Economics 310
Spring 2004
UW-Madison

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Social Sciences 7418

Problem Set 1 Answers

[Problems from textbook] (corrections to Problem 3.20 a,b on 2/16; 3.20 f on 3/1)

2.16 a. The highest proportion of test scores (.25) fell in the measurement class 7.5–9.5

b. The proportion of scores between 3.5 and 5.5 is .15

c. The proportion of test scores higher than 11.5 is .10 + .05 + .05 = .20

d. The proportion of the 100 students who scored less than 5.5 is .05 + .15 = .20

2.40 a. Using MINITAB, the stem-and-leaf display is:

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Stem-and-leaf of PAF            N=17
Leaf Unit = 1.0

 6   0  000009
 8   1  25
(2)  2  45
 7   3  13
 5   4  0
 4   5
 4   6  2
 3   7  057
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b. The median is the middle number once the data are arranged in order. The data arranged in order are: 0, 0, 0, 0, 0, 9, 12, 15, 24, 25, 31, 33, 40, 62, 70, 75, 77.

The middle number or the median is 24.

c. The mean of the data is $\bar{x} = \frac{\sum x}{n} = \frac{77 + 33 + 75 + \cdots + 31}{17} = \frac{473}{17} = 27.82$

d. The number occurring most frequently is 0. The mode is 0.

e. The mode corresponds to the smallest number. It does not seem to locate the center of the distribution. Both the mean and the median are in the middle of the stem-and-leaf display. Thus, it appears that both of them locate the center of the data.

2.72 a. From the information given, we have $\bar{x} = 375$ and $s = 25$. From Chebyshev's Rule, we know that at least three-fourths of the measurements are within the interval:

$\bar{x} \pm 2s$, or (325, 425)

Thus, at most one-fourth of the measurements exceed 425. In other words, more than 425 vehicles used the intersection on at most 25% of the days.

b. According to the Empirical Rule, approximately 95% of the measurements are within the interval:

$\bar{x} \pm 2s$, or (325, 425)

This leaves approximately 5% of the measurements to lie outside the interval. Because of the symmetry of a mound-shaped distribution, approximately 2.5% of these will lie below 325, and the remaining 2.5% will lie above 425. Thus, on approximately 2.5% of the days, more than 425 vehicles used the intersection.

2.84 a. The mean and standard deviation are:

$$\bar{x} = \frac{\sum x}{n} = \frac{138.8}{12} = 11.567$$

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} = \frac{14{,}725.04 - \frac{(138.8)^2}{12}}{12-1} = \frac{13{,}119.58667}{11} = 1{,}192.6897$$

$$s = \sqrt{1{,}192.6897} = 34.5353$$

b. The z-scores are:

Coca-Cola: $z = \frac{x - \bar{x}}{s} = \frac{2.4 - 11.567}{34.5353} = -.27$

Flowers: $z = \frac{x - \bar{x}}{s} = \frac{120.5 - 11.567}{34.5353} = 3.15$

Smithfield: $z = \frac{x - \bar{x}}{s} = \frac{-5.0 - 11.567}{34.5353} = -.48$

c. The sales growth of Coca-Cola is .27 standard deviations below the mean. The sales growth of Flowers Industries is 3.15 standard deviations above the mean. The sales growth of Smithfield Foods is .48 standard deviations below the mean.

3.8 a. The sample points of this experiment correspond to each of the 8 possible types of commodities. Suppose we introduce notation to make the listing of the sample points easier.

A: {carload contains agricultural products}
CH: {carload contains chemicals}
CO: {carload contains coal}
F: {carload contains forest products}
MO: {carload contains metallic ores and minerals}

MV: {carload contains motor vehicles and equipment}
N: {carload contains nonmetallic minerals and products}
O: {carload contains other}

The eight sample points are: A CH CO F MO MV N O

b. The probability of each sample point is found by dividing the number of carloads for each sample point by the total number of carloads. The probabilities are:

$P(A) = 41{,}690 / 335{,}770 = .124$

$P(CH) = 38{,}331 / 335{,}770 = .114$

$P(CO) = 124{,}595 / 335{,}770 = .371$

$P(F) = 21{,}929 / 335{,}770 = .065$

$P(MO) = 34{,}521 / 335{,}770 = .103$

$P(MV) = 22{,}906 / 335{,}770 = .068$

$P(N) = 37{,}416 / 335{,}770 = .111$

$P(O) = 14{,}382 / 335{,}770 = .043$

c. $P(MV) = .068$

$$\begin{aligned} P(\text{nonagricultural products}) &= P(CH) + P(CO) + P(F) + P(MO) + P(MV) + P(N) + P(O) \\ &= .114 + .371 + .065 + .103 + .068 + .111 + .043 = .875 \end{aligned}$$

d. $P(CH) + P(CO) = .114 + .371 = .485$

e. Since there were 335,770 carloads that week, the probability of selecting any one in particular would be $1 / 335{,}770 = .00000298$. Thus, the probability of selecting the carload with the serial number 1003642 is .00000298.

3.12 a. Let I = Infiniti, St = Saturn, and Sb = Saab. All possible rankings are as follows, where the first dealer listed is ranked first, the second dealer listed is ranked second, and the third dealer listed is ranked third:

I, St, Sb $\quad$ I, Sb, St $\quad$ St, Sb, I $\quad$ St, I, Sb $\quad$ Sb, I, St $\quad$ Sb, St, I

b. If each set of rankings is equally likely, then each has a probability of 1/6.

The probability that Saturn is ranked first = $P(St, Sb, I) + P(St, I, Sb) = 1/6 + 1/6 = 2/6 = 1/3$.

The probability that Saturn is ranked third = $P(I, Sb, St) + P(Sb, I, St) = 1/6 + 1/6 = 2/6 = 1/3$.

The probability that Saturn is ranked first and Infiniti is second = $P(St, I, Sb) = 1/6$.

3.20

a. $P(A^c) = P(E_3) + P(E_6) + P(E_8) = .20 + .30 + .03 = .53$ (corrected 2/16)
b. $P(B^c) = P(E_1) + P(E_7) + P(E_8) = .10 + .06 + .03 = .19$ (corrected 2/16)
c. $P(A^c \cap B) = P(E_3) + P(E_6) = .2 + .3 = .5$

d. $P(A \cup B) = P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) + P(E_7)$
$= .10 + .05 + .20 + .20 + .06 + .30 + .06 = .97$

e. $P(A \cap B) = P(E_2) + P(E_4) + P(E_5) = .05 + .20 + .06 = .31$

f. $P(B^c \cup B^c) = P(E_1) + P(E_7) + P(E_3) + P(E_6) + P(E_8) = .10 + .06 + .20 + .30 + 0.03 = .69$ (corrected 3/1)

g. No. A and B are mutually exclusive if $P(A \cap B) = 0$. Here, $P(A \cap B) = .31$.

Solutions to Supplementary Problems of Assignment 1

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Problem X.1

a)

$$\frac{\sum_{i=1}^{4} x_i}{\sum_{i=1}^{4} y_i} = \frac{7+10+0+14}{1+5-3+4} = \frac{31}{7} \approx 4.4286.$$

b)

$$\frac{\sum_{i=1}^{6} |x_i|}{|\sum_{i=1}^{4} y_i|} = \frac{|7|+|10|+|0|+|14|+|-2|+|2|}{|1+5-3+4|} = \frac{35}{7} = 5.$$

c)

$$\sum_{i=1}^{4} \frac{x_i}{y_i} = \frac{7}{1} + \frac{10}{5} + \frac{0}{-3} + \frac{14}{4} = \frac{25}{2} = 12.5.$$

Problem X.2

a)

$$\bar{x} = \frac{3+5+4+6+10+5+6+9+2+8}{10} = \frac{58}{10} = 5.8$$

b) We first rank these 10 measurements in ascending order: 2, 3, 4, 5, 5, 6, 6, 8, 9, 10. It is easy to calculate that $m = (5+6)/2 = 5.5$

c) 5 and 6 both appear twice–hence we have two modes: 5 and 6.

Problem X.3

a)

$$\bar{x} = \frac{4+1+3+1+3+1+2+2}{8} = \frac{17}{8} = 2.125.$$

b)

$$\begin{aligned} s^2 &= \frac{1}{n-1}\sum_{i=1}^{8}(x_i - \bar{x})^2 \\ &= \frac{1}{7}[(4-2.125)^2 + 3\times(1-2.125)^2 + 2\times(3-2.125)^2 + 2\times(2-2.125)^2] \\ &= \frac{71}{56} \approx 1.2678 \end{aligned}$$

c)

$$\begin{aligned}
s^2 &= \frac{1}{n-1}\left(\sum_{i=1}^{8} x_i^2 - n\bar{x}^2\right) \\
&= \frac{1}{7}[(16+1+9+1+9+1+4+4) - 8 \times 2.125^2] \\
&= \frac{71}{56} \approx 1.2678 \\
s &= \sqrt{\frac{71}{56}} \approx 1.126
\end{aligned}$$

The results of part (b) and (c) are identical.

<u>Problem X.4</u>

a)

$$\binom{8}{3} = \frac{8!}{3!(8-3)!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

b)

$$\binom{6}{4} = \frac{6!}{4!(6-4)!} = \frac{6 \times 5}{2 \times 1} = 15$$

c)

$$\binom{4}{0} = \frac{4!}{0!(4-0)!} = 1$$

d)

$$\binom{9}{8} = \frac{9!}{8!(9-8)!} = \frac{9}{1} = 9$$

<u>Problem X.5</u>

Define A to be the event that a car has been involved in an accident in the past year and B to be the event that a car has antilock brakes.

a) $P(A) = 0.03 + 0.12 = 0.15$

b) $P(\bar{A} \cap B) = 0.40$

c) $P(B|A) = P(A \cap B)/P(A) = 0.03/0.15 = 0.2$