Economics 310
Fall 2003
University of Wisconsin-Madison

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Social Sciences 7418

Midterm Exam 1 Answer Key

Answer all questions in your bluebook. Make certain you write your name, your student ID number, and your TA's name on your bluebook.
Point allocations are proportional to time allocations.
Be sure to show your work; partial credit will be awarded.

Yellow text denotes answers that can be obtained without a calculator. Green text denotes specific numerical answers that you were not expected to obtain, and are provided for your information only.

1. (8 minutes) The customer service department of a large company monitors the incoming phone calls to determine how long a customer must wait on hold before receiving the assistance they need. The company claims that the average time on hold is 2 minutes and the standard deviation is .6 minutes. Would you expect to wait on hold for more than 5 minutes when calling the customer service department described above? Explain.

   The z-score is $z = \frac{x - \mu}{\sigma}$
   For a waiting time of 5 minutes, z = (5-2)/0.6 = 5.0
   A waiting time of 5 minutes falls 5 standard deviations above the mean, which is very unlikely to be observed. We would not expect to see a waiting time that is more than 5 minutes.

2. (8 minutes) The University Book Store received a late shipment of 25 statistics books from the publisher. Packaged with each of the books is a copy of the statistical software that is being used in the statistics classes. Unknown to the book store is that five of the disks shipped contained an error that will not allow the student access to the software. Suppose you and your friend both purchase books from the late shipment of 25. Find the probability that at least one of your disks contains the error.

   P(At least one disk contains the error)
   = 1 - P(both do not contain the error)
   = 1 - P(disk 1 does not contain the error)
   x P(disk 2 does not contain the error | disk 1 does not contain the error)
   = 1 – 20/25 x 19/24 = 1 – 380/600
   = 0.3667

3. (8 minutes) A local newspaper claims that 60% of the items advertised in its classified advertisement section are sold within 1 week of the first appearance of

the ad. To check the validity of the claim, the newspaper randomly selected n = 25 advertisements from last year's classified advertisements and contacted the people who placed the ads. They found that x = 13 of the 25 items sold within a week. Based on this claim, is it likely to observe $x \leq 13$ who sold their item within a week?

Let x = the number of the 25 ads that resulted in the item being sold within a week.
Then x is a binomial random variable with n = 25 and p = .60.
$P(x \leq 13) = 0.268$ (from Table II in Appendix B)

4. (8 minutes) A small life insurance company has determined that the variance of the death claims it receives is 5 per day. Find the probability that the company receives more than seven death claims on a randomly selected day, assuming the process follows one of the discrete random variables discussed in class.

Let x = the number of death claims received per day.
Then x is a Poisson random variable with $\lambda = 5$.
$P(x > 7) = 1 - P(x \leq 7) = 1 - 0.867 = 0.133$ (from Table III in Appendix B)

5. (8 minutes) In a certain community, 36 percent of the families own a dog, and 22 percent of the families that own a dog also own a cat. In addition, 30 percent of the families own a cat. What is the conditional probability that a randomly selected family owns a dog given that it owns a cat?

$P(Dog|Cat) = P(Dog \cap Cat)/P(Cat)$
But $P(Dog \cap Cat) = P(Dog) \times P(Cat|Dog) = 0.36 \times 0.22 = 0.0792$
$P(Dog|Cat) = 0.36 \times 0.22 / 0.30$
$= 0.264$

6. (12 minutes total) Consider this table of the probabilities of quarterly growth rates of GDP.

Tabulation of DY and DYLAG
Sample: 1967:1 2003:2
Included observations: 146

Tabulation Summary

Variable	Categories
DY	2
DYLAG	2
Product of Categories	4

		DYLAG		
		[-0.05, 0)	[0, 0.05)	Total
DY	[-0.05, 0)	0.05	0.10	0.15
	[0, 0.05)	0.10	0.75	0.85
	Total	0.15	0.85	1.00

a) (4 minutes) What is the probability of positive growth next quarter if this quarter's growth rate is negative?

Note that this table is in the same format as the Table handed out in the 9/24 lecture. The *only* changes are the values of the probabilities, and that the probabilities are expressed in decimal form instead of percentages.

$P(dy[t+1]>0|dy[t]<0) = P(dy[t]>0|dy[t-1]<0)$
$P(dy[t]>0|dy[t-1]<0) = P(dy[t]>0 \cap dy[t-1]<0)/P(dy[t-1]<0)$
$= (0.10/0.15)$
$= 2/3$

b) (8 minutes) What is the probability of the growth rate being positive both next quarter and the quarter thereafter, if this quarter's growth rate is negative?

$P(dy[t+2]>0|dy[t+1]>0)\times P(dy[t+1]>0|dy[t]<0) = ?$

$P(dy[t+2]>0|dy[t+1]>0) = P(dy[t]>0|dy[t-1]>0) =$
$= P(dy[t]>0 \cap dy[t-1]>0)/P(dy[t-1]>0) = 0.75/0.85$

Hence

$P(dy[t+2]>0|dy[t+1]>0)\times P(dy[t+1]>0|dy[t]<0) = (.75/.85)(2/3) = 0.5582$

7. (8 minutes) Of eight missiles produced at a particular factory, test runs revealed four of them to be defective. Nevertheless, four of the eight missiles were shipped to country A and the other four to country B. Country A received all four defective missiles. What is the probability of this event occurring if, in fact, the missiles were shipped randomly? *Reduce the expression to the simplest fraction.*

# of combinations = $\binom{8}{4}$

$= 8!/(4!(8-4)!) = 8\times7\times6\times5/4\times3\times2 = 70$
since this combination of FFFF occurs only once, then the probability is 1/70.

14.10.2003 corrected typo 15.10.2003