

Midterm Exam 1

This exam is 60 minutes long, although you will be given 70 minutes to complete it. Points are allocated in proportion to the time allocated. Answer all questions in your bluebook. Make certain you write your name, your student ID number, and your TA's name on your bluebook.

Some expressions are provided below. Tables are also provided.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)(x-\mu)^2} \quad p(x) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} \quad p(x) = \binom{n}{x} p^x q^{n-x} \quad p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Be sure to show your work, "boxing in" your final answer; partial credit will be awarded.

1. [10 minutes] Consider two events A and B such that $P(A) = .4$ and $P(B) = .3$. Compute $P(A|B)$ for each of the following cases:
- when $P(A \cap B) = .1$
 - when $P(A \cup B) = .5$
 - when B is a subset of A
 - when events A and B are independent
 - when events A and B are mutually exclusive

$$(a) P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.1}{.3} = 1/3$$

(b) The definition of the union is: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. This implies that

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = .4 + .3 - .5 = .2$$

$$\text{By definition of conditional probability } P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.2}{.3} = 2/3$$

(c) If B is a subset of A , then A must occur when B occurs. This implies $P(A|B) = 1$

(d) If A and B are independent, $P(A \cap B) = P(A)P(B) = .12$

$$\text{By definition of conditional probability } P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.12}{.3} = .4$$

(e) By definition of mutually exclusive events $P(A \cap B) = 0$

By definition of conditional probability $P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{.3} = 0$

2. (6 minutes) As part of a promotion, both you and your roommate are given free cellular phones from a batch of 14 phones. Unknown to you, 5 of the phones are faulty and do not work. Find the expression for the probability that you and your roommate's phones are both faulty. Be sure to reduce the expression to the simplest form (do not leave !'s in the expressions).

$$p(x) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{5}{x} \binom{14-5}{2-x}}{\binom{14}{2}} = \frac{\binom{5}{x} \binom{9}{2-x}}{\binom{14}{2}}$$

P(both faulty phones) = P(x=2) = p(2)

$$p(2) = \frac{\binom{5}{2} \binom{9}{0}}{\binom{14}{2}} = \frac{\frac{5!}{2!3!} \times \frac{9!}{0!9!}}{\frac{14!}{2!12!}} = \frac{5 \times 4 \times 1}{14 \times 13}$$

3. [8 minutes] A new drug has been synthesized that is designed to reduce a person's blood pressure. Twelve randomly selected hypertensive patients receive the new drug. Suppose the probability that a hypertensive patient's blood pressure drops if he or she is untreated is 0.5. Then what is the probability of observing 10 or more blood pressure drops in a random sample of 12 treated patients if the new drug is in fact ineffective in reducing blood pressure? Note: $(0.5)^{12} = 0.000244$.

Let x = the number of the 12 hypertensive patients whose blood pressure drops.

Then X is a binomial random variable with n = 12 and p = .5.

$$P(x \geq 10) = P(x = 10) + P(x = 11) + P(x = 12)$$

$$= 0.019287$$

4. [4 minutes] The number of traffic accidents that occurs on a particular stretch of road during a month follows a Poisson distribution with a mean of 7.8. Find the probability that fewer than two accidents will occur on this stretch of road during a randomly selected month.

Let x = the number of accidents that occur on the stretch of road during a month.

Then x is a Poisson random variable with $\lambda = 7.8$.

$$P(x < 2) = P(x \leq 1) = 0.004 \text{ from Table III.}$$

5. [8 minutes total] Consider this table of the probabilities of annualized quarterly growth rates of GDP (*GDP_GROWTH*).

Tabulation of DY and DYLAG
 Date: 09/22/04 Time: 12:49
 Sample: 1967:1 2004:2
 Included observations: 150

Tabulation Summary

Variable	Categories
DY	2
DYLAG	2
Product of Categories	4

% Total		DYLAG		Total
		[-0.2, 0)	[0, 0.2)	
DY	[-0.2, 0)	0.0405	0.0946	0.1351
	[0, 0.2)	0.0946	0.7703	0.8649
Total		0.1351	0.8649	1.0000

- 5.1 (5 minutes) What is the conditional probability of negative output growth next quarter if this quarter's growth rate is negative?

Note that this table is in the same format as the Table handed out in the 9/22 lecture, and that the probabilities are expressed in decimal form instead of percentages.

$$\begin{aligned}
 P(dy[t+1]<0|dy[t]<0) &= P(dy[t]<0|dy[t-1]<0) \\
 &= P(dy[t]<0 \cap dy[t-1]<0)/P(dy[t-1]<0) \\
 &= (0.0405/0.1351) \\
 &= 0.2998
 \end{aligned}$$

- 5.2 (3 minutes) Is the quarterly growth rate of GDP independent of the previous quarter's growth rate?

No, since according to calculation in part (a) and part (b), $P(dy[t]<0|dy[t-1]<0) \neq P(dy[t]<0)$.

6. [4 minutes] In a set of 10 bunkers, 2 bunkers contain WMDs. Suppose 2 bunkers are selected at random. What is the probability of choosing 2 bunkers containing WMDs out of a pool of 10 bunkers?

$$\# \text{ of combinations} = \binom{10}{2}$$

$$= 10!/(2!(10-2)!) = (10 \times 9)/2 = 45$$

since this combination of WW occurs only once, then the probability is 1/45.

7. [10 minutes] The probability of exposure to the flu during an epidemic is .6. Experience has shown that the flu vaccine is 80% successful in preventing an inoculated person from acquiring the flu, if exposed to it. A person not inoculated faces a probability .9 of acquiring the flu if exposed to it. Two persons, one inoculated and one not, perform a highly specialized task in a business. Assume that they are not in the same location, are not in contact with the same people and cannot expose each other to the flu. What is the probability that at least one will get the flu?

Let 1 be the person not inoculated and 2 be the inoculated person. The information given in the problem implies that the probability of their becoming ill is independent.

$$\begin{aligned} P(\text{at least one gets flu}) &= 1 - P(\text{neither gets flu}) \\ &= P(1 \text{ does not get flu} \cap 2 \text{ does not get flu}) \\ &= P(1 \text{ does not get flu}) * P(2 \text{ does not get flu}) \\ &\quad \text{by independence} \end{aligned}$$

Now, we can calculate

$$\begin{aligned} P(1 \text{ does not get flu}) &= P(1 \text{ does not get flu} \cap \text{exposed}) + P(1 \text{ does not get flu} \cap \text{not exposed}) \\ &= P(1 \text{ does not get flu} | \text{exposed}) * P(\text{exposed}) + P(1 \text{ does not get flu} | \text{not exposed}) * P(\text{not exposed}) \\ &= (.1)(.6) + (1)(.4) \\ &= .46 \end{aligned}$$

Similarly,

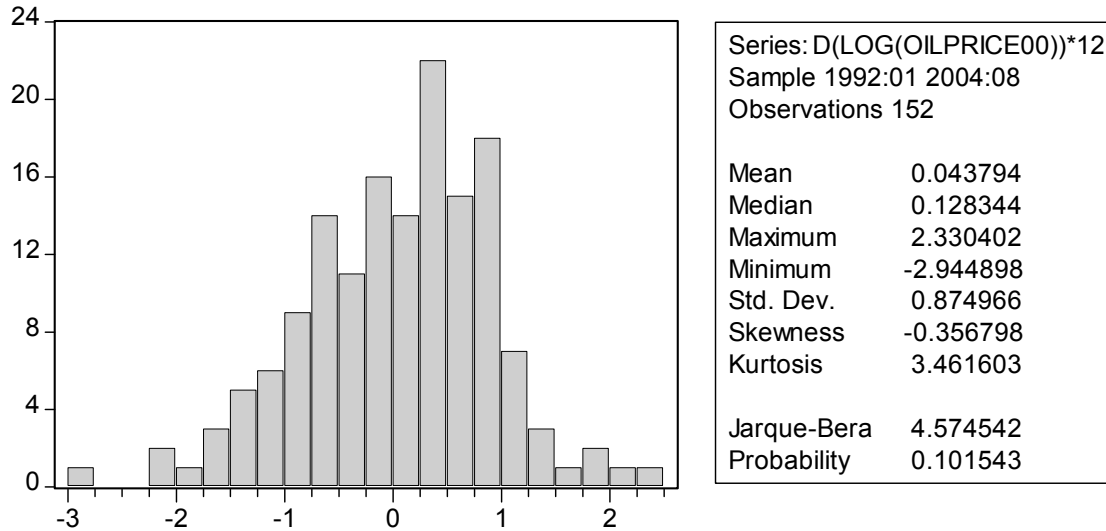
$$\begin{aligned} P(2 \text{ does not get flu}) &= P(2 \text{ does not get flu} \cap \text{exposed}) + P(2 \text{ does not get flu} \cap \text{not exposed}) \\ &= P(2 \text{ does not get flu} | \text{exposed}) * P(\text{exposed}) + P(2 \text{ does not get flu} | \text{not exposed}) * P(\text{not exposed}) \\ &= (.8)(.6) + (1)(.4) \\ &= .88 \end{aligned}$$

As shown above:

$$\begin{aligned} P(\text{at least one gets flu}) &= 1 - P(1 \text{ does not get flu}) * P(2 \text{ does not get flu}) \\ &= 1 - (.46)(.88) \\ &= .5952 \end{aligned}$$

This problem can also be solved by adding $P(\text{both get the flu}) + P(\text{only one gets the flu}) + P(\text{only two gets the flu})$, where these probabilities can be calculated in a similar manner to above.

8. [10 minutes total] Given the following histogram of month-on-month percent changes in oil prices:



- 8.1. (5 minutes) *Following the assumptions we made in lecture on 10/13 regarding the distribution of this random variable, find an algebraic expression for x_0 such that there is a 10% probability that oil price changes (in annualized percent) equal or exceed that x_0 . You do not need to solve this expression for x_0 .*

$$P(z > 1.28) = 0.10 \rightarrow 1.28 = (X_0 - 0.044) / 0.875$$

- 8.2. (5 minutes) The annualized month-on-month percent change in oil prices in August was 127%. In September, the price of oil hovered around \$50, compared to \$42.2 in August. This implied an (approximate) annualized month-on-month percent 200% increase. If the corresponding z-scores for August and September values are 1.40 and 2.25, and these changes can be treated as independent, calculate the likelihood of this event (up to three significant digits).

This question was mis-phrased, so that technically speaking, the answer is zero. Everyone was given credit for this part of the exam, as a consequence.

However, if the question were phrased as:

If the corresponding z-scores for August and September values are 1.40 or greater and 2.25 or greater, and these changes can be treated as independent, what is the likelihood of both these events occurring (up to three significant digits).

$$P(z > 1.40) \cap P(z > 2.25) = (0.5 - 0.4192)(0.5 - 0.4878) = (0.0808)(0.0122) = 0.0010$$