Economics 310
Fall 2003
University of Wisconsin-Madison

Menzie D. Chinn
Social Sciences 7418

# Midterm Exam 2 Answers

This exam is 60 minutes long. There are **8** questions on the exam – be sure to check that you answer **all 8** questions. Points are allocated in proportion to the time allocated. Answer all questions in your bluebook. Make certain you write your name, your student ID number, and your TA's name on your bluebook.

Be sure to show your work, **"boxing in"** your final answer (don't "box-in" more than one answer!); partial credit will be awarded *for showing your work*. Note: there may be some fractions that cannot be simplified, but in general, you should be able to solve for exact numerical solutions using simple multiplication or long division.

1. (8 minutes) 6.3-5 (Situation 6.2) A local bank reported to the federal government that its 5,246 savings accounts have a mean balance of \$1,000 and a standard deviation of \$240. Government auditors have asked to randomly sample 64 of the banks accounts to assess the reliability of the mean balance reported by the bank. If the bank's information is correct in the situation above, find the probability that the sample mean balance would be less than \$1,060.

The sampling distribution of $\bar{x}$ has the following properties:
1. $\mu_{\bar{x}} = \mu = 1{,}000$
2. $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 240 / \sqrt{64} = 30$
3. The sampling distribution of $\bar{x}$ is approximately normally distributed.

$$P(\bar{x} < 1060) = P\left[\frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} < \frac{1060 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right]$$

$$= P\left[z < \frac{1060 - 1000}{30}\right] = P(z > 2.0)$$

= .5 + .4772
= .9772

2. (8 minutes) (7.3-2) It is desired to estimate the population mean to within 50 units with 90% reliability. The population standard deviation is estimated to be 250 units. What size sample should be selected? Note: The number of observations must be an integer. Hints: $(1.645)^2 = 2.71$; $(1.96)^2=3.84$; $(2.575)^2 = 6.63$.

To determine the sample size necessary to estimate μ we use $n = \left[\frac{z_{\alpha/2}}{B}\right]^2 \sigma^2$

For confidence coefficient .90 → 1 - α = .90 → α = 1 - .90 = .10. α/2 = .10/2 = .05. $z_{\alpha/2} = z_{.05}$ = 1.645.

$$n = \left[\frac{1.645}{50}\right]^2 250^2 = (1.645)^2\left(\frac{250}{50}\right)^2 = 2.71 \times 25 = 67.75$$

Round up to n = 68.

3. (7.3-3) (8 minutes) The U.S. Commission on Crime wishes to estimate the fraction of crimes related to firearms in an area with one of the highest crime rates in the country. The commission randomly selects 600 files of recently committed crimes in the area and finds 400 in which a firearm was reportedly used. Find a 90% confidence interval for p, the true fraction of crimes in the area in which some type of firearm was reportedly used.

Let p = the true fraction of crimes in the area in which some type of firearm was reportedly used.

$$\hat{p} = \left(\frac{400}{600}\right) = 0.6667, \hat{q} = 1 - \hat{p} = 1 - 0.6667 = 0.3333$$

The confidence interval for p is $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$.

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$$.6667 \pm 1.645\sqrt{\frac{.6667 \times .3333}{600}}$$

4. 4. (8 minutes) (8.3-8) (Situation 8.19) The Chronicle of Higher Education Almanac (Sept. 1990) reported that for the 1989-1990 academic year, 4-year private colleges charged students an average of \$8,446 for tuition and fees, whereas at 4-year public colleges the average was \$1,781. Suppose that for 1990-1991 a random sample of 25 colleges yielded the following data on tuition and fees: $\bar{x}$ = \$9,446 and *s* = \$2,000. Assume that \$8,446 is the population mean for 1989-1990. Calculate the test statistic for the test of hypothesis that the random sample comes from the same population as that of private colleges.

The test statistic for the test is $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \approx \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{9{,}446 - 8{,}446}{2{,}000 / \sqrt{25}} = 2.5$

5. (8 minutes) (8.3-19) (Situation 8.24) A company purchases large quantities of naphtha in 50-gallon drums. Because the purchases are ongoing, small shortages in the drums can represent a sizable loss to the company. Suppose the company samples the contents of n = 25 drums and tests the hypothesis that the average fill per 50-gallon drum is less than 50 gallons. The company provides the following summary statistics: $\bar{x}$ = 49.7 gallons and s = 1/3 gallon. Find and interpret the p-value for the test of interest to the company.

The test statistic for the test is $t = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \approx \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}} = \dfrac{49.7 - 50}{.33333 / \sqrt{25}} = -4.5$

The p-value is found using P(t < -4.5) with n - 1 = 20 - 1 = 19 degrees of freedom. Using the t-table, we find that p < .0005.

For any $\alpha < .0005$, there is sufficient evidence to reject $H_0$. There is sufficient evidence to indicate the average fill per drum is less than 50 gallons.

For these True/False/Explain questions, indicate your answer, and then explain in one or two sentences your answer, using equations or a graph is necessary.

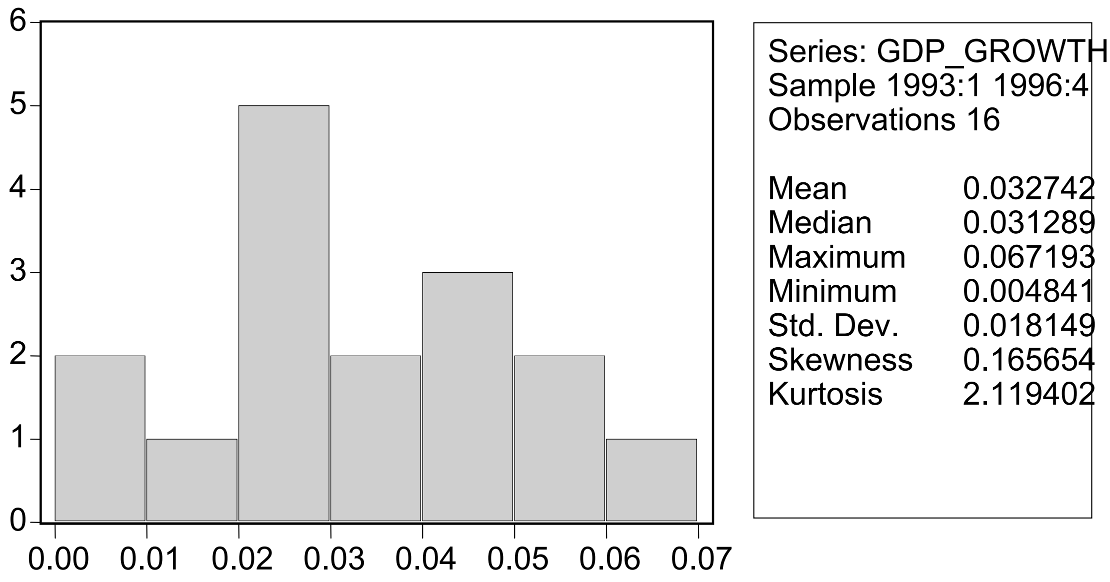
6. True/False/Explain. (5 points) “A 90% confidence interval is the interval bounded by two numbers for which one has a 90% probability of encompassing the true population parameter.”

False. A 90% confidence interval is a formula that tells one how to use sample data to calculate an interval that estimates a population parameter, and that encompasses the population parameter 90% of the time it is calculated.

7. True/False/Explain. (5 points) “The power of a test is equal to one minus the probability of Type I error.”

False. The power of a test is defined as 1-P(type II error)

8. (10 points total) Consider the following data regarding US GDP over the 1993q1 1996q4 period.




Where GDP_GROWTH is the annualized quarter-on-quarter growth of real GDP.

a. (3 minutes) Write out how you would calculate out the 95% confidence interval for annualized GDP growth.
b. (3 minutes) State any assumptions you need to make in order to calculating the confidence interval.
c. (4 minutes) Calculate the p-value for a two sided test if the null hypothesis is that output growth is 2.52% per annum (0.0252 in decimal form); show your work. (Hint: $0.0181/\sqrt{16} \approx 0.0096/2.13$).

a. $\bar{x} \pm t_{\alpha/2}\sigma_{\bar{x}} = 0.0327 \pm 2.131 \times (0.0181/\sqrt{16})$

b. Need to assume *x*'s normally distributed.

c. $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{0.0327 - 0.0252}{0.0181/\sqrt{16}} = \dfrac{0.0075}{0.00453} = 1.657$

Note $t_{.05} = 1.753$ for 15 d.f., and $t_{.100} = 1.341$ so the p-value is smaller than 0.20, but larger than 0.10 (closer to 0.10 than to 0.20).

7.4.2004