Economics 310
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University of Wisconsin-Madison

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Social Sciences 7418

Midterm Exam 2 Answer Key

Answer all questions in your bluebook. Make certain you write your name, your student ID number, and your TA's name on your bluebook.

The exam is written for a total of 70 minutes. Point allocations are proportional to time allocations. Partial credit will be awarded if the written material indicates understanding of how to answer the question (i.e., gibberish will not be given credit).

1. (9 minutes) The Environmental Protection Agency (EPA) estimated that the 1991 G-car obtains a mean of 35 miles per gallon on the highway, and the company that manufactures the car claims that it exceeds the EPA estimate in highway driving. To support its assertion, the company randomly selects 36 1991 G-cars and records the mileage obtained for each car over a driving course similar to that used by the EPA. The following data resulted: $\bar{x}$ = 37 miles per gallon, s = 6.0 miles per gallon. Find the observed significance level for testing H_0: $\mu = 35$ vs. H_a: $\mu > 35$.

The test statistic is

$$z = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{37 - 35}{6 / \sqrt{36}} = 2$$

The observed significance level for the test is p = P(z > 2) = .5 - P(0 < z < 2) = .5 - .4772 = .0228.

2. (10 minutes) Suppose a department store wishes to estimate μ, the average age of the customers of its contemporary apparel department, correct to within 2 years with probability equal to 0.95. Approximately how large a sample would be required if the estimated standard deviation of the customers' ages is 10 years? Solve explicitly for the sample size n.

To determine the sample size necessary to estimate μ, we use $n = \left(\frac{z_{\alpha/2}}{B} \right)^2 \sigma^2$.

For confidence coefficient $0.95 \rightarrow 1 - \alpha = 0.95 \rightarrow \alpha = 1 - .95 = 0.05.$
$\alpha / 2 = 0.05 / 2 = 0.025; z_{\alpha/2} = z_{.025} = 1.96$

$n = \left[\frac{1.96}{2} \right]^2 10^2 = 96.04$ [note: you could *easily* divide 1.96 by 2, then square, multiply by 100]

Round up to n = 97.

3. (10 minutes) The board of examiners that administers the real estate broker's examination in a certain state found that the mean score on the test was 435 and the

standard deviation was 72. If the board wants to set the passing score so that only the best 10% of all applicants pass, what is the passing score? Assume that the scores are normally distributed.

Let x be a score on this exam. Then x is a normally distributed random variable with $\mu = 435$ and $\sigma = 72$.
We want to find the value of x_0, such that $P(x > 0) = .10$. The z-score for the value $x = x_0$ is $z = (x_0 - \mu)/\sigma = (x_0 - 435)/72$

$P(x > x_0) = P(z > (x_0 - 435)/72) = .10$
From Table IV, we find $(x_0 - 435)/72 \approx 1.28$
$x_0 - 435 = 1.28(72)$; $x_0 = 435 + 1.28(72) = 527.16$

4. (Situation 7.16) (10 minutes) Following a year-long investigation, a congressional subcommittee concluded that inflated home appraisals are responsible, in part, for many defaulted home mortgages. One insurer sampled 400 defaulted home mortgages and found that 150 of them involved defective appraisals. Based on the information above, estimate the true proportion of all defaulted home mortgages that had defective appraisals using a 90% confidence interval.

Let p = the true proportion of defaulted home mortgages that had defective appraisals.

$\hat{p} = 150/400 = 0.375$ and $\hat{q} = 1 - \hat{p} = 1 - 0.375 = 0.625$

The confidence interval for p is $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$.

For confidence coefficient .90 → 1 - α = .90 → α = 1 - .90 = .10. α/2 = .10/2 = .05.
$z_{\alpha/2} = z_{.05} = 1.645$. The 90% confidence interval is:

$$0.375 \pm 1.645\sqrt{\frac{(150/400)(250/400)}{400}} = 0.375 \pm 1.645\frac{\sqrt{15 \times 25}}{4\sqrt{4}} \times (1/100)$$

(without a calculator, it is difficult to reduce the fractions further).

$$0.375 \pm 1.645\sqrt{\frac{(0.375)(0.625)}{400}} \rightarrow 0.375 \pm 0.040$$

→ (.335, .415)

5. (10 minutes) Electric power plants that use water for cooling their condensers sometimes discharge heated water into rivers, lakes or oceans. It is known that water heated above certain temperatures has a detrimental effect on the plant and animal life in the water. Suppose it is known that the increased temperature of the heated water discharged by a certain power plant on any given day has a distribution with a mean of 5° Centigrade and a standard deviation of 5° Centigrade. Suppose a sample of n = 49 days was collected.

In the situation above, what is the approximate probability that the average increase in temperature of the discharged water exceeds 6° C? (Hint: 7/5=1.4)

By the Central Limit Theorem, the sampling distribution of x is approximately normal with

$\mu_{\bar{x}} = \mu = 5$ and $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 5 / \sqrt{49} = 0.7143$

The z-score for $\bar{x} = 6$ is $(\bar{x} - \mu_{\bar{x}}) / \sigma_{\bar{x}} = (6-5) / 0.7143 = 1.4$

$P(\bar{x} > 6) = P(z > 1.4) \approx 0.5 - 0.4192 = 0.0808$

The following are True/False/Explain questions. To gain full credit, a correct explanation must be provided. The explanation should be no longer than one or two sentences and/or equations.

6. (True/False/Explain) (7 minutes) Using a random sample of 25 parents, Pascal wants to test the hypothesis: "The average number of children parents have is 1." To do this, he uses a T-test with the null hypothesis that the mean equals 1. This distribution is asymmetric and displays extreme rightward skew. Pascal's T-test is appropriate.

FALSE - T-tests, especially with small samples, are only valid if the underlying distribution is approximately normal.

7. (True/False/Explain) (7 minutes) Judicial principles are different in France than they are in the United States. In particular, trials in France follow the principle that the accused is "presumed guilty until proven innocent." In this context, convicting a person who is actually innocent would be a Type II error in a French trial.

TRUE - If H_0 = accused is guilty, then convicting an innocent person is accepting H_0 when H_0 is false. This is the definition of a type II error

8. (True/False/Explain) (7 minutes) To raise the confidence level of a confidence interval, we need to make the confidence interval narrower.

FALSE - If we want to increase the confidence level, we are going to have to include MORE potential outcomes in our interval - this means making the interval wider.