1. Lucas Supply Curve

Suppose a single representative firm has a supply curve of the following form:

\[ Y_i = h(P_i - P) + Y^*_i \quad (15.1) \]

Where \( h > 0 \), as shown in Figure 15.1, at left.

Suppose firms don’t know exactly what the price level will be; then they guess:

\[ Y_i = h(P_i - P^e) + Y^*_i \quad (15.2) \]

How do firms “guess”? They use historical correlations between the average price level and their own firm’s price:

\[ P^e = \hat{P} + \hat{b}(P_i - \hat{P}) \quad (15.3) \]

Where \( \hat{b} \) is the regression coefficient of past price level on past own price level, \( \hat{b} = 0 \) implies the firm perceives all changes in own-firm-goods-price as changes in relative prices; \( \hat{b} = 1 \) implies the firm think relative prices never change when its own-firm-goods-price changes.

Substitute (15.3) into (15.1):

\[ Y_i = h(P_i - [\hat{P} - \hat{b}(P_i - \hat{P})]) + Y^*_i \quad (15.4) \], rearranging

\[ Y_i = h(1 - \hat{b})(P_i - \hat{P}) + Y^*_i \quad (15.5) \]

Aggregate over all \( n \) firms:

\[ Y = nh(1 - \hat{b})(P - \hat{P}) + Y^* \quad (15.6) \]

Invert solving for \( P \):

\[ P = \hat{P} + \hat{c}(Y - Y^*) \text{ where } \hat{c} \equiv \frac{1}{nh(1 - \hat{b})} \quad (15.7) \]

To examine monetary policy, assume a simplified AD equation: \( Y = \hat{k}_0 + \hat{k}_1(M - P) \quad (15.8) \)
If the increase in money is a surprise, the output will rise as firms mistakenly believe the increase in their own goods prices reflect largely (depending on $\tilde{b}$) the change in relative prices of their own firm’s goods price. After the fact, they will realize that the price level rose, rather than their own goods price. (Notice that if $\tilde{b} = 1$ then $\tilde{c} = 0$, and the slope of the Lucas Supply curve is infinite.)

However, over time, if firms think the central bank will try the same trick, then next time around they may push up $\hat{P}$ in proportion to what they think the central bank will do. If they guess correctly, the Lucas supply curve will shift up, resulting – on average – in no increase in output. To the extent that the firms mis-guess, output is either slightly above or below $Y^*$.

2. Staggered Wage Setting

Prices are sticky, partly due to the nature of nominal wage contracts.

Let’s formalize the effect of this phenomenon:

$$W_t = (1/2)(X_t + X_{t-1}) \quad (15.9)$$
What does the staggering of contracts imply? When workers and firms negotiate their multiperiod contracts, they will take into account the future:

\[ X_t = \frac{1}{2}(W_t + W_{t+1}^e) - \left( \frac{\tilde{d}}{2} \right) \left[ (U_t - U^*) + (U_{t+1}^e - U^*) \right] \] (15.10)

Rewrite (15.10) substituting in (15.9):

\[ X_t = \frac{1}{2}\left[ \frac{1}{2}(X_t + X_{t-1}) + \frac{1}{2}(X_{t+1}^e + X_t) \right] - \left( \frac{\tilde{d}}{2} \right) \left[ (U_t - U^*) + (U_{t+1}^e - U^*) \right] \] (15.11)

Solving for \( X_t \):

\[ X_t = \frac{1}{2}(X_{t-1} + X_{t+1}^e) - \tilde{d}\left[ (U_t - U^*) + (U_{t+1}^e - U^*) \right] \] (15.12)

Notice that the firm negotiates contract wages as a function of past contract wages and expected future contract wages. In addition future, as well as current, unemployment matters.

In steady state, with expected inflation equaling actual, and the change in the contract wage equal to 10, then (15.12) becomes:

\[ \frac{1}{2}(X_t - X_{t-1}) = \frac{1}{2}(X_{t+1}^e - X_t) - \tilde{d}\left[ (U_t - U^*) + (U_{t+1}^e - U^*) \right] \]

\[ \frac{1}{2} \times 10 = \frac{1}{2}(10) - \tilde{d} \times [0] \] (15.13)

Notice this relationship, in combination with price-cost markup, is approximately equal to:

\[ \frac{\Delta W_t}{W_{t-1}} \approx \pi_t = fY_{t-1} + \pi_t^e + Z_t \] (15.14)
Where $\hat{Y}_t \equiv \left( \frac{Y_t - Y^*}{Y^*} \right)$, (a.k.a., the “output gap”).

Where does price-cost markup come from? In general, profit maximization, combined with downward sloping demand curves, leads to:

$$P = \left( \frac{e}{e - I} \right) MC = \lambda MC$$  \hspace{1cm} (1)$$

Hence the price-cost markup is a function of the demand curve elasticity. Notice what happens if $e$ is infinite; and if $e$ is substantially less than infinite.

Now let the following describe marginal cost,

$$MC = \left[ \left( \frac{W}{APL} \right)^r \times P_{input}^{\lambda} \right]$$  \hspace{1cm} (2)$$

and think of this firm representing the whole economy: Then (2) can be rewritten as:

$$P = \lambda \left[ \left( \frac{W}{APL} \right)^r \times P_{input}^{\lambda} \right]$$ \hspace{1cm} (3)$$

where $APL$ is average labor productivity, $Q/N$. Log this equation (and add time subscripts),

$$p_t = \ln(\lambda) + \left[ \Gamma w_t \Gamma apl_t + (1 - \Gamma) P_{input,t} \right]$$ \hspace{1cm} (4)$$

Taking the derivative with respect to time, and holding constant the price-cost markup and $APL$ yields:

$$\pi_t = \Gamma(\Delta W_t/W_{t-1}) + (1 - \Gamma) \pi_{input,t}$$
\hspace{1cm} f(\hat{Y}_{t-1}) + \pi_t + \pi_t^e + Z_t$$

where

$$\Gamma(\Delta W_t/W_{t-1}) \equiv f(\hat{Y}_{t-1}) + \pi_t^e$$
(1 - \Gamma) \pi_{input,t} \equiv Z_t$$ \hspace{1cm} (5)$$

Notice that in (5), the familiar equation one obtains from the textbook requires several assumptions, including a constant price-cost markup, and constant $APL$. 

E302_staggered_f10  4.12.2010