Economics 310
Fall 2003
University of Wisconsin-Madison

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Social Sciences 7418

Midterm Exam 1 Answers

This exam is 60 minutes long, although you will be given 70 minutes to complete it. Points are allocated in proportion to the time allocated. Answer all questions in your bluebook. Make certain you write your name, your student ID number, and your TA's name on your bluebook. Answers are in Arial. Highlighting indicates numerical answers you were not expected to get, since you did not have access to a calculator.

Be sure to show your work, "boxing in" your final answer; partial credit will be awarded.

1. (6 minutes) An investor believes the rate of return for on a proposed investment can be best described by a normal distribution with mean 10% and standard deviation 2%. What is the probability that the rate of return for the investment will be at least 7.5%?

   Let x be the internal rate of return. Then x is a normal random variable with μ = 10% and σ = 2%. To determine the probability that x is at least 7.5%, we need to find the z-value for x = 15.5%

   The z-score is z = $z = \frac{x-\mu}{\sigma} = \frac{7.5-10}{2} = -1.25$

   $P(x \geq 7.5\%) = P(z \geq -1.25) = .5000 + .3944$ (from Table IV)
   $= 0.8944$

2. (8 minutes) (Situation 3.17) A manufacturer of 35-mm cameras knows that a shipment of 30 cameras sent to a large discount store contains eight defective cameras. The manufacturer also knows that the store will choose two of the cameras at random, test them, and accept the shipment if neither is defective. Find the probability that both the cameras selected are defective.

   Let D_i = the event that camera i is defective.
   $P(D_1 \text{ and } D_2) = P(D_1) \times P(D_2 \mid D_1) = (8/30) \times (7/29)$ = .0644

3. (8 minutes) (sbe8 4.3-4) An automobile manufacturer has determined that 30% of all gas tanks that were installed on its 1988 compact model are defective. If 15 of these cars are independently sampled, what is the probability that more than half need new gas tanks?

   Let x = the number of the 15 cars with defective gas tanks.
   Then x is a binomial random variable with n = 15 and p = .30.

   $P(\text{more than half}) = P(x > 7.5) = P(x \geq 8) = 1 - p(x \leq 7)$
   $= 1 - .950 = .050$
   (from Table II in Appendix B)

4. (8 minutes) (Situation 4.7) The number of hurricanes that are formed during each hurricane season can be modeled using the Poisson probability distribution. History suggests that the average number of hurricanes formed during a year is 6.0 hurricanes. Find the probability of between three and seven hurricanes being formed during a randomly selected year.

Let x = number of hurricanes.
x is a Poisson random variable with $\lambda = 6.0$
$P(3 \le x \le 7) = P(x \le 7) - P(x \le 2) = .744 - .062 = .682$ (from Table III).

5. (12 minutes total) Consider this table of the probabilities of quarterly growth rates of GDP (*GDP_GROWTH*) and of payroll employment (*PAYEMPL_GROWTH*) (both annualized).

Tabulation of GDP_GROWTH and PAYEMPL_GROWTH
Date: 02/23/04 Time: 20:35
Sample: 1953:2 2003:4
Included observations: 203

Tabulation Summary

Variable	Categories
GDP_GROWTH	2
PAYEMPL_GROWTH	2
Product of Categories	4

Proportion of Total		PAYEMPL_GROWTH [-0.5, 0)	PAYEMPL_GROWTH [0, 0.5)	Total
GDP_GROWTH	[-0.2, 0)	0.0985	0.0591	0.1576
	[0, 0.2)	0.1133	0.7291	0.8424
	Total	0.2118	0.7882	1.0000

5.1 (2 minutes) What is the unconditional probability of negative payroll employment growth.

0.2118

5.2 (7 minutes) What is the conditional probability of negative payroll employment growth this quarter if this quarter's growth rate is positive?

Note that this table is in the same format as the Table handed out in the 2/4 lecture, except contemporaneous employment growth is used instead of lagged GDP growth, and that the probabilities are expressed in decimal form instead of percentages. Let dn be growth rate of employment, and dy be the growth rate of output.

$P(dn<0|dy>0) = P(dn<0 \cap dy>0)/P(dy>0)$
$= (0.1133/0.8424)$
$= 0.1345$

5.3 (3 minutes) Is the quarterly growth rate of GDP independent of the quarterly growth rate of payroll employment?

No, since according to calculation in part (a) and part (b), $P(dn<0|dy>0) \neq P(dn<0)$.

6. (4 minutes) In a pool of 6 students, 4 students are not US citizens. What is the probability of choosing 4 non-US citizen students out of the pool of 6 students?

# of combinations = $\binom{6}{4}$

$= 6!/(4!(6-4)!) = 6 \times 5/2 = 15$
since this combination of NNNN occurs only once, then the probability is 1/15.

7. (4 minutes) It is generally believed that the life length of a light bulb follows an exponential distribution. Suppose we know that the expected life length for a certain type of bulb is 1 year. What's the exact probability that such a bulb's life length exceeds 4 years?

To calculate this probability exactly, $P(x>4) = e^{(-4)} = 0.018$.

8. (10 minutes) At a psychiatric clinic the social workers are so busy that, on the average, only 60% of the potential new patients that telephone are able to talk immediately with a social worker when they call. The other 40% are asked to leave their phone numbers. About 75% of the time a social worker is able to return the call on the same day, and the other 25% of the time the caller is contacted on the following day. Experience at the clinic indicates that the probability a caller will actually visit the clinic for consultation is0.8 if the caller was able speak to a social worker, whereas it is 0.6 and 0.4, respectively, if the patient's call was returned the same day or the following day.

8.1 (5 minutes) What percentage of people that telephone visit the clinic for consultation?

8.1 Define the events V, I, S, F by

V: caller visits the clinic for consultation.
I: caller immediately speaks to a social worker.
S: caller is contacted later on the same day.
F: caller is contacted on the following day.

Then
$P(V)=P(V|I)P(I)+P(V|S)P(S)+P(V|F)P(F)$
$=(.8)(.6)+(.6)(.4)(.75)+(.4)(.4)(.25) = .70$

where we have used the facts that $P(S)=(.4)(.75)$ and $P(F)=(.4)(.25)$

8.2 (5 minutes) What percentage of patients that visit the clinic did not have to have their telephone calls returned?

$P(I|V)=P(V|I)P(I) / P(V)=(.8)(.6)/(.7) \approx .686$
Hence approximately 69% of the patients that visit the clinic had their phone call immediately answered by a social worker.

8.3.2004