

### The Keynesian Model of Income Determination

This set of notes outlines the Keynesian model of national income determination in closed and open economy. It then shows how to solve for multipliers.

#### 1. An Expanded Model and Equilibrium

<u>Eq.No.</u>	<u>Equation</u>	<u>Description</u>
(1)	$Y = AD$	Output equals aggregate demand, an equilibrium condition
(2)	$AD = C + I + G + X$	Definition of aggregate demand
(3)	$C = a_o + bY_d$	Consumption function, $b$ is the mpc
(4)	$Y_d \equiv Y - T$	Definition of disposable income
(5)	$T = TA_o + tY$	Tax function; $TA_o$ is lump sum taxes, $t$ is marginal tax rate.
(6)	$I = IN_o$	Investment function, exogenous
(7)	$G = GO_o$	Government spending on goods and services, exogenous
(8)	$X = g_o$	Net Exports, exogenous

Note: the marginal propensity to consume out of disposable income is  $b = \frac{\partial C}{\partial(Y - T)}$

Substitute (3)-(8) into (2), and substitute (2) into (1):

$$(9) \quad Y = AD = (a_o + bY_d) + (IN_o) + (GO_o) + (g_o)$$

$$Y = AD = (a_o + b(Y - (TA_o + tY))) + (IN_o) + (GO_o) + (g_o)$$

Collect up terms:

$$(10) \quad Y = b(1 - t)Y + A_o \text{ where } A_o = a_o - b(TA_o) + IN_o + GO_o + g_o$$

Shift "Y" terms to the left hand side:

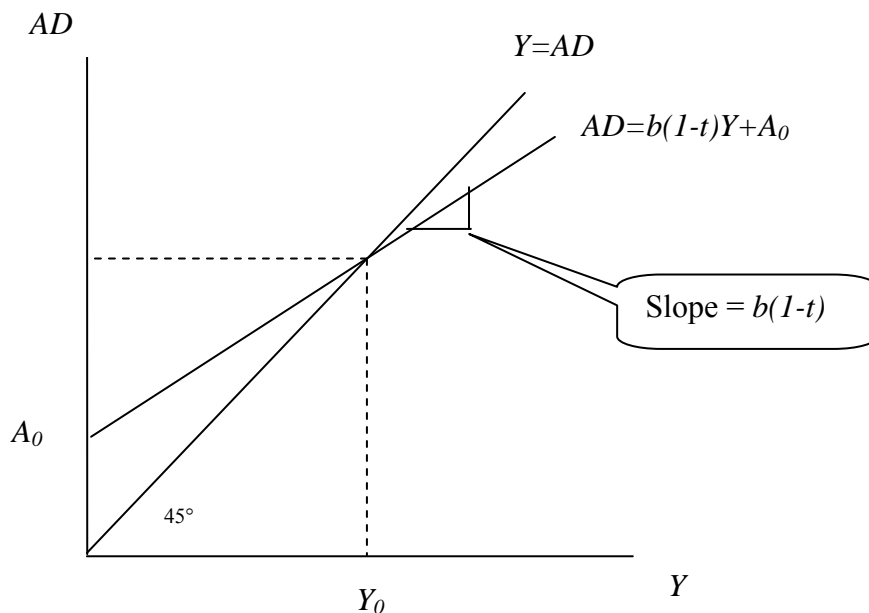
$$(11) \quad Y - b(1 - t)Y = A_o \rightarrow Y[1 - b(1 - t)] = A_o$$

Divide both sides by the term in the square bracket to obtain equilibrium income,  $Y_o$ :

$$(12) \quad \boxed{Y_0 = \bar{\alpha}A_0} \quad \text{let } \bar{\alpha} = \frac{1}{[1-b(1-t)]}$$

Notice that if there are no lump taxes, i.e.,  $TA_0 = 0$ , then equation (12) is the same as equation 7.5 in the textbook.

Interpretation of (12): equilibrium income is a multiple of the amounts of “autonomous” spending. The higher the level of autonomous spending, the higher the equilibrium level of income. Notice also that lump sum taxes enter in negatively, so the higher lump sum taxes, the lower equilibrium income is.



**Figure 1:** Equilibrium in the Keynesian Cross

## 2. Effects of Changes in Autonomous Spending and Multipliers

To think about how changes in autonomous spending – government spending ( $GO_0$ ), investment spending ( $IN_0$ ), net export spending ( $g_0$ ) – affect equilibrium income, consider a change of income ( $\Delta Y$ ) as being attributable to changes in each of those autonomous spending components. Take equation (12):

$$(13) \quad \Delta Y = \bar{\alpha}\Delta A$$

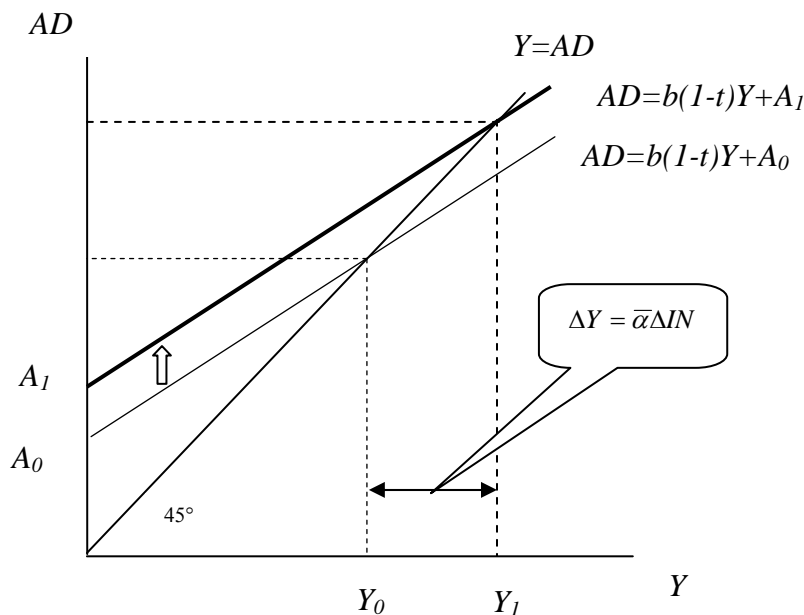
(holding constant the tax *rate*). Remember, however, that  $A_0 \equiv a_0 - b(TA_0) + IN_0 + GO_0 + g_0$ . So if, for instance, the only autonomous spending component that changes is government spending (so  $\Delta A = \Delta GO$ ), then:

$$(14) \quad \Delta Y = \bar{\alpha} \Delta GO$$

Notice that a similar expression occurs if one holds government spending, lump sum taxes, and net exports constant, then:

$$(15) \quad \Delta Y = \bar{\alpha} \Delta IN$$

This can be interpreted in the following figure.



**Figure 2:** Change in income due to a change in autonomous spending

Note that  $Y_1 = Y_0 + \Delta Y$ . You should also understand that  $A_1 = A_0 + \Delta A$ .

Returning to (14), consider what the change in income for a change in government spending is. That can be obtained by dividing both sides by  $\Delta GO$ :

$$(16) \quad \frac{\Delta Y}{\Delta GO} = \bar{\alpha} \equiv \frac{1}{1 - b(1 - t)}$$

This is the same expression as equation (7.6), verifying that this object is the government spending multiplier. See if you can solve for the lump sum tax multiplier.

### 3. The Open Economy Multiplier

One important point to realize is that there are different multipliers corresponding to different variables changing (consider the government spending vs. tax lump multipliers). More importantly, the multiplier for the same variable will be different if the *model* changes. In the previous section, net exports depended upon an exogenously determined amount. Suppose we assume net exports, the difference between exports and imports, depend upon income. In particular, suppose when income rises, consumption rises, and some of that consumption falls on imported goods. Hence, imports (which enter in negatively in the calculation of net exports) rise with income. This assumption can be incorporated into the export equation thus:

$$(7') \quad X = g_0 - mY$$

Which corresponds to equation (7.7) in the textbook. Note that  $m = \frac{\partial \text{imports}}{\partial Y}$ , and hence

$-m = \frac{\partial X}{\partial Y}$ . If one goes through the same steps as in Section 1, one obtains:

$$(17) \quad Y = AD = (a_0 + b(Y - T)) + (IN_0) + (GO_0) + (g_0 - mY)$$

$$(18) \quad Y = (a_0 + b(Y - (TA_0 + tY))) - mY + (IN_0) + (GO_0) + (g_0)$$

Solving for equilibrium, one obtains:

$$(19) \quad Y_0 = \left( \frac{1}{1 - b(1 - t) + m} \right) A_0 \quad \text{where} \quad A_0 \equiv a_0 - b(TA_0) + IN_0 + GO_0 + g_0$$

In this case the, government spending multiplier is:

$$(20) \quad \frac{\Delta Y}{\Delta GO} = \frac{1}{1 - b(1 - t) + m}$$

In this open economy model, the government spending multiplier is less than that in the closed economy.