device works properly}

From the statement of the problem, we know

$P(A) = .91$ and $P(B) = .87$

Furthermore, since the activation devices work independently, we conclude that

$P(A \cap B) = P(A)P(B) = (.91)(.87) = .7917$

Now, if a fire starts near a sprinkler head, the sprinkler will be activated if either the first activation device or the second activation device, or both, operates properly. Thus,

$$P(\text{Sprinkler head will be activated}) = P(A \cup B) = P(A) + P(B) - P(A \cap B) = .91 + .87 - .7917 = .9883$$

b. The event that the sprinkler head will not be activated is the complement of the event that the sprinkler will be activated. Thus,

$$P(\text{Sprinkler head will not be activated}) = 1 - P(\text{Sprinkler head will be activated}) = 1 - .9883 = .0117$$

c. From part **a**, $P(A \cap B) = P(A)P(B) = .7917$

d. In terms of the events we have defined, we wish to determine

$$P(A \cap B^c) = P(A)P(B^c) \text{ (by independence)} = .91(1 - .87) = .91(.13) = .1183$$

Problem Z.1

Let A1 denote the event that the policyholder will have an accident within a year of purchase; and let A denote the event that the policyholder is accident-prone. Acomplement is the complement of the event A.

a) Hence the desired probability, P(A1), is given by

P(A1)=P(A1|A)P(A)+P(A1|Acomplement)P(Acomplement)=(.4)*(.3)+(.2)*(.7)=.26

b) The desired probability is P(A|A1), which is given by

P(A|A1)=P(A,A1)/P(A1)=P(A)*P(A1|A)/P(A1)=(.3)*(.4)/(.26)=6/13

4.18 a. Yes. For all values of x, $0 \leq p(x) \leq 1$ and $\sum p(x) = .01 + .02 + .03 + .05 + .08 + .09 + .11 + .13 + .12 + .10 + .08 + .06 + .05 + .03 + .02 + .01 + .01 = 1.00$.

b. $P(x = 16) = .06$

c. $P(x \leq 10) = p(5) + p(6) + p(7) + p(8) + p(9) + p(10)$
$= .01 + .02 + .03 + .05 + .08 + .09 = .28$

d. $P(5 \leq x \leq 15) = p(5) + p(6) + p(7) + p(8) + p(9) + p(10) + p(11) + p(12) + p(13) + p(14) + p(15)$
$= .01 + .02 + .03 + .05 + .08 + .09 + .11 + .13 + .12 + .10 + .08$
$= .82$

4.68 a. $\mu = \sum xp(x) = 10(.2) + 12(.3) + 18(.1) + 20(.4) = 15.4$

$\sigma^2 = \sum (x - \mu)^2 p(x)$
$= (10 - 15.4)^2(.2) + (12 - 15.4)^2(.3) + (18 - 15.4)^2(.1) + (20 - 15.4)^2(.4) = 18.44$

$\sigma = \sqrt{18.44} \approx 4.294$

b. $P(x < 15) = p(10) + p(12) = .2 + .3 = .5$

c. $\mu \pm 2\sigma = 15.4 \pm 2(4.294) \Rightarrow (6.812, 23.988)$

d. $P(6.812 < x < 23.988) = .2 + .3 + .1 + .4 = 1.0$

4.82 a. For company A,

$$E(x) = \sum_{\text{All } x} xp(x) = 2(.05) + 3(.15) + 4(.20) + 5(.35) + 6(.25)$$
$$= .10 + .45 + .80 + 1.75 + 1.50 = 4.60$$

For company B,

$$E(x) = \sum_{\text{All } x} xp(x) = 2(.15) + 3(.30) + 4(.30) + 5(.20) + 6(.05)$$
$$= .30 + .90 + 1.20 + 1.00 + .30 = 3.70$$

b. The expected profit equals the expected value of x times the profit for each job.

For company A, 4.6($10,000) = $46,000

For company B, 3.7($15,000) = $55,500

c. For company A,

$$\sigma^2 = \sum_{\text{All } x} (x - \mu)^2 p(x) = (2 - 4.6)^2.05 + (3 - 4.6)^2.15 + (4 - 4.6)^2.20$$
$$+ (5 - 4.6)^2.35 + (6 - 4.6)^2.25$$
$$= .338 + .384 + .072 + .056 + .49 = 1.34$$

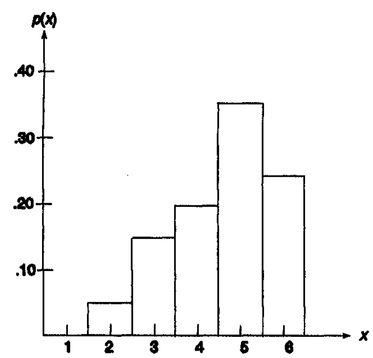
$$\sigma = \sqrt{\sigma^2} = \sqrt{1.34} = 1.16$$

For company B,

$$\sigma^2 = \sum_{\text{All } x} (x - \mu)^2 p(x) = (2 - 3.7)^2.15 + (3 - 3.7)^2.30 + (4 - 3.7)^2.30$$
$$+ (5 - 3.7)^2.20 + (6 - 3.7)^2.05$$
$$= .4335 + .147 + .027 + .338 + .2645 = 1.21$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{1.21} = 1.10$$

d. For company A, the graph of $p(x)$ is given here.

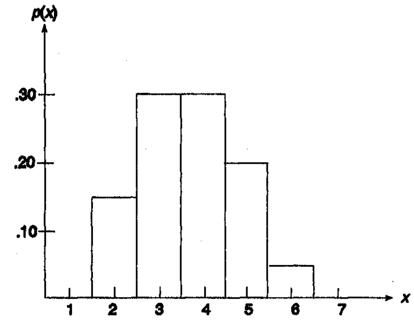


For company A,

$\mu \pm 2\sigma \Rightarrow 4.6 \pm 2(1.16) \Rightarrow 4.6 \pm 2.32 \Rightarrow (2.28, 6.92)$

$P(2.28 < x < 6.92) = p(3) + p(4) + p(5) + p(6) = .15 + .20 + .35 + .25 = .95$

For company B, the graph of $p(x)$ is given here.



For company B,

$\mu \pm 2\sigma \Rightarrow 3.70 \pm 2(1.10) \Rightarrow 3.67 \pm 2.2 \Rightarrow (1.5, 5.9)$

$P(1.5 < x < 5.9) = p(2) + p(3) + p(4) + p(5) = .15 + .30 + .30 + .20 = .95$

Problem Z.2

The probability distribution for $n = 5$ and $p = .5$ is shown in the following tables (see Exercise 4.4), along with the partial sums, $\sum_{x=0}^{a} p(x)$.

x	$p(x)$	a	$\sum_{x=0}^{a} p(x)$
0	.03125	0	$p(0) = .03125$
1	.15625	1	$p(0) + p(1) = .18750$
2	.31250	2	$p(0) + p(1) + p(2) = .50000$
3	.31250	3	$p(0) + p(1) + p(2) + p(3) = .8125$
4	.15625	4	$p(0) + p(1) + p(2) + p(3) + p(4) = .96875$
5	.03125	5	$p(0) + p(1) + p(2) + p(3) + p(4) + p(5) = 1$

Problem Z.3

Use Table 1, Appendix II.

a $P[x < 12] = P[x \leq 11] = .748$

b $P[x \leq 6] = .610$

c $P[x > 4] = 1 - P[x \leq 4] = 1 - .633 = .367$

d $P[x \geq 6] = 1 - P[x \leq 5] = 1 - .034 = .966$

e $P[3 < x < 7] = P[x \leq 6] - P[x \leq 3] = .828 - .172 = .656$

Correction to Z.3.b: P[x<=6]=0.095
[answer given was for p=0.40, not p=0.60]

4.46 Define x as the number of physically healthy patients that seek medical assistance. The random variable x is a binomial random variable (the patients are independently chosen with two possible outcomes).

a. When $n = 15$ and $p = .1$,

$$P(x \geq 5) = 1 - P(x \leq 4)$$
$$= 1 - .987 \quad \text{(Table II, Appendix B)}$$
$$= .013$$

b. When $n = 15$ and $p = .4$,

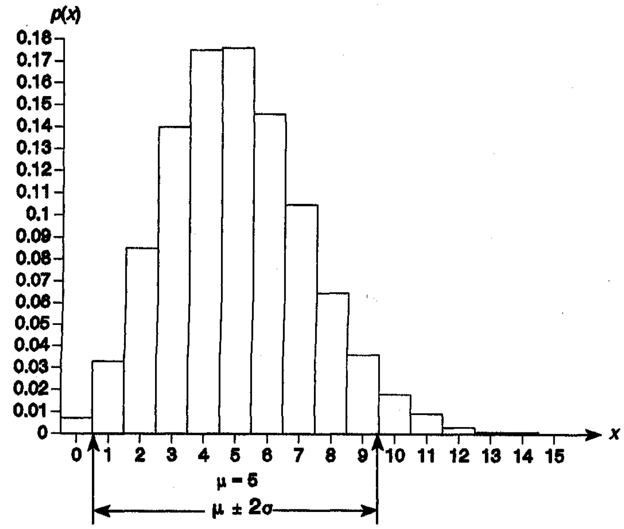
$$P(x \geq 5) = 1 - P(x \leq 4)$$
$$= 1 - .217 \quad \text{(Table II, Appendix B)}$$
$$= .783$$

c. We did find 5 of 15 patients seeking medical assistance when they were physically healthy. In part **a**, we found the probability of finding 5 or more was only .013 when $p = .10$. Since this did occur, p is probably larger than .10.

4.56 a. To graph the Poisson probability distribution with $\lambda = 5$, we need to calculate $p(x)$ for $x = 0$ to 15. Using Table III, Appendix B,

$$p(0) = .007$$
$$p(1) = P(x \leq 1) - P(x \leq 0) = .040 - .007 = .033$$
$$p(2) = P(x \leq 2) - P(x \leq 1) = .125 - .040 = .085$$
$$p(3) = P(x \leq 3) - P(x \leq 2) = .265 - .125 = .140$$
$$p(4) = P(x \leq 4) - P(x \leq 3) = .440 - .265 = .175$$
$$p(5) = P(x \leq 5) - P(x \leq 4) = .616 - .440 = .176$$
$$p(6) = P(x \leq 6) - P(x \leq 5) = .762 - .616 = .146$$
$$p(7) = P(x \leq 7) - P(x \leq 6) = .867 - .762 = .105$$
$$p(8) = P(x \leq 8) - P(x \leq 7) = .932 - .867 = .065$$
$$p(9) = P(x \leq 9) - P(x \leq 8) = .968 - .932 = .036$$
$$p(10) = P(x \leq 10) - P(x \leq 9) = .986 - .968 = .018$$
$$p(11) = P(x \leq 11) - P(x \leq 10) = .995 - .986 = .009$$
$$p(12) = P(x \leq 12) - P(x \leq 11) = .998 - .995 = .003$$
$$p(13) = P(x \leq 13) - P(x \leq 12) = .999 - .998 = .001$$
$$p(14) = P(x \leq 14) - P(x \leq 13) = 1.000 - .999 = .001$$
$$p(15) = P(x \leq 15) - P(x \leq 14) = 1.000 - 1.000 = .000$$

The graph is shown at right:



b. $\mu = \lambda = 5$

$\sigma = \sqrt{\lambda} = \sqrt{5} = 2.2361$

$\mu \pm 2\sigma \Rightarrow 5 \pm 2(2.2361) \Rightarrow 5 \pm 4.4722 \Rightarrow (.5278, 9.4722)$

c. $P(.5278 < x < 9.4722) = P(1 \leq x \leq 9) = P(x \leq 9) - P(x = 0)$

$= .968 - .007 = .961$

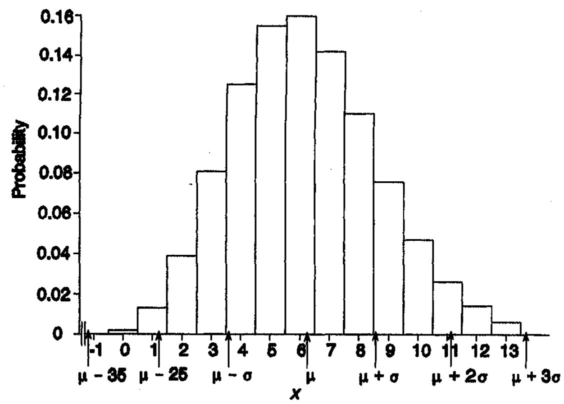
d. First, we need to find the mean number of customers per hour. If the mean number of customers per 10 minutes is 6.2, then the mean number of customers per hour is $6.2(6) = 37.2 = \lambda$.

$\mu = \lambda = 37.2$ and $\sigma = \sqrt{\lambda} = \sqrt{37.2} = 6.099$
$\mu \pm 3\sigma \Rightarrow 37.2 \pm 3(6.099) \Rightarrow 37.2 \pm 18.297 \Rightarrow (18,903, 55.498)$

Using Chebyshev's Rule, we know at least 8/9 or 88.9% of the observations will fall within 3 standard deviations of the mean. The number 75 is way beyond the 3 standard deviation limit. Thus, it would be very unlikely that more than 75 customers entered the store per hour on Saturdays.

4.58 a. Using Table III and $\lambda = 6.2$, $P(x = 2) = P(x \le 2) - P(x \le 1) = .054 - .015 = .039$
$P(x = 6) = P(x \le 6) - P(x \le 5) = .574 - .414 = .160$
$P(x = 10) = P(x \le 10) - P(x \le 9) = .949 - .902 = .047$

b. The plot of the distribution is:



c. $\mu = \lambda = 6.2$, $\sigma = \sqrt{\lambda} = \sqrt{6.2} = 2.490$
$\mu \pm \sigma \Rightarrow 6.2 \pm 2.49 \Rightarrow (3.71, 8.69)$
$\mu \pm 2\sigma \Rightarrow 6.2 \pm 2(2.49) \Rightarrow 6.2 \pm 4.98 \Rightarrow (1.22, 11.18)$
$\mu \pm 3\sigma \Rightarrow 6.2 \pm 3(2.49) \Rightarrow 6.2 \pm 7.47 \Rightarrow (-1.27, 13.67)$

See the plot in part **b**.

d. First, we need to find the mean number of customers per hour. If the mean number of customers per 10 minutes is 6.2, then the mean number of customers per hour is $6.2(6) = 37.2 = \lambda$.

$\mu = \lambda = 37.2$ and $\sigma = \sqrt{\lambda} = \sqrt{37.2} = 6.099$
$\mu \pm 3\sigma \Rightarrow 37.2 \pm 3(6.099) \Rightarrow 37.2 \pm 18.297 \Rightarrow (18,903, 55.498)$

Using Chebyshev's Rule, we know at least 8/9 or 88.9% of the observations will fall within 3 standard deviations of the mean. The number 75 is way beyond the 3 standard deviation limit. Thus, it would be very unlikely that more than 75 customers entered the store per hour on Saturdays.

5.10 a. If x is uniformly distributed over the interval \$10,000 to \$15,000, then d = \$15,000 and c = \$10,000.

$$E(x) = \frac{c + d}{2} = \frac{\$10,000 + \$15,000}{2} = \$12,500$$

The average monthly reimbursements to the employees is \$12,500.

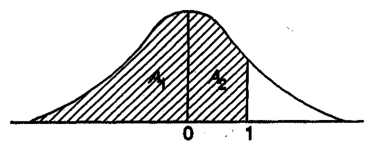
b. $$f(x) = \begin{cases} \frac{1}{d - c} = \frac{1}{\$15,000 - \$10,000} = \frac{1}{\$5,000} = .0002 & (\$10,000 \le x \le \$15,000) \\ 0 & \text{otherwise} \end{cases}$$

$P(x > \$12,000) = (\$15,000 - \$12,000).0002 = .6$

c. $P(x > a) = .20 \Rightarrow (\$15,000 - a).0002 = .20$
$\Rightarrow 3 - .0002a = .2$
$\Rightarrow .0002a = 2.8$
$\Rightarrow a = \$14,000$

5.16 a. $P(z = 1) = 0$, since a single point does not have an area.

b. $P(z \le 1) = P(z \le 0) + P(0 < z \le 1)$
$= A_1 + A_2$
$= .5 + .3413$
$= .8413$
(Table IV, Appendix B)

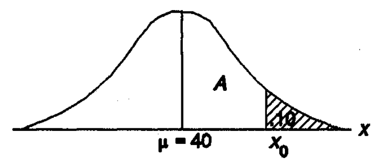


c. $P(z < 1) = P(z \le 1) = .8413$ (Refer to part **b.**)

d. $P(z > 1) = 1 - P(z \le 1) = 1 - .8413 = .1587$ (Refer to part **b.**)

5.78 x is normal random variable with $\mu = 40$, $\sigma^2 = 36$, and $\sigma = 6$.

a. $P(x \geq x_0) = .10$

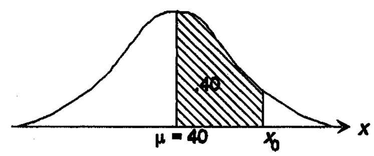


So, $A = .5000 - .1000 = .4000$.
$z_0 = 1.28$ (See part **a.**)

To find x_0, substitute the values into the z-score formula:

$$z_0 = \frac{x_0 - \mu}{\sigma} \Rightarrow 1.28 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 1.28(6) + 40 = 47.68$$

b. $P(\mu \leq x \leq x_0) = .40$

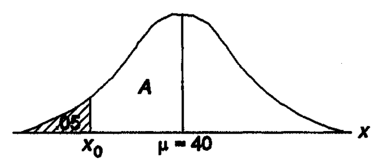


Look up the area .4000 in the body of Table IV, Appendix B; (take the closest value) $z_0 = 1.28$.

To find x_0, substitute the values into the z-score formula:

$$z_0 = \frac{x_0 - \mu}{\sigma} \Rightarrow 1.28 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 40 + 6(1.28) = 47.68$$

c. $P(x < x_0) = .05$



So, $A = .5000 - .0500 = .4500$.

Look up the area .4500 in the body of Table IV, Appendix B; $z_0 = -1.645$. (.45 is halfway between .4495 and .4505; therefore, we average the z-scores

$$\frac{1.64 + 1.65}{2} = 1.645$$

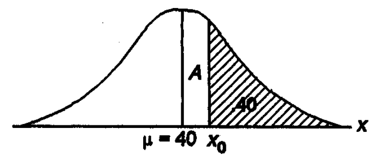
z_0 is negative since the graph shows z_0 is on the left side of 0.

To find x_0, substitute the values into the z-score formula:

$$z_0 = \frac{x_0 - \mu}{\sigma} \Rightarrow -1.645 = \frac{x_0 - 40}{6} \Rightarrow x_0 = -1.645(6) + 40 = 30.13$$

d. $P(x > x_0) = .40$

So, $A = .5000 - .4000 = .1000$.

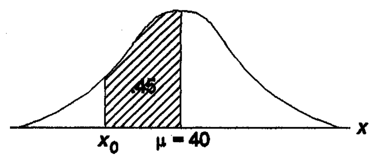


Look up the area .1000 in the body of Table IV, Appendix B; (take the closest value) $z_0 = .25$.

To find x_0, substitute the values into the z-score formula:

$$z_0 = \frac{x_0 - \mu}{\sigma} \Rightarrow .25 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 40 + 6(.25) = 41.5$$

e. $P(x_0 \leq x < \mu) = .45$



Look up the area .4500 in the body of Table IV, Appendix B; $z_0 = -1.645$. (.45 is halfway between .4495 and .4505; therefore, we average the z-scores

$$\frac{1.64 + 1.65}{2} = 1.645$$

z_0 is negative since the graph shows z_0 is on the left side of 0.

To find x_0, substitute the values into the z-score formula:

$$z_0 = \frac{x_0 - \mu}{\sigma} \Rightarrow -1.645 = \frac{x_0 - 40}{6} \Rightarrow x_0 = 40 - 6(1.645) = 30.13$$

5.32 a. Using Table IV, Appendix B, and $\mu = 75$ and $\sigma = 7.5$,

$$P(x > 80) = P\left(z > \frac{80 - 75}{7.5}\right) = P(z > .67) = .5 - .2486 = .2514$$

Thus, 25.14% of the scores exceeded 80.

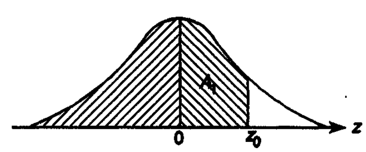
b. $P(x \leq x_0) = .98$. Find x_0.

$$P(x \leq x_0) = P\left(z \leq \frac{x_0 - 75}{7.5}\right) = P(z \leq z_0) = .98$$

$A_1 = .98 - .5 = .4800$

Looking up area .4800 in Table IV, $z_0 = 2.05$.

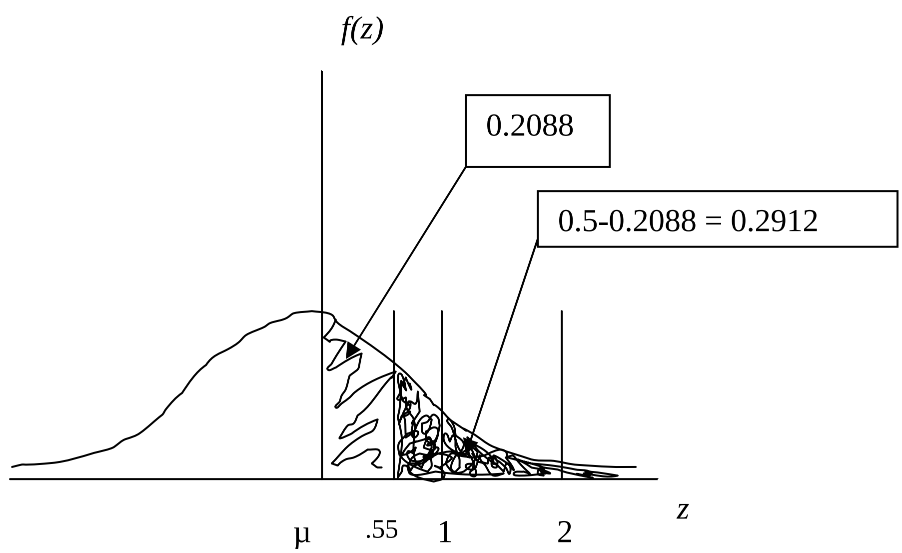
$$z_0 = \frac{x_0 - 75}{7.5} \Rightarrow 2.05 = \frac{x_0 - 75}{7.5} \Rightarrow x_0 = 90.375$$



Z.4

a. Calculate z:

$$z = \frac{0.03 - \mu}{\sigma} = \frac{0.030 - 0.018}{0.022} \approx 0.5454 \approx 0.55$$



So the probability that z > 0.55 is **0.2912**.

b. The probability that payroll employment growth is greater than 3% at annual rate for each of the next four quarters is given by:

$$= P(x > 0.03) \times P(x > 0.03) \times P(x > 0.03) \times P(x > 0.03)$$
$$= P(x > 0.03)^4$$
$$= (0.2912)^4 = 0.0072$$

or less than one percent.

Note, this answer assumes independence of trials (i.e., in this context, employment growth one quarter is independent of employment growth in another quarter; this is not true in general, so the true probability is likely to be greater than this).