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Fall 2010 Social Sciences 7418
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Problem Set 1 Answers

Due ***in lecture*** on Monday, September 27th. Be sure to put your name on your problem set. Put "boxes" around your answers to the algebraic questions.

1. Chain-Weighting
Suppose that the agrarian economy of Simpsonia consists only of two sectors: private consumption and private investment. The following figures give total production and prices for both sectors in 2050 and 2051. The base year is 2050

CONSUMPTION

	POTATOES		**RICE**	
	Quantity	**Price**	**Quantity**	**Price**
2050	100	$3	150	$5
2051	100	$4	400	$1

INVESTMENT

	TRACTORS		**SHOVELS**	
	Quantity	**Price**	**Quantity**	**Price**
2050	4	$100	13	$25
2051	5	$130	15	$30

1.1 Calculate nominal consumption, investment and GDP for 2050 and 2051.

Answer: Nominal consumption for 2050 is
(100 potatoes)*$3 + (150 rice)*$5 = $1050
Nominal consumption for 2051 is:
(100 potatoes)*$4 + (400 rice)*$1 = $800
Using similar calculations, nominal investment is $725 in 2050 and $1100 in 2051. Nominal GDP, therefore, is $1775 in 2050 and $1900 in 2051.

1.2 Using the traditional method, calculate real consumption for 2051.

The traditional method values 2051 production at 2050 prices (the base year) to get a time-comparable real measure. So, 2051 real consumption:
(100 potatoes)*$3 + (400 rice)*$5 = $2300

1.3 Using the traditional method, calculate real investment for 2051.

In the same way as 2.2, 2051 real investment is $875.

1.4 Using the traditional method, calculate real GDP for 2051.

Real GDP is everything in 2051 production, but valued at 2050 prices. This is $3175, and is equal to the sum of the two components of real GDP.

1.5 Does 2051 real GDP equal the sum of real consumption and real investment in 2051 when calculated using the traditional method?

Yes.

1.6 Using the chain-weighted method, calculate real consumption in 2051.

Real quantity of potatoes grew 0% between 2050 and 2051. Real quantity of rice grew (400-150)/150 = 1.67, or 167% between 2050 and 2051.
Now, using 2051 prices, $800 was spent on consumption in 2051; of this $400 was on potatoes and $400 on rice. So, potatoes represent proportion .5 of expenditures, as does rice. The weighted average growth rate (where weights are determined by expenditure shares) is:

=weight_potatoes*growth_potatoes + weight_rice*growth_rice
=.5*0 + .5*1.67 = .835

So, cumulated real consumption in 2051 using the chain-weighted method is an 83.5% increase over real consumption in 2050 (note that real 2050 consumption equals nominal 2050 consumption from (a), since 2050 is the base year):

Real_2051 = $1050*(1+.835) = $1926.75

1.7 Using the chain-weighted method, calculate real investment in 2051.

Using similar methods to 2.6, quantity of tractors grew 25% and shovels grew 15.4% between 2050 and 2051. $1100 was spent on investment in 2051. Of this, $650 – 59.1% - was on tractors, and the other 40.9% was spent on shovels. Therefore, the weighted average growth rate is:

=weight_tactors*growth_tractors + weight_shovels*growth_shovels
=.591*.25 + .409*.154 = .211

Consequently, cumulated real investment in 2051 is 21.1% higher than in 2050:

Real_2051 = $725*(1+.211) = $877.98

1.8 Using the chain-weighted method, calculate real GDP in 2051 (note: develop weights for *all four* goods and take a weighted average of the growth rates).

If we look at the whole economy in 2051 ($1900), potatoes account for $400 – or 21.1%, rice accounts for 21.1%, tractors for 34.2% and shovels for 23.7%.
Using the growth rates for each good from 2.6 and 2.7, the weighted average growth rate for the whole economy (all 4 goods) is:

=.211*0 + .211*1.67 + .342*.25 + .237*.154 = .474

So, real GDP in 2051 is 47.4% higher than in 2050, using the chain weighted method.

Real_2051 = 1775(1+.474) = $2616.35

1.9 Does 2051 real GDP equal the sum of real consumption and real investment in 2051 when using the chain-weighted method? Explain why or why not.

Notice that if we sum chain-weighted real consumption and real investment, calculated separately for each sector in 2.6 and 2.7, we get $2804.73. This is not the same as the answer we got by chain-weighting the whole economy in 2.8.

This is one caveat when using figures adjusted for inflation with chain-weighted methods. Chain-weighted GDP does NOT generally equal the sum of the chain-weighted figures for each sector separately. However, this property IS true for real GDP calculated the traditional way (see 2.4). Why is this? Unlike the traditional method, where goods are aggregated in a linear way, the aggregation across goods for chain-weighted GDP is nonlinear. Consequently, there is no reason to expect it to be additively separable across sectors.

2. This problem requires obtaining data from various sources. You can access the latest GDP data from the BEA at http://www.bea.gov/newsreleases/national/gdp/2010/pdf/gdp2q10_2nd.pdf (August 2010 release). The Consumer Price Index figures can be obtained from the St. Louis Fed website at
http://research.stlouisfed.org/fred2/series/CPIAUCSL?cid=9 and
http://research.stlouisfed.org/fred2/series/CPILFESL?cid=9
.
2.1. Calculate the *annualized* quarterly growth rate of real GDP in each of the last four quarters. Is the economy expanding or contracting? **Show your work!**

Here is the data from Table 3:

Billions of chained (2005) dollars					
	Seasonally adjusted at annual rates				
2009	2009			2010	
	II	III	IV	I	II r
12,880.6	12,810.0	12,860.8	13,019.0	13,138.8	13,191.5

Use the formula:

$$annualizedgrowthrate = \left(\frac{Z_t}{Z_{t-1}}\right)^4 - 1$$

$$\left(\frac{12860.8}{12810.0}\right)^4 - 1 = 0.01596 \Rightarrow 1.6\%$$

$$\left(\frac{13019.0}{12860.6}\right)^4 - 1 = 0.05012 \Rightarrow 5.0\%$$

$$\left(\frac{13138.8}{13019.0}\right)^4 - 1 = 0.03732 \Rightarrow 3.7\%$$

$$\left(\frac{13191.5}{13138.8}\right)^4 - 1 = 0.01614 \Rightarrow 1.6\%$$

which matches up with the reported statistics in Table 1:

)09		2010	
III	IV	I r	II r
1.6	5.0	3.7	1.6

2.2. Calculate the annual rate of change of the GDP deflator, and the Personal Consumption Expenditure deflator, from the second quarter of 2010 to the second quarter of 2010. **Show your work!** Are they the same value?

From Table 5 of the GDP release:

	2007	2008	2009	Seasonally adjusted				
				2009			2010	
				II	III	IV	I	II r
Gross domestic product	104.672	104.672	101.917	101.358	101.760	103.012	103.960	104.376
Personal consumption expenditures	105.335	105.057	103.797	103.379	103.885	104.126	104.608	105.117

Use the formula:

$$annual\_growth\_rate = \left(\frac{Z_t}{Z_{t-4}}\right) - 1$$

For GDP price index,

$$\left(\frac{104.376}{101.358}\right) - 1 = 0.02978 \Rightarrow 3.0\%$$

For the Personal Consumption expenditures price index,

$$\left(\frac{105.117}{103.379}\right) - 1 = 0.01681 \Rightarrow 1.7\%$$

They are not the same value. In general the two inflation rates will differ since the baskets of goods and services differ.

2.3 Calculate the annual rate of change in the Consumer Price Index - All, and the Consumer Price Index excluding food and energy, from July 2009 to July 2010 (using seasonally adjusted data). **Show your work!** Are the rates identical?

obs	CPI-ALL	CPI-CORE
2009M06	214.5580	219.2650
2009M07	**214.7740**	**219.5330**
2009M08	215.5660	219.6870
2009M09	215.9110	220.0350
2009M10	216.3570	220.4590
2009M11	216.8590	220.5460
2009M12	217.2240	220.7640
2010M01	217.5870	220.4630
2010M02	217.5910	220.5790
2010M03	217.7290	220.6640
2010M04	217.5790	220.7680
2010M05	217.2240	221.0370
2010M06	216.9290	221.3880
2010M07	**217.5970**	**221.6760**
2010M08	218.1500	221.7790

For CPI-All,

$$\left(\frac{217.597}{214.774}\right) - 1 = 0.01314 \Rightarrow 1.3\%$$

For the Core CPI,

$$\left(\frac{221.676}{219.533}\right) - 1 = 0.00976 \Rightarrow 1.0\%$$

3. Consider the following economy.

Eq.No.	Equation	Description
(1)	$Y = AD$	Output equals aggregate demand, an equilibrium condition
(2)	$AD = C + I + G + X$	Definition of aggregate demand
(3)	$C = a_o + bY_d$	Consumption function, a_0 = 1200, b = 0.8
(4)	$Y_d \equiv Y - T$	Definition of disposable income
(5)	$T = TA_0 + tY$	Tax function; TA_0 = -800, t = 0.15
(6)	$I = IN_0$	Investment function, IN_0 = 1000
(7)	$G = GO_0$	Government spending, GO_0 = 1000
(8)	$X = g_0$	Net Exports, g_0 = 200

3.1 Express, in algebraic symbols, the equilibrium level of income ($\boldsymbol{Y_0}$) in this economy. Show your work.

$Y = AD = C + I + G + X$; substitute in for C, I, G, X

$Y = a_0 + bY_d + IN_0 + GO_0 + g_0$; substitute in for Y_d

$Y = a_0 + b(Y - T) + IN_0 + GO_0 + g_0$; substitute in for tax, transfers functions

$Y = a_0 + b(Y - TA_0 - tY) + IN_0 + GO_0 + g_0$; bring the "$Y$" terms to left hand side.

$Y - b(Y - tY)\tilde{}\ Y(1-b(1-t)) = a_0 - bTA_0 + IN_0 + GO_0 + g_0$

divide both sides by $(1-b(1-t))$, let $A_0\tilde{}\ a_0 - bTA_0 + IN_0 + GO_0 + g_0$

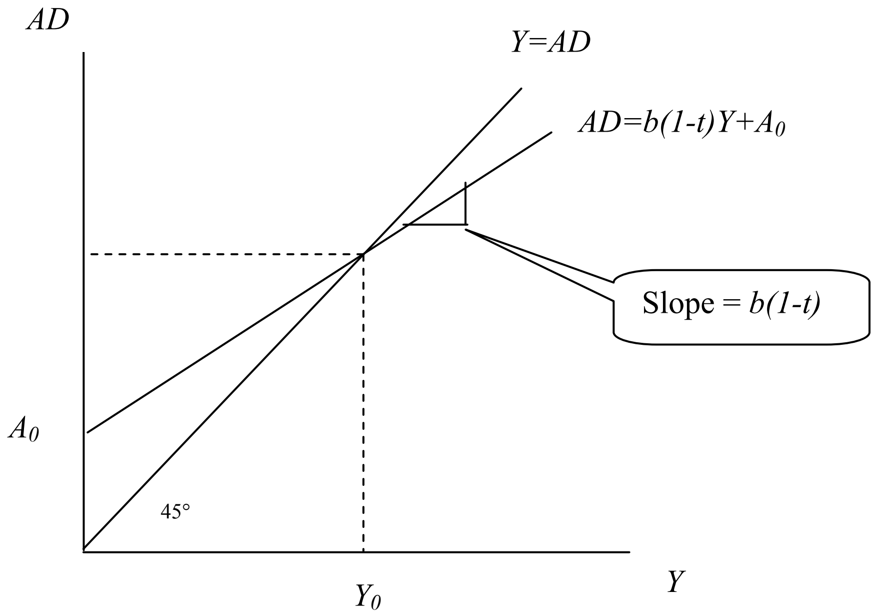
$\boxed{Y_0 = \overline{\alpha} A_0}$ let $\overline{\alpha} = \dfrac{1}{[1 - b(1-t)]}$

3.2. Substituting in the numerical values given above, indicate the numerical value of equilibrium income (in this and future subsequent numerical answers, round off your answer at two decimal places).

$$Y_0 = \left(\frac{1}{1 - 0.8(1 - 0.15)}\right) \times [1200 - (0.8) \times (-800) + 1000 + 1000 + 200]$$

$$\boxed{Y_0 = 12625}$$

3.3. Using the Keynesian Cross diagram, illustrate your answer in part (4.1), with all relevant curves, intercepts and slopes indicated clearly.



3.4. Once again, using algebraic symbols, calculate the government spending multiplier in this economy? What is the government *transfers* multiplier (recall that a government transfer is the opposite of taxes)? Why are they different?

Recall that equilibrium income is:

$$Y_0 = \overline{\alpha} A_0$$

Taking the total differential yields:

$$\Delta Y = \overline{\alpha}[\Delta a - b\Delta TA + \Delta IN + \Delta GO + \Delta g]$$

To obtain the government spending multiplier, set all the Δ terms, except for ΔGO, equal to zero, and solve for $\Delta Y/\Delta GO$). To obtain the government transfers multiplier, recall that transfers are the negative of taxes. Set all the Δ terms, except for ΔTA, equal to zero, and solve for $\Delta Y/\Delta TA$. This yields:

$$\boxed{\begin{aligned} &\Delta Y = \overline{\alpha}\Delta GO \Rightarrow \frac{\Delta Y}{\Delta GO} = \overline{\alpha} \\ &\Delta Y = -\overline{\alpha} b \Delta TA \Rightarrow \frac{\Delta Y}{\Delta TA} = -b\overline{\alpha} \end{aligned}}$$

so that the government transfers multiplier is αc1. To express this mathematically, define lump sum component $TA_0 \equiv TX_0 - TR_0$, where TX is lump sum taxes and TR is lump sum transfers. Then

$$\boxed{\Delta Y = \overline{\alpha} b \Delta TR \Rightarrow \frac{\Delta Y}{\Delta TR} = b\overline{\alpha}}$$

The two multipliers are different because in the case of the government spending multiplier, the initial spending (on goods and services) is immediately counted in GDP. However, in the case of government transfers, the initial effect is to increase income, of which only *b(1-t)* is directed to consumption spending, which is then counted as part of GDP.

3.5. Using the answer to part (4.2), what is the level of consumption spending in this economy?

Recall ***C = a₀ + b(Y - T)*** = ***a₀ + b(1-t)Y – bTA₀*** = 1200 + (.8)(.85)(12625) + .8(800); hence,

$$\boxed{C_0 = 10425}$$

3.6 If the level of investment spending were to fall to 800, what would be the equilibrium level of income?

There are two ways of solving this. One way is to just substitute the value of ***I*** = 800, instead of 1000, into the expression for part 4.2. The other way is to recognize that the multiplier for changes in autonomous investment, *viz.*, **ΔY/ΔIN**, is $\overline{\alpha}$, so the new income level is:

$$Y_1 = Y_0 + \overline{\alpha}\Delta IN = 12625 + 3.125 \times (800 - 1000)$$

$$\boxed{Y_1 = 12000}$$

4. Using the same economy as described in question 4, answer the following, given that the budget surplus is:

$$BuS \equiv T - G = TA_0 + tY - GO_0$$

Assuming there is no government debt.

4.1. What is the value of the budget surplus when investment spending is 1000?

BuS **≡** *(T - G)*; substituting in the functions given in question 4, one finds:

$$BuS = \overline{TA} + tY - \overline{GO} = -800 + 0.15 \times 12625 - 1000$$

$$\boxed{BuS_0 = 93.75}$$

4.2. What is the budget surplus when *I* falls to IN_1 = 800?

The new income level corresponding to I = 800 is 12000 (from 4.6 above):

$$BuS_1 = -800 + 0.15 \times 12000 - 1000$$

$$\boxed{BuS_1 = 0}$$

4.3. What accounts for the change in the budget surplus from part (4.1) to (4.2)?

The budget surplus is endogenous. When income changes (in this case because investment spending changes) then tax receipts fall, and so too does the budget surplus. In symbols:

$$\Delta BuS = \Delta TA + t(\Delta Y) - \Delta GO = t(\overline{\alpha}\Delta IN)$$

4.4. Suppose potential GDP (or "full-employment GDP") Y^* is 13000. What is the full-employment, or structural, budget surplus, *BuS**, when *I* = 1000? 800?

$BuS^* = TA_0 + tY^* - GO_0$ Substituting in the given numerical values, one finds:

$$BuS_0^* = -800 + 0.15 \times 13000 - 1000$$

$$\boxed{BuS_0^* = 150}$$

4.5. Can you write out what the *BuS* depends upon, *algebraically* (i.e., using the symbols rather than the numbers)? What variables affect *BuS*? What variables affect the full-employment budget surplus, *Bus**?

This is answered in parts 4.3 and 4.4 above. BuS depends on TA_0, GO_0, Y, while BuS* depends on TA_0, GO_0, Y^*.

5. Suppose the government spending function is different: $G = GO_0 - \theta Y$ where θ is a parameter. This means that as the economy grows, government spending on goods and services (such building roads and buying tanks) decline. (For purposes of answering the below questions, assume the rest of the economy is the same as in question 4.)

5.1. Solve out for equilibrium income using algebraic symbols.

$Y = AD = C + I + G + X$; substitute in for C, I, G, X

$Y = a_0 + bY_d + IN_0 + GO_0 - \theta Y + g_0$; substitute in for Y_d

$Y = a_0 + b(Y - T) + IN_0 + GO_0 - \theta Y + g_0$; substitute in for tax, transfers functions

$Y = a_0 + b(Y - TA_0 - tY) + IN_0 + GO_0 - \theta Y + g_0$; bring the "Y" terms to left hand side.

$Y(1-b(1-t)+\theta) = a_0 - bTA_0 + IN_0 + GO_0 + g_0$

divide both sides by $(1-b(1-t))$, let $A_0 \tilde{} \; a_0 - bTA_0 + IN_0 + GO_0 + g_0$

$$\boxed{Y_0 = \tilde{\alpha}A_0} \quad \text{let} \quad \tilde{\alpha} = \frac{1}{[1 - b(1-t) + \theta]} < \overline{\alpha}$$

5.2. What is the new government spending multiplier, $\Delta Y/\Delta GO$ (algebraically)?

$$\boxed{\frac{\Delta Y}{\Delta GO} = \tilde{\alpha} = \frac{1}{[1 - b(1-t) + \theta]}}$$

5.3. Why is the new multiplier less than the standard one, intuitively?

The new multiplier is less than the standard one, because at each stage of income augmentation-spending, instead of an additional respending of $b(1-t)$ dollars of each incremental dollar, there is $b(1-t)-\theta$ dollars of respending. That's because not only is the government taxing away t proportion of each dollar income, it's also spending θ proportion less on final goods and services.

5.4. Substituting in the parameter values, what is the numerical value of the multiplier for θ = 0.10?

If θ = 0.10, then

$$\boxed{\tilde{\alpha} = \left(\frac{1}{1-0.8(1-0.15)+0.10}\right) \approx 2.38}$$

5.5. In this new economy, what are (i) the parameters; (ii) the exogenous variables; (iii) the endogenous variables?

In this new economy:
(i) the parameters are a_0, b, t, θ, (and TA, GO)
(ii) Exogenous variables are I, Y^* (and BuS^*)
(iii) Endogenous variables are Y, AD, C, Y_d, T, G (and BuS).

6. National savings identity and the Keynesian Model

Suppose equation 8 in the model in problem (4) looks like:

(8') $X = g_0 - mY$ Net Exports

6.1. Solve for the impact of a (lump sum) tax increase on the trade balance or net exports, algebraically.

Taking the total differential,

$$\Delta X = \Delta g - m\Delta Y$$

But one knows that the change in income resulting from a change in lump sum taxes is given by:

$$\Delta Y = -\overline{\alpha} b \Delta TA$$

Substituting this into the first expression yields:

$$\Delta X = -m \times (-\overline{\alpha} b \Delta TA)$$

$$\boxed{\frac{\Delta X}{\Delta TA} = m\overline{\alpha} b > 0}$$

6.2. Using the definition of the budget surplus in problem 5, solve for the impact of a tax increase on the budget balance, algebraically.

$\Delta BuS = \Delta TA + t(\Delta Y) - \Delta GO$ holding government spending constant.

$$\Delta BuS = \Delta TA + t(-\overline{\alpha} b \Delta TA) = (1 - \overline{\alpha} bt)\Delta TA$$

$$\boxed{\frac{\Delta BuS}{\Delta TA} = (1 - \overline{\alpha} bt) > 0}$$

6.3. Will the budget and trade balances move in the same direction in response to a tax increase?

Yes, since both the trade balance and the budget balance move in the same direction in response to a (lump sum) tax increase.

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