## Notes on Open Economy Macroeconomics

## 1. Model Description

### 1.1 Algebraic Approach

Open economy macroeconomics can be analyzed by incorporating several changes in the closed economy IS-LM framework. First redefine aggregate demand, which in the closed economy looks like this:
$Y=\alpha[A-d R]$
$R=\left(\frac{\mu_{0}}{h}\right)-\left(\frac{1}{h}\right)\left(\frac{M_{0}}{P}\right)+\left(\frac{k}{h}\right) Y$
$<$ LM $>$

Where

$$
\begin{aligned}
& A_{0} \equiv a_{0}+e_{0}+G O_{0}-b T A_{0} \\
& \alpha \equiv \frac{1}{1-b(1-t)}
\end{aligned}
$$

The IS equation must be revised to include net exports:

$$
A D \equiv C+I+G+X \quad<\text { Open Economy } A D>
$$

Where net exports behave as follows (equation 12.1):

$$
\begin{equation*}
X=g_{0}-m Y-n\left(\frac{E P}{P_{w}}\right) \tag{1}
\end{equation*}
$$

Solving for Y yields a revised IS equation:

$$
\begin{gather*}
Y=\bar{\alpha}\left[\widetilde{A}_{0}-d R-n\left(\frac{E P}{P_{w}}\right)\right] \quad<\text { Revised IS }>  \tag{0’}\\
\bar{\alpha} \equiv \frac{1}{1-b(1-t)+m} ; \widetilde{A}_{0} \equiv a_{0}+e_{0}+G O_{0}-b\left(T A_{0}\right)+g_{0}
\end{gather*}
$$

(The LM equation remains the same.)
In this interpretation of the IS curve, the real exchange rate $\left(E P / P_{w}\right)$ is a shift variable. It is sometimes more convenient to express this real exchange rate effect in terms of R, since the exchange rate actually depends upon the interest rate.

Unfortunately, this exchange rate - interest rate relationship is quite complicated. It is derived in the following manner:

First assume "uncovered interest parity" (UIP), the condition that the interest differential equals the expected rate of depreciation.

$$
\begin{equation*}
R-R_{w}=-\frac{\Delta E_{+1}^{e}}{E} \equiv-\frac{E_{+1}^{e}-E}{E}<U I P> \tag{2}
\end{equation*}
$$

Next assume that the actual exchange rate converges to the long run exchange rate (called "purchasing power parity" in Hall and Papell, p.338) at a rate $\Theta$ :

$$
\begin{equation*}
-\Delta E_{+1}^{e} / E=\Theta\left[\left(\frac{E}{E_{L R}}\right)-1\right] \quad<\text { Overshooting }> \tag{3}
\end{equation*}
$$

Combining UIP and "overshooting" yields:

$$
\begin{equation*}
R-R_{w}=\Theta\left[\left(\frac{E}{E_{L R}}\right)-1\right] \tag{4}
\end{equation*}
$$

Now assume that the rest of the world interest rate is a function of the US interest rate. This assumption is equivalent to the assertion that the US is a large country in the context of the world financial capital markets:

$$
\begin{equation*}
R_{w}=\overline{R_{w}}+\gamma R \tag{5}
\end{equation*}
$$

(This is assumed at the bottom of page 337 in Hall and Papell). Subtracting this expression from R yields another expression for the interest differential:

$$
\begin{equation*}
R-R_{w}=(1-\gamma) R-\overline{R_{w}} \tag{6}
\end{equation*}
$$

Substituting (4) into (6):

$$
\begin{equation*}
\Theta\left(\frac{E}{E_{L R}}\right)-\Theta=(1-\gamma) R-\overline{R_{w}} \tag{7}
\end{equation*}
$$

Solving for the nominal exchange rate, E :

$$
\begin{equation*}
E=\left(\frac{E_{L R}}{\Theta}\right)\left[(1-\gamma) R-\overline{R_{w}}\right]+E_{L R} \tag{8}
\end{equation*}
$$

The trade balance responds to the real, not nominal, exchange rate. To obtain an expression for the real exchange rate, multiply both sides of (8) by $\mathrm{P} / \mathrm{P}_{\mathrm{w}}$ :

$$
\begin{equation*}
\frac{E P}{P_{\mathrm{w}}}=\left(\frac{E_{L R} P}{P_{\mathrm{w}}}\right) \times\left(\frac{(1-\gamma)}{\Theta} R-\frac{\overline{R_{w}}}{\Theta}\right)+\frac{E_{L R} P}{P_{\mathrm{w}}} \tag{9}
\end{equation*}
$$

Assume Purchasing Power Parity in the long run:

$$
\begin{equation*}
E_{L R}=P_{W} / P \quad<\text { LR PPP }> \tag{10}
\end{equation*}
$$

And setting $(1-\tau) / \Theta \equiv v$ and $-(1 / \Theta) \mathrm{R}_{\mathrm{w}}+1 \equiv q_{0}$ yields equation (12-3):

$$
\begin{equation*}
\frac{E P}{P_{w}}=q_{0}+v R \tag{12.3}
\end{equation*}
$$

Substituting (12.3) into (12-5) yields:

$$
\begin{gather*}
Y=\bar{\alpha}\left[\hat{A}_{0}-d R-n(q+v R)\right] \\
Y=\bar{\alpha}\left[\hat{A}_{0}-(d+n v) R\right] \quad \text { where } \hat{A}_{0} \equiv a_{0}+e_{0}+G O_{0}-b T A_{0}+g_{0}-n q_{0} \tag{11}
\end{gather*}
$$

or

$$
\begin{equation*}
R=\frac{\widehat{A}_{0}}{d+n v}-\left(\frac{1-b(1-t)+m}{d+n v}\right) Y \tag{12.7’}
\end{equation*}
$$

### 1.2. Graphical Depiction



Y

The slope of the IS curve is different from the closed economy case, but ambiguously so. That is, the slopes are:

$$
\frac{1-b(1-t)}{d}>\frac{1-b(1-t)+m}{d+n v}
$$

The marginal propensity to import ( $m$ ) increases the slope, because of the leakage out of the domestic economy of each dollar's worth of spending of $m$ amount. However, the $n v$ term makes the IS flatter.

It is instructive to examine why the $n$ and $v$ coefficients enter into the IS slope. First, why is the IS curve downward sloping? In the open economy case, one can trace out a dual chain of events:

$$
\begin{aligned}
& \quad=\mathrm{d}=>\mathrm{I} \uparrow=\alpha=>\mathrm{AD}, \mathrm{Y} \uparrow \\
& \mathrm{R} \downarrow \\
& =\mathrm{v}=>\mathrm{E} \downarrow===>\left(\mathrm{EP} / \mathrm{P}_{\mathrm{w}}\right) \downarrow=\mathrm{n}=>\mathrm{X} \uparrow=\alpha=>\mathrm{AD}, \mathrm{Y} \uparrow
\end{aligned}
$$

The greater $v$ is, the greater the drop in the real value of the dollar for a given drop in the interest rate; the greater $n$ is, the greater the increase in net exports for that drop in the real exchange rate, and hence the larger the increase in output. Notice that the composite parameter " $n v$ " occupies a role analogous to the " $d$ " term. Both summarize the sensitivity of a component of aggregate demand to interest rates.

## 2. Policy in an Open Economy Model

### 2.1 Policy Multipliers

Derive the multipliers in the usual manner -- by first solving for equilibrium income.

$$
Y=\bar{\alpha}\left[\hat{A}_{0}-(d+n v) R\right] \quad<R e-\text { revised IS }>
$$

Substitute in the LM curve, and solve, to obtain:

$$
\begin{gathered}
Y=\bar{\alpha}\left[\hat{A}_{0}-(d+n v)\left(\frac{k}{h} Y-\frac{1}{h} \frac{M_{0}}{P}+\frac{\mu_{0}}{h}\right)\right] \\
Y\left(1-b(1-t)+m+\frac{(d+n v) k}{h}\right)=\hat{A}_{0}+\frac{(d+n v)}{h}\left(\frac{M_{0}}{P}\right)-\frac{(d+n v) \mu_{0}}{h} \\
Y_{0}=\widetilde{\alpha}\left[\hat{A}_{0}+\frac{(d+n v)}{h}\left(\frac{M_{0}}{P}\right)-\frac{(d+n v) \mu_{0}}{h}\right] \\
\text { where } \widetilde{\alpha} \equiv \frac{1}{1-b(1-t)+m+(d+n v) k / h}
\end{gathered}
$$

Solve for the partials with respect to $G O$ and $(M / P)$ :

$$
\begin{gathered}
\frac{\Delta Y}{\Delta G O}=\widetilde{\alpha} \\
\frac{\Delta Y}{\Delta(M / P)}=\widetilde{\alpha}(d+n v) / h
\end{gathered}
$$

To assess how the multipliers change with variations in parameter magnitudes, take the limit of them as $n$ and/or $v$ approach infinity:

$$
\begin{gathered}
\operatorname{Lim}_{n, v \rightarrow \infty}\left(\frac{\Delta Y}{\Delta G O}\right)=0 \\
\operatorname{Lim}_{n, v \rightarrow \infty}\left(\frac{\Delta Y}{\Delta(M / P)}\right)=1 / k
\end{gathered}
$$

### 2.2 Policy Interpretation

When fiscal policy is implemented, $\Delta G$, then the IS shifts out. The equilibrium income and interest rate rises as autonomous spending rises. Since imports rise with income, net exports decline. The interest rate increase exacerbates the trade balance deterioration, since it attracts an incipient capital inflow that appreciates the dollar. The dollar appreciation in turn makes U.S. goods relatively less competitive on both world and domestic markets, thereby worsening the trade balance.

A monetary expansion has more ambiguous consequences. It shifts the LM out, increasing income but reducing the equilibrium interest rate. The latter effect depreciates the dollar. (In fact it depreciates it so much, that it must be expected to appreciate over the future enough to satisfy the uncovered interest parity condition.) The increase in income has a negative effect on net exports, while the dollar depreciation has a positive effect, so that the overall effect is ambiguous.

To verify this assertion, it is easiest to think of the net export equation in functional

$$
X=X\left[Y(G O, M / P) ; E P / P_{w}(R(G O, M / P))\right]
$$

terms:
Where X[. ; .] denotes the net export function
$\mathrm{Y}(. ;$.) denotes the equilibrium income function
$\mathrm{EP} / \mathrm{P}_{\mathrm{w}}($.$) denotes the equilibrium real exchange rate function$
$\mathrm{R}(. ;$.) denotes the equilibrium interest rate function
Taking the total differential of $X$, setting $\Delta G=0$, yields:

$$
\Delta X=(\partial X / \partial Y)\left(\frac{\partial Y}{\partial(M / P)}\right) \Delta(M / P)+\left(\frac{\partial X}{\partial\left(E P / P_{w}\right)}\right)\left(\frac{\partial\left(E P / P_{w}\right)}{\partial R}\right)\left(\frac{\partial R}{\partial(M / P)}\right) \Delta(M / P)
$$

One can then substitute in for each of these partial derivatives:

$$
\begin{gathered}
\Delta X=(-m)\left(\frac{\widetilde{\alpha}(d+n v)}{h}\right) \Delta(M / P)+(-n)(v)(-\Gamma) \Delta(M / P) \\
\text { Where }-\Gamma \equiv-\frac{(1-b(1-t)+m) / k}{d+n v+(1-b(1-t)+m) k / h} \\
\frac{\Delta X}{\Delta(M / P)}>{ }_{0} \text { if } \begin{array}{l}
m[\widetilde{\alpha}(d+n v) / h]>n v \Gamma \\
m[\widetilde{\alpha}(d+n v) / h]<n v \Gamma
\end{array} \\
\begin{array}{l}
\text { effect due to } \\
\text { increased imports } \quad \begin{array}{l}
\text { effect due to } \\
\text { depreciated dollar }
\end{array}
\end{array}
\end{gathered}
$$

## 3. Appendix on Overshooting Model of the Exchange Rate

In the below time line, every variable $(\mathrm{M}, \mathrm{P},(1 / \mathrm{E}))$ is normalized to one. Consider what happens when the money supply is increased at time T (from 1.0 to 1.5 ). Then, in the short run, with prices fixed, real money balances increases 1.5 . That in turn means that domestic interest rates fall. If the exchange rate immediately depreciated to its long run value of 1.5 , then future expected depreciation would be 0 , and equation (2) ("uncovered interest parity") could not hold. Hence, it must be that the exchange rate depreciates below the long run value of the exchange rate, such that in the long run it is expected to appreciate over the long run so as to satisfy UIP.

le302_open_s10.doc 7.4.2010

